Solution to Test 2(A)

MAT1320B, Fall 2015
Total = 20 marks

This test covers sections 3.4, 3.5, 3.6, 3.9, 3.10, 4.9, 5.1– 5.5, 7.1, 7.2.

Part I. Multiple-choice Questions: (14 marks)

Answer. ECCADBA

1. Let $y = \arctan\sqrt{x}$. Then $y'(4) =$

(A) 1; (B) $\frac{1}{2}$; (C) $\frac{1}{5}$; (D) $\frac{1}{15}$; (E) $\frac{1}{20}$.

Solution. (E) Let $u = \sqrt{x}$. Then $y = \arctan u$. $y'_u = \frac{1}{1+u^2} = \frac{1}{1+x}$, and $u'_x = \frac{1}{2\sqrt{x}}$. Then $y'_x = \left(\frac{1}{1+x}\right)\left(\frac{1}{2\sqrt{x}}\right)$, and $y'(4) = \frac{1}{20}$.

2. The slope of the tangent line of the graph of the equation $y^3 - x^2 + xy = 5$ at the point (3, 2) is

(A) $\frac{2}{3}$; (B) $\frac{4}{7}$; (C) $\frac{4}{15}$; (D) $\frac{5}{13}$; (E) $\frac{3}{5}$.

Solution. (C) $3y^2y' - 2x + y + xy' = 0$. $12y' - 6 + 2 + 3y' = 0$. $y' = 4 / 15$.

3. The derivative of the function $y = (x + 1)^x$ at $x = 1$ is

(A) 2; (B) ln 2; (C) 2 ln 2 + 1; (D) ln 2 + 2; (E) ln 2 + 1 / 2.

Solution. (C) $\ln y = x \ln (x + 1)$. $y' / y = \ln (x + 1) + \frac{x}{x+1}$. $y(1) = 2$.
$y'(1) = 2 (\ln 2 + 1 / 2) = 2 \ln 2 + 1$.

4. $\int_0^{\pi/4} \tan^2 x \sec^4 x dx$.

(A) 8 / 15; (B) 1 / 2; (C) 2 / 3; (D) 2 / 5; (E) 7 / 12.

Solution. (A) Let $u = \tan x$. Then $u' = \sec^2 x$.

$$\int_0^{\pi/4} \tan^2 x \sec^4 x dx = \int_0^1 \tan^2 x \sec^2 x du = \int_0^1 u^2(1+u^2)du = \left[\frac{1}{3}u^3 + \frac{1}{5}u^5\right]_{u=0}^1 = \frac{8}{15}.$$

5. $\int_1^4 \frac{x+1}{\sqrt{x}} dx =$

(A) $\frac{11}{8}$; (B) $\frac{15}{8}$; (C) $\frac{16}{3}$; (D) $\frac{20}{3}$; (E) $\frac{11}{5}$.

Solution. (D) $\int_1^4 \frac{x+1}{\sqrt{x}} dx = \int_1^4 (x^{1/2} + x^{-1/2})dx = \left[\frac{2}{3}x^{3/2} + 2x^{1/2}\right]_{x=1}^4 = \frac{16}{3} + 4 - \frac{2}{3} - 2 = \frac{20}{3}$.

6. The area between the graph of function $y = \frac{1}{\sqrt{x}+1}$ and the x-axis in interval [0, 4] is

(A) ln 3 − ln 2; (B) 4 – 2 ln 3; (C) 2 – 2 ln 3;
(D) 4 + 2 ln 3; (E) 2 – 4 ln 3 + 2 ln 2.

Solution. (B) Let $u = 1 + \sqrt{x}$. Then $\sqrt{x} = u - 1$ and $u' = \frac{1}{2\sqrt{x}}$.

$$\int_0^4 \frac{1}{\sqrt{x}+1} dx = \int_1^3 \frac{2\sqrt{x}}{1+\sqrt{x}} du = 2\int_1^3 \frac{u-1}{u} du = 2\left[u - \ln u\right]_{u=1}^3 = 4 - 2 \ln 3.$$

7. $\int_1^e x \ln x dx =$

(A) $\frac{1}{4}(e^2 + 1)$; (B) $\frac{1}{4}(e^2 - 1)$; (C) $\frac{1}{2}(e^2 + 1)$;
(D) $\frac{1}{2}(e^2 - 1)$; (E) $\frac{1}{2}e^2 + \frac{1}{4}$.

Solution. (A)

$$\int_1^e x \ln x dx = \frac{1}{2}\int_1^e \ln x d(x^2) = \frac{1}{2}\left(\left[x^2 \ln x\right]_{x=1}^e - \int_1^e x dx\right) = \frac{1}{2}\left(e^2 - \frac{1}{2}(e^2 - 1)\right) = \frac{1}{4}(e^2 + 1).$$

Part II. Detailed Answer Question

1. (4 marks) A monitor is located on top of a pole 12 meters high. A man is walking away from the pole. When the man is 16 meters away from the pole, the monitor shows that the distance between the man and the monitor is increasing at a speed 1.2 meters per second. Find the speed of the man.

Solution. Let the distance between the man and the pole be x, and let the distance between the man and the monitor be y. Both x and y are functions of time t, and $y'_t = 1.2$. These functions are related by the equation $x^2 + 12^2 = y^2$.

Taking the derivative of this equation with respect to t, $2xx' = 2yy'$. When $x = 16$ meters, $y = \sqrt{16^2 + 12^2} = \sqrt{256 + 144} = 20$ meters. $x'_t = 2yy' / x = 20 \times 1.2 / 16 = 1.5$ meter / second.

2. (2 marks) Find the derivative of the function $f(x) = \int_{x^2}^{1} \cos(t^2)\,dt$.

Solution. Let $u = h(x)$, where $h(x) = x^2$. Let $g(u) = \int_{u}^{1} \cos(t^2)\,dt$. Then $f(x) = (g \circ h)(x)$. By the fundamental theorem of calculus, the derivative of $g(u)$ to u is $-\cos(u^2)$. The derivative of u to x is $2x$. Hence, $f'(x) = (-\cos(u^2))(2x) = -2x \cos(u^2) = -2x \cos(x^4)$.