Part B: Short Answers (60 marks)

1. (20 marks) Assume that a country's production function is $Y = AK^{.3}L^{.7}$. The ratio of capital to output is 3, the growth rate of output is 3 percent, and the depreciation rate is 4 percent. Capital is paid its marginal product.
   a. What is the marginal product of capital in this situation? (*Hint:* The marginal product of capital may be computed using calculus by differentiating the production function and using the capital-output ratio or by using the fact that capital's share equals *MPK* multiplied by *K* divided by *Y*.) (5 marks)
   b. If the economy is in a steady state, what must be the saving rate? (Hint: The saving rate multiplied by *Y* must provide for gross growth of $(\delta + n + g)K$, where δ is the depreciation rate.) (5 marks)
   c. If the economy decides to achieve the Golden Rule level of capital and actually reaches it, what will be the marginal product of capital? (5 marks)
   d. What must the saving rate be to achieve the Golden Rule level of capital? (5 marks)

2. (20 marks) Assume the following model of the economy, with the price level fixed at 1.0:

$C = 0.8(Y - T)$	$T = 1{,}000$
$I = 800 - 20r$	$G = 1{,}000$
$Y = C + I + G$	$M^s/P = M^d/P = 0.4Y - 40r$
$M^s = 1{,}200$	

   a. Write a numerical formula for the *IS* curve, showing *Y* as a function of *r* alone. (*Hint*: Substitute out *C*, *I*, *G*, and *T*.) (4 marks)
   b. Write a numerical formula for the *LM* curve, showing *Y* as a function of *r* alone. (*Hint*: Substitute out *M/P*.) (4 marks)
   c. What are the short-run equilibrium values of *Y*, *r*, $Y - T$, *C*, *I*, private saving, public saving, and national saving? Check by ensuring that $C + I + G = Y$ and national saving equals *I*. (4 marks)
   d. Assume that *G* increases by 200. By how much will *Y* increase in short-run equilibrium? What is the government-purchases multiplier (the change in *Y* divided by the change in *G*)? (4 marks)
   e. Assume that *G* is back at its original level of 1,000, but M^s (the money supply) increases by 200. By how much will *Y* increase in short-run equilibrium? What is the multiplier for money supply (the change in *Y* divided by the change in M^s)? (4 marks)

4. (20 marks)This question asks you to give a step-by-step effect of a policy change on the economy. Suppose the economy is in a long-run equilibrium. Now, suppose that the government increases expenditure by ΔG. The Central Bank policy is to hold money-supply constant.

   a. What is the effect of the increase in government spending in the Keyensian Cross? (5 marks)
   b. What is the effect of the increase in government spending in the IS-LM model? Is it bigger or smaller than your answer in part b? (5 marks)
   c. What would your answer for part (b) be if the Central Bank policy was to hold interest rates constant? (5 marks)
   d. What would your answer for part (c) be if the Central Bank policy was to hold output constant? (5 marks)

Ans. to Ques 1

(a) Marginal Product of Capital

Given: $Y = AK^{0.3}L^{0.7}$; capital to output ratio $\frac{K}{Y} = 3$

$$MPK = \frac{dY}{dK} = 0.3 \times AK^{0.3-1}L^{0.7}$$

$$= 0.3 \times \frac{AK^{0.3}L^{0.7}}{K}$$ ← Note: this is the same as Y

$$= 0.3 \times \frac{Y}{K}$$

$$= 0.3 \times \frac{1}{3} = 0.1$$ Ans

(b) Steady-State:

$$\Delta k = s \cdot f(k) - (\delta + n + g) \cdot k = 0$$ [in per capita terms]

$$\Rightarrow s \cdot \frac{Y}{LE} - (\delta + n + g)\frac{K}{LE} = 0$$

$$\Rightarrow s \cdot Y = (\delta + n + g)K$$ [eliminating LE from both sides]

$$\Rightarrow s = (\delta + n + g) \frac{K}{Y}$$

Recall: $y = \frac{Y}{L \times E} \Rightarrow Y = y \times L \times E$

at SS, growth rate of $y = f(k)' = 0$, since growth rate of $k = 0$

But, growth rate of Y = growth rate of y + growth rate of L + growth rate of E

$= 0 + n + g = (n+g)$

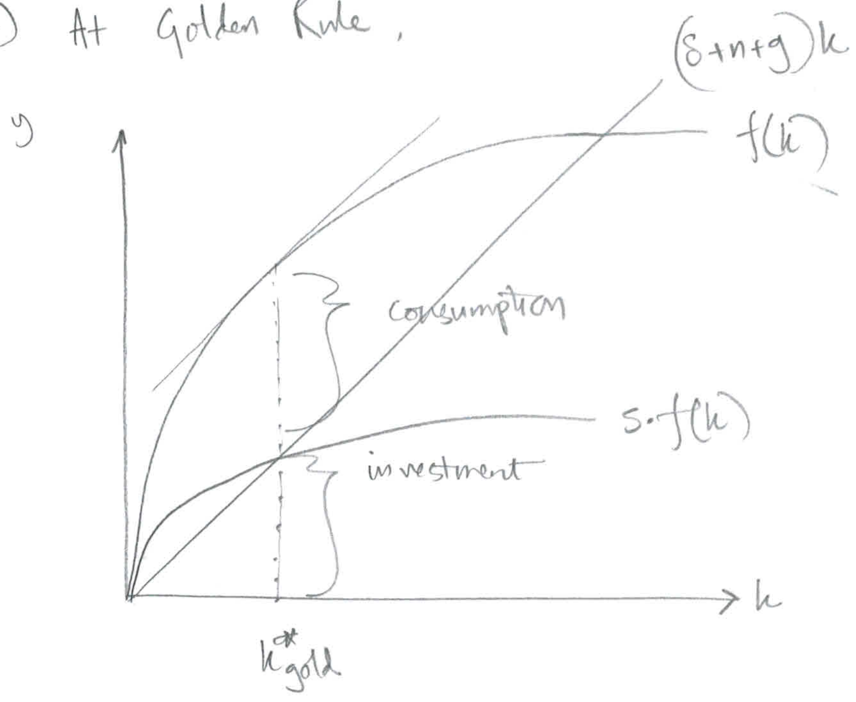
Question says, growth rate of $Y = n+g = 3$ percent $= 0.03$

depreciation rate $\delta = 4$ percent $= 0.04$

So, $s = (0.04 + 0.03) \times \frac{K}{Y}$

$= 0.07 \times 3 = 0.21$ Ans

(c) At Golden Rule,



to maximize consumption, we need to find the k^* that gives the largest consumption $[f(k) - (\delta+n+g)k]$

this is achieved when

$f'(k) - (\delta+n+g) = 0$

or $f'(k) = (\delta+n+g)$

Note: $f(k) = \frac{Y}{LE}$ and $k = \frac{K}{LE}$

So, $f'(k) = \frac{d f(k)}{dk} = \frac{d \frac{Y}{LE}}{d \frac{K}{LE}} = \frac{dY}{dK} = MPK.$

So, at Golden Rule, $MPK = \delta + n + g = 0.07$ Ans

(d) Now, $MPK = 0.3 \times \left(\frac{Y}{K}\right)_{gold}$ [from question a]

So at Golden Rule. $MPK = 0.3 \times \left(\frac{Y}{K}\right)_{gold} = \delta + n + g = 0.07$

$$\Rightarrow \left(\frac{Y}{K}\right)_{gold} = \frac{0.07}{0.3}$$

from question (b) $s = 0.07 \times \left(\frac{K}{Y}\right)_{gold} = 0.07 \times \frac{0.3}{0.07}$

$= 0.3$ Ans

Ans to Ques 2

(a) $Y = C + I + G$ sub in C, I, G

$$= 0.8(Y - 1000) + 800 - 20r + 1000$$

$$= 0.8Y - 800 + 800 - 20r + 1000$$

$$\Rightarrow 0.2Y = 1000 - 20r$$

$$\Rightarrow Y = 5000 - 100r \quad \leftarrow \text{IS curve}$$

(b) $\frac{M^d}{P} = 0.4Y - 40r = M^s = 1200$ ← from money demand = money supply

$$\Rightarrow 0.4Y = 1200 + 40r$$

$$\Rightarrow Y = 3000 + 100r \quad \leftarrow \text{LM curve}$$

(c) In the short run equilibrium IS and LM intersect:

$$Y = 3000 + 100r = 5000 - 100r$$

$$\Rightarrow 200r = 2000 \Rightarrow r = 10$$

Plugging in: $Y = 3000 + 100 \times 10 = 4000$

$$C = 0.8 \times (4000 - 1000) = 2400$$

$$I = 800 - 20 \times 10 = 600$$

private saving $= (1 - 0.8) \times (Y - T) = 0.2 \times (4000 - 1000) = 600$

public saving $= T - G = 1000 - 1000 = 0$

national saving $= 600 + 0 = 600$ = investment [checked]

(d) When G increases by 200, the IS curve shifts to the right by $\left(\frac{1}{1-mpc}\right) \times \Delta G$

So, $IS_1 \Rightarrow \quad Y = 5000 - 100r + \left(\frac{1}{1-0.8}\right) \times \Delta G$

$= 5000 - 100r + 5 \times 200$

$= 6000 - 100r$

LM stays the same: $Y = 3000 + 100r$

New equilibrium $r \Rightarrow \quad 6000 - 100r = 3000 + 100r$

$\Rightarrow r = \frac{3000}{200} = 15$

So, new equilibrium $Y = 6000 - 100 \times 15 = 4500$

So, $\Delta Y = 4500 - 4000 = 500$ Ans

(↑ new Y; ↑ previous Y)

So, government purchases multiplier $\frac{\Delta Y}{\Delta G} = \frac{500}{200} = 2.5$ Ans

Note that you can get the new LM by setting $G = 1000 + \Delta G$
$= 1200$

and re-doing question (a). It should give you the same answer.

(e) $\Delta M^s = 200$ [Note: P is fixed at 1.0].

So, $M_1^s = 1,200 + 200 = 1400$ [New money supply]

So, new LM_1 : $\frac{M^d}{P} = 0.4Y - 40r = \frac{M_1^s}{P} = 1400$

$\Rightarrow 0.4Y = 1400 + 40r$

$\Rightarrow Y = 3500 + 100r$

New equilibrium Y : $IS_{old} = LM_{new}$

$\Rightarrow 5000 - 100r = 3500 + 100r$

$\Rightarrow 200r = 1500$

$\Rightarrow r = 7.5$

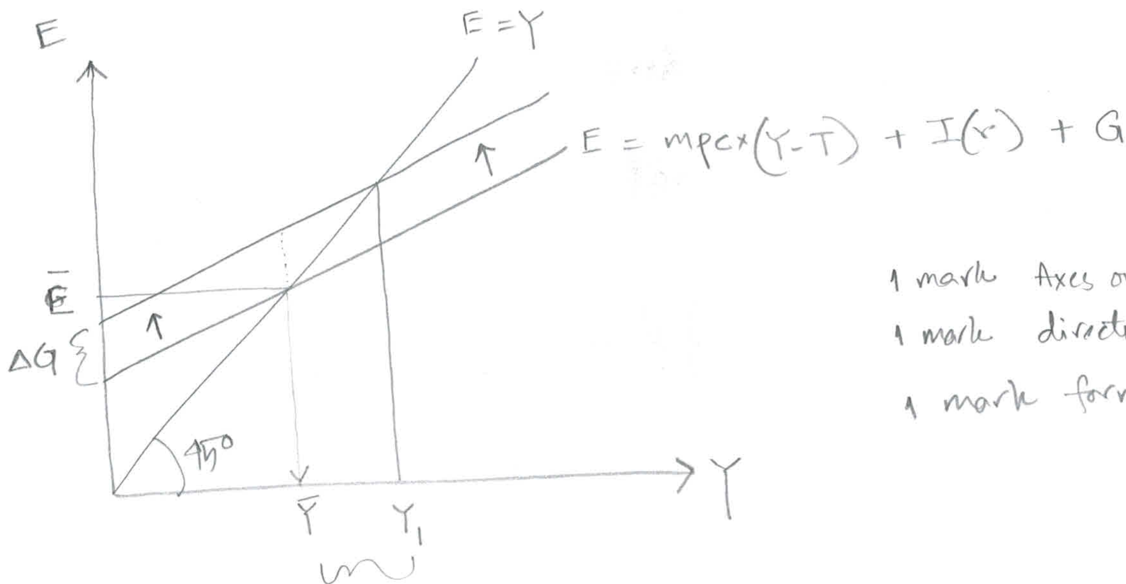
$\Rightarrow Y = 5000 - 750 = 4250$

So $\Delta Y = 4250 - 4000 = 250$ Ans

Money-supply multiplier $\frac{\Delta Y}{\Delta M^s} = \frac{250}{200} = 1.25$ Ans

Ans. to Ques No: 3

(a)



$$\Delta Y = \left(\frac{1}{1-mpc}\right) \times \Delta G$$

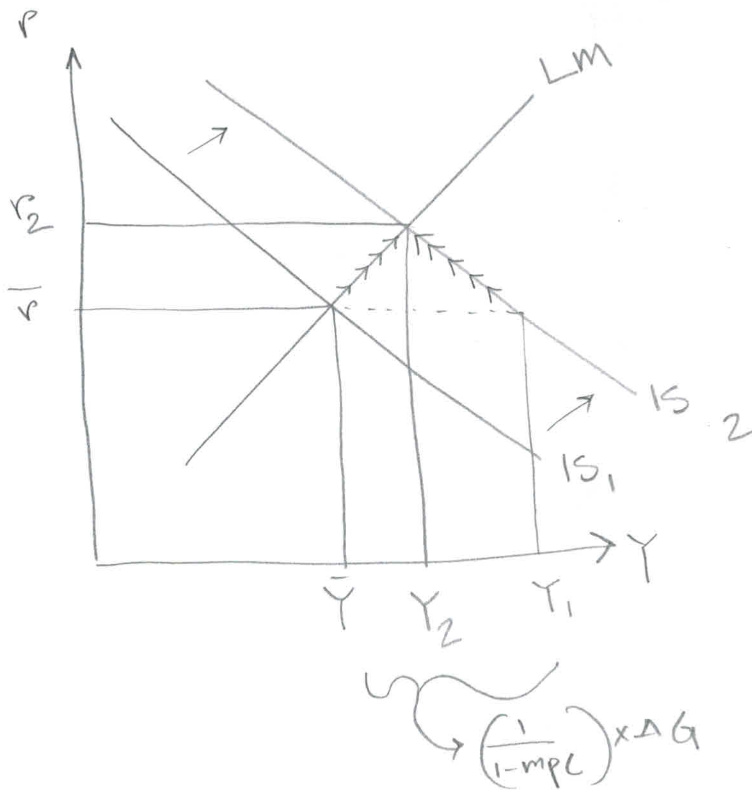
1 mark Axes on the graph
1 mark direction of the shift
1 mark formulae.

Explanation (2 marks):

The planned expenditure curve in the Keynesian cross equates planned spending E to planned consumption C, planned investment I and exogenous government G spending, G. As government spending goes up by ΔG, the E curve shifts up by the same amount. At the old equilibrium, $\bar{Y}$, there is now a fall in actual inventory (decumulation). To bring actual inventory back to its planned level, firms produce more goods. As a result, workers' income increases, and so does their consumption. Through the multiplier effect, equilibrium output goes up by

$\Delta Y = \left(\frac{1}{1-mpc}\right) \times \Delta G$ where $\left(\frac{1}{1-mpc}\right)$ is the government spending multiplier in the Keynesian Cross model.

(b)



1 mark axes on the graph
1 mark direction of shift
1 mark formulae / arrows / labelling

Explanation (2 marks):

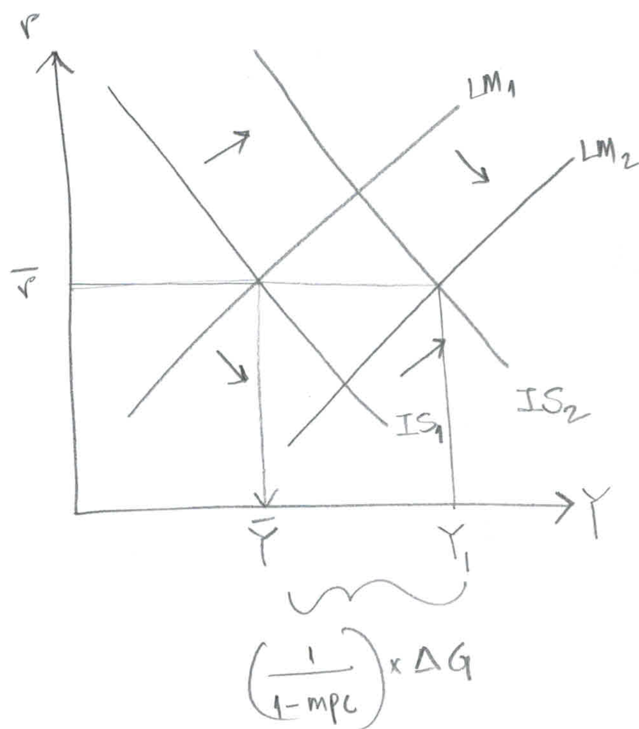
A rise in government spending by ΔG shifts the IS curve out from IS_1 to IS_2 by the same amount $\left(\frac{1}{1-mpc}\right) \times \Delta G$ as the shift in Y in the Keynesian Cross Model.

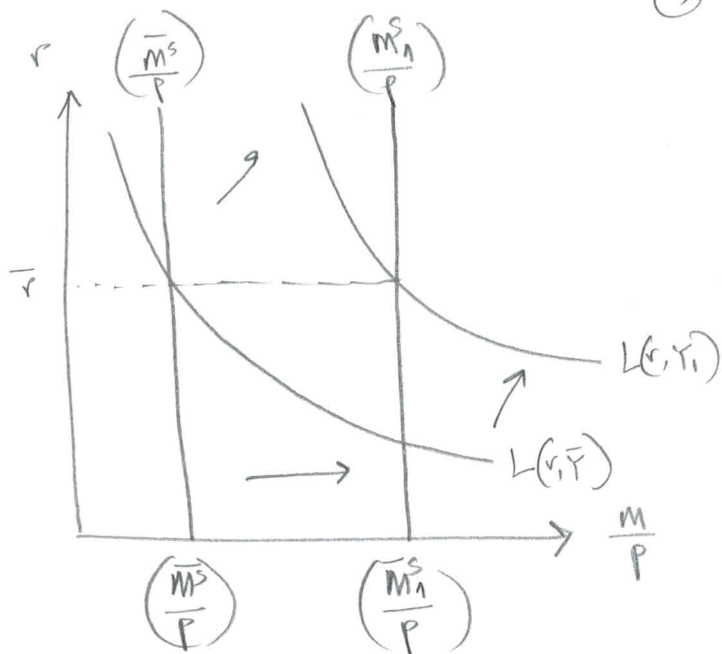
However, now, there is a feedback from the money market.

As output goes up, people want to hold more money to facilitate the higher volume of transactions. This means that banks have to raise interest rates to keep money demand equal to money supply. So, as Y goes up, so does r. This is a movement along and up the LM curve.

The rise in interest rates reduce investment (crowding out), and output goes up only to Y_2 instead of Y_1. As a result, ΔY in the IS-LM model is SMALLER than ΔY in the Keynesian Cross.

(c)





No penalty if this graph is omitted.

1 mark for axes on this graph
1 mark direction of shift
1 mark formulae / arrows / labelling

Explanation (2 marks)

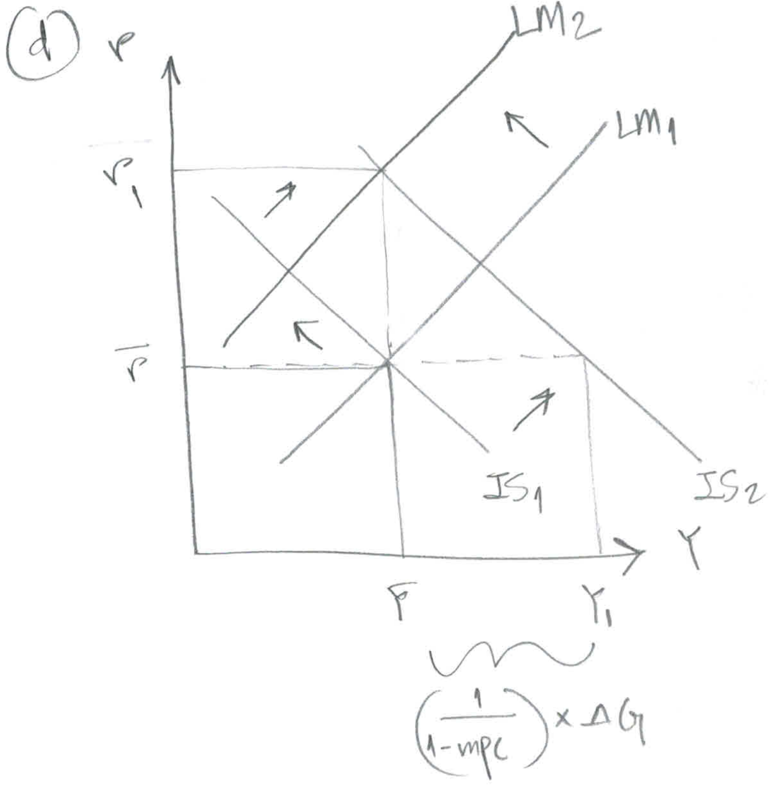
If Central Bank policy is to hold interest rate constant, then the LM curve shifts appropriately.

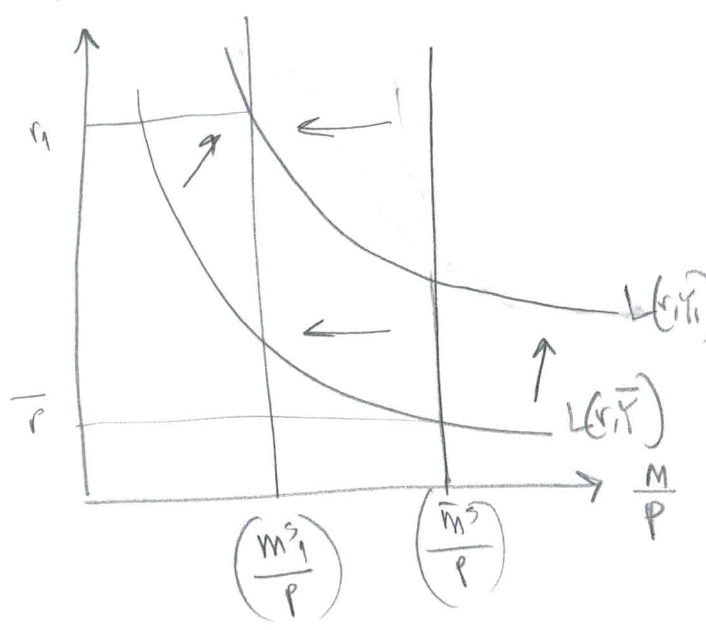
ΔG shifts the IS curve out by $\left(\frac{1}{1-mpc}\right) \times \Delta G$ same as before.

However, now, as money demand increases [from $L(r, \bar{Y})$ to $L(r, Y_1)$] for higher transaction purposes, the Central Bank also increases the money supply to keep interest rates constant at $\bar{r}$.

As a result, LM shifts to the right by the same amount as the shift in IS. Investment is <u>NOT</u> crowded out any more, since there is no change in r. Y goes up by the full amount $\left(\frac{1}{1-mpc}\right) \times \Delta G$ as envisioned in the Keynesian Cross.

(d)





Explanation (2 marks)

When the Central Bank policy is to hold output constant, then the LM curve shifts appropriately (this time, to the left).

ΔG shifts the IS curve out and to the right by $\left(\frac{1}{1-mpc}\right) \times \Delta G$ same as before.

However, the Central Bank now REDUCES money supply to counteract the rise in money demand for transaction purposes. The resulting rise in interest is much higher than in part (b), and the decline in investment COMPLETELY crowds out the rise in Y envisioned in the Keynesian Cross model. The final result is no change in Y and a very high interest rate.