

FAMILY NAME (<u>Capitals</u>)	_____
First Name	_____
Signature	_____
Student Number	_____

Mat 1348 Midterm Test

Professor: P. J. Scott

Date: Feb. 11, 2010

Time: 80 minutes

Instructions:

- (a) *No notes or papers allowed.*
- (b) Answer all questions on the test itself (you may write on the backs of pages). There are extra work pages. Write neatly and clearly show your answer.
- (c) **Note:** In multiple choice questions, circle your chosen answer
- (d) Be precise and explain what you are doing in long-answer questions.
- (e) The test consists of 10 questions (5 multiple choice, 5 long answer). You may write on the backs of pages. Multiple Choice are worth 2 points each. The others have the indicated marks. Total: 30 points.

Reserved for the Professor

Total (30 pts)

/ 2	/ 2	/ 2	/ 2	/ 2
1	2	3	4	5
/ 4	/ 3	/ 5	/ 4	/ 4
6	7	8	9	10

1. The truth table of a propositional formula φ (whose atoms are A , B and C) is given by:

A	B	C	φ
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	F
F	T	F	T
F	F	T	F
F	F	F	T

Which of the following formulas is a CNF equivalent to φ ?

- (a) $((A \wedge B \wedge \neg C) \vee (A \wedge B \wedge \neg C) \vee (\neg A \wedge B \wedge \neg C))$
- (b) $((\neg A \wedge B \wedge \neg C) \vee (\neg A \wedge \neg B \wedge \neg C) \vee (A \wedge B \wedge C))$
- (c) $((\neg A \wedge B \wedge C) \vee (A \wedge \neg B \wedge \neg C) \vee (A \wedge B \wedge \neg C))$
- (d) $((A \vee \neg B \vee \neg C) \wedge (\neg A \vee B \vee \neg C) \wedge (A \vee \neg B \vee \neg C))$
- (e) $((\neg A \vee B \vee C) \wedge (A \vee \neg B \vee \neg C) \wedge (A \vee B \vee \neg C))$
- (f) $((A \vee B \vee C) \wedge (\neg A \vee B \vee C) \wedge (\neg A \vee \neg B \vee C))$

2. Consider the set $A = \{1, \{1\}, \{1, 2\}, 2\}$. Among the following sentences, only one is false. **Circle the one false answer.**
- (a) The cardinality of the power set of A is 16.
 - (b) $\emptyset \subseteq A$
 - (c) $\{1, \{1\}\} \subseteq A$
 - (d) $\{\{1\}, \{2\}\} \subseteq A$
 - (e) $\{2, \{1, 2\}\} \subseteq A$
 - (f) $\{1, 2\} \in A$

3. Consider the following atoms:

P := The program outputs a value.

B := The program enters an infinite loop.

S := The output value is positive

V := The program is infected with a virus.

Which of the following formulas is a translation of the following:

In order that the output value is positive and that the program is not infected with a virus, it is sufficient that the program does not output a value unless it enters an infinite loop.

- (a) $((S \wedge \neg V) \rightarrow (\neg P \vee B))$
- (b) $((\neg P \wedge B) \rightarrow (S \vee V))$
- (c) $((S \vee V) \rightarrow (\neg P \wedge B))$
- (d) $(\neg(S \vee V) \rightarrow (\neg P \vee B))$
- (e) $((\neg P \vee B) \rightarrow (S \wedge \neg V))$
- (f) None of the above answers.

4. Which of the following statements are **true** ?

- (1). If X is true, Y is false and Z is false, then $(X \vee (Y \wedge Z))$ is true.
- (2). The two formulas $(p \rightarrow q)$ and $(\neg p \rightarrow \neg q)$ are logically equivalent.
- (3). The two formulas $(p \wedge (q \vee r))$ and $((p \wedge q) \vee (p \wedge r))$ are logically equivalent.
- (4). The formula $(A \rightarrow (\neg A \vee B))$ is a tautology.

Circle the statement which is true

- (a) (3) and (4)
- (b) (1) only
- (c) (4) only
- (d) (2) and (3)
- (e) (1) and (3)
- (f) (3) only

5. Let A, B be two finite sets, $f : A \rightarrow B$ and $g : B \rightarrow A$ two functions. Among the following statements, only one is **false**. **Circle the false statement.**

- (a) If $g \circ f$ is injective, then f is injective
- (b) If $|A| > |B|$, then f cannot be surjective (recall, $|X|$ is the cardinality of X).
- (c) If $g \circ f$ is surjective, then g is surjective
- (d) If f is bijective then $|A| = |B|$
- (e) If $|A| > |B|$, then f cannot be injective

6. Use whatever methods you prefer from class to find a DNF equivalent to the following formula:

$$(\neg(R \rightarrow P) \leftrightarrow (Q \wedge \neg P)).$$

7. Let A, B, C be three subsets of a universal set \mathcal{U} . Using Boolean algebra operations, prove

$$C \setminus (\bar{A} \cap B) = (C \cap A) \cup (C \setminus B)$$

8. Consider the following argument. Using any method we gave in class, verify if the argument is valid or not. If it is not valid, give all values of the atoms which falsify the argument.

$$\frac{\begin{array}{l} (Q \leftrightarrow R) \\ (\neg(R \rightarrow P) \leftrightarrow P) \\ (\neg R \vee P) \end{array}}{\neg(P \rightarrow Q)}$$

9. Let $f : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R} \times \mathbb{R}$ be defined by: $f(x, y) = (2x - y, x)$. Prove f is bijective.

10. You are on the island of knights and knaves. You encounter two individuals A and B who say the following:

A: *If there is not a hotel on the island then B is a knave.*

B: *There is a hotel on the island and at least one of us is a knave.*

Questions:

- (i) Is it possible to determine the nature of A and B (knights or knaves)?
- (ii) Is it possible to determine if there is a hotel on the island.

Work Sheet

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