Université d'Ottawa · University of Ottawa

Faculté des sciences
Mathématiques et de statistique

Faculty of Science
Mathematics and Statistics

Discrete Mathematics for Computing MAT1348A

Midterm Examination — Version α

13 February 2014

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Instructions:

- This is an 80-minute *closed-book* exam; no notes are allowed. Calculators are *not* allowed.
- The exam consists of 12 questions on 10 pages. Page 10 is for additional work. *Please do not detach it.*
- Questions 1-4 are multiple-choice. You must enter the letter corresponding to each correct answer in the table preceding Question 1. No partial marks will be given for other work.
- Questions 5-9 are short-answer. You need not show your work if your answer is correct, but may receive partial marks for showing work.
- Questions 10-12 are long-answer. You must clearly show all relevant steps and justify your solution to receive full marks. Clearly indicate the final answer.
- Be sure to read carefully and follow the instructions for the individual problems.
- For rough work, you may use the back pages. Do not use scrap paper of your own.
- Use proper mathematical notation and terminology.
- If you require clarification, raise your hand.
- Good luck!

Last name:_____

First name:_____

Student number:_____

Signature:_____

Question	1 – 4	5 – 9	10	11	12	Total
Max	4×2	$4 \times 2 + 3$	4	5	5	33
Marks						

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Questions 1–4 are multiple choice. Enter the **letter** corresponding to each correct answer in the appropriate box below.

Question	1	2	3	4
Answer				

[2pts] 1. Which of the following statements are **false**?

(i) The compound proposition $(a \to b) \vee a$ is a tautology.
(ii) The compound propositions $a \to \neg b$ and $b \to \neg a$ are logically equivalent.
(iii) If a is false, b is false, and c is true, then $(a \wedge b) \vee c$ is true.
(iv) The compound propositions $\neg(a \to \neg b)$ and $a \wedge b$ are logically equivalent.
(v) If the set of premises of an argument is inconsistent, then the argument is valid.

A. only (iii) **B.** only (iv) **C.** only (i) **D.** (ii) and (v)

E. none **F.** only (ii)

[2pts] 2. Let $S = \{1, 2, \{1, \emptyset\}, \{\emptyset\}\}$. Which of the following statements are **false**?

(i) $\{1, \{\emptyset\}\} \subseteq S$
(ii) $\{1, 2\} \in S$
(iii) $\{1, \emptyset\} \subseteq S$
(iv) $\{1, \emptyset\} \in S$
(v) $\{\emptyset\} \in S$

A. only (iii) **B.** (i) and (iii) **C.** only (v) **D.** (ii) and (iii)

E. (ii) and (v) **F.** only (iv)

[2pts] 3. Which of the following arguments (rules of inference) are **invalid**?

(i) $\dfrac{a \vee b \quad \neg a \vee c}{\therefore b \wedge c}$ (ii) $\dfrac{a \vee b \quad \neg a \vee c}{\therefore b \vee c}$ (iii) $\dfrac{a \to b \quad \neg b}{\therefore a}$

(iv) $\dfrac{a \vee b \quad \neg b}{\therefore a}$ (v) $\dfrac{a \to b \quad \neg b}{\therefore \neg a}$ (vi) $\dfrac{a \to b \quad \neg a \to c}{\therefore \neg b \to c}$

A. (i) and (iii) **B.** (iv) and (v) **C.** (ii) and (iii) **D.** (i), (ii), and (vi)

E. only (i) **F.** (i) and (v)

4. Below, let A and B be finite sets, and $f : A \to B$ a function. Furthermore, let
[2pts] $g : \mathbb{R}^+ \to \mathbb{R}^+$ be a function defined by $g(x) = x^2 + 3$.
Which of the following statements are **true**?

(i) If $|A| < |B|$, then f can not be one-to-one.
(ii) If $|A| = |B|$, then f is a bijection.
(iii) If f is onto, then $|A| \geq |B|$.
(iv) g is one-to one.
(v) g is onto.

A. (iii) and (iv) **B.** (i) and (iii) **C.** (ii) and (iii) **D.** (ii) and (iv)

E. (iv) and (v) **F.** (ii) and (v)

In each of the following five questions, write your final answer in the answer box.
Show your work below the answer box to receive partial marks.

5. Let A and B be finite sets with $|A| = 3$. If the **cardinality of the power set of** $A \times B$
[2pts] is 4096, what is the cardinality of B? *Hint:* $2^{10} = 1024$.

$\|B\| =$

6. The truth table of a compound proposition P with atomic propositions x, y, and z is shown below. Give a **disjunctive normal form** of P. *You do not need to simplify your answer.* [2pts]

x	y	z	P
T	T	T	F
T	T	F	T
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	T

DNF of P:

7. On the Island of Knights and Knaves (where knights always tell the truth, and knaves always lie) we meet two inhabitants A and B. Person A says: "B is a knave if and only if I am a knight." What is person B? [2pts]

Answer: B is a

[2pts] 8. Define the following atomic propositions:

H: "The tiger hides."

F: "The hunt is finished soon."

E: "The hunter is eaten by the tiger."

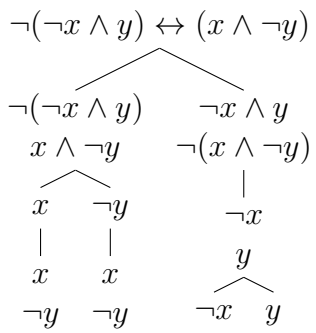
N: "The hunt is happening at night."

Translate the following sentence into a compound proposition (parentheses are there to help you):

(The tiger does not hide or the hunter is eaten by the tiger) only if (the hunt is happening at night and is not finished soon).

Compound proposition:

[3pts] 9. Let P be the compound proposition $\neg(\neg x \wedge y) \leftrightarrow (x \wedge \neg y)$. Below is a complete truth tree for P. Answer the questions about P in the answer box below.



P is a contradiction *(circle)*: YES NO

If NO, give a counterexample:

P is a tautology *(circle)*: YES NO

If NO, give a counterexample:

Give a DNF for P:

10. Let A, B, and C be subsets of the universal set $\mathcal{U}$. Use properties of set operations and
[4pts] set identities to prove the following equality. *You need not name the identities used.*

$$A - (B - \overline{C}) = (A - B) \cup (A - C)$$

11. Let n be an integer. Give an **indirect proof** of the following theorem.

[5pts] *If $n^2 + 4n - 1$ is odd, then n is even.*

12. Use any method you know to determine whether or not the argument below is valid. If
[5pts] the argument is not valid, give **all counterexamples**. *Fully justify your answer.*

$\neg(A \leftrightarrow B)$
$(A \to C) \to \neg B$
$\neg(B \vee C)$

$\therefore \neg A \wedge \neg B$

Additional work space. Do not detach this page.