Econ 302
Suggested answers for tutorial 1

1. (a) For the given cost function identify the variable and the fixed cost, as wells as the $AVC(y)$, $AFC(y)$, $ATC(y)$, $MC(y)$.

- $c_v(y) = 10y + 0.5y^2$
- $F = 800$
- $AVC(y) = \frac{c_v(y)}{y} \Rightarrow AVC(y) = \frac{10y + 0.5y^2}{y} = 10 + 0.5y$
- $AFC(y) = \frac{F}{y} \Rightarrow AFC(y) = \frac{800}{y}$
- $ATC(y) = \frac{c(y)}{y} \Rightarrow ATC(y) = \frac{800 + 10y + 0.5y^2}{y} = \frac{800}{y} + 10 + 0.5y$, or just $ATC(y) = AFC(y) + AVC(y) \Rightarrow ATC(y) = \frac{800}{y} + 10 + 0.5y$
- $MC(y) = \frac{\partial c(y)}{\partial y} \Rightarrow MC(y) = \frac{\partial}{\partial y}\left(800 + 10y + 0.5y^2\right) = 10 + y$

(b) The shut-down price is given by the minimum value of the average variable cost ($i.e.$, $AVC_{MINIMUM}$). To find the minimum there are two ways: first, straight minimization of the AVC function; second, by setting the AVC equal to MC.

AVC minimization:

$$\min_y \{AVC(y) = 10 + 0.5y\}$$

The FOC is

$$\frac{\partial AVC(y)}{\partial y} = 0 \Rightarrow \frac{\partial}{\partial y}(10 + 0.5y) = 0 \Rightarrow 0.5 = 0 \qquad \text{(impossible for any value of } y\text{)}$$

Since $\frac{\partial}{\partial y}(10 + 0.5y) = 0.5 > 0$ and we are looking for minimization, y has to take its lowest possible value, i.e., in order to minimize AVC it must be $y = 0$. When $y = 0$ we get

$$AVC_{MINIMUM} = AVC(0) = 10.$$

AVC equal to MC:

$$MC(y) = AVC(y) \Rightarrow 10 + y = 10 + 0.5y \Rightarrow 0.5y = 0 \Rightarrow y = 0$$

When $y = 0$ we get

$$AVC_{MINIMUM} = AVC(0) = 10.$$

Therefore, the shut-down price is \$10

(c) The break-even price is given by the minimum value of the average total cost (*i.e.*, $ATC_{MINIMUM}$). To find the minimum there are two ways: first, straight minimization of the ATC function; second, by setting the ATC equal to MC.

ATC minimization:

$$\min_y \left\{ ATC\left(y\right) = \frac{800}{y} + 10 + 0.5y \right\}$$

The FOC is

$$\frac{\partial ATC\left(y\right)}{\partial y} = 0 \Rightarrow \frac{\partial}{\partial y}\left(\frac{800}{y} + 10 + 0.5y\right) = 0 \Rightarrow -\frac{800}{y^2} + 0.5 = 0 \Rightarrow y = 40$$

When $y = 40$ we get

$$ATC_{MINIMUM} = ATC\left(40\right) = \frac{800}{40} + 10 + 0.5\left(40\right) = 50$$

ATC equal to MC:

$$MC\left(y\right) = ATC\left(y\right) \Rightarrow 10 + y = \frac{800}{y} + 10 + 0.5y \Rightarrow 0.5y = \frac{800}{y} \Rightarrow y = 40$$

When $y = 40$ we get

$$ATC_{MINIMUM} = ATC\left(40\right) = \frac{800}{40} + 10 + 0.5\left(40\right) = 50$$

Therefore, the break-even price is \$50

(d) The supply is the part of MC that lies above the $AVC minimum$. In this case we have $MC - AVC = 10 + y - (10 + 0.5y) = 0.5y > 0$ for all $y > 0$. Therefore, the entire MC is the supply of the firm. Specifically, we have

$$MC\left(y\right) = p \Rightarrow 10 + y = p \Rightarrow y = p - 10$$

Therefore, the supply of the firm is $y = p - 10$, provided that $p \geq \$10$.

(e) For $p = \$60$, the firm will supply (using the supply function we found in part b)

$$y = p - 10 \Rightarrow y = 60 - 10 \Rightarrow y = 50.$$

Therefore, the profit is

$$\Pi = py - c\left(y\right) \Rightarrow$$
$$\Pi = py - \left(800 + 10y + 0.5y^2\right) \Rightarrow$$
$$\Pi = 60\left(50\right) - \left(800 + 10\left(50\right) + 0.5\left(50\right)^2\right) \Rightarrow$$
$$\Pi = \$450.$$

Consequently, the producer surplus is

$$PS = \Pi + F \Rightarrow$$
$$PS = 450 + 800 = \$1,250$$

(f) For $p = \$40$, the firm will supply (using the supply function we found in part b)

$$y = p - 10 \Rightarrow y = 40 - 10 \Rightarrow y = 30.$$

Therefore, the profit is

$$\Pi = py - c(y) \Rightarrow$$
$$\Pi = py - \left(800 + 10y + 0.5y^2\right) \Rightarrow$$
$$\Pi = 40(30) - \left(800 + 10(30) + 0.5(30)^2\right) \Rightarrow$$
$$\Pi = -350.$$

Consequently, the producer surplus is

$$PS = \Pi + F \Rightarrow$$
$$PS = -350 + 800 = 450$$

2.(a) Consider a competitive firm with a cost function given by $c[y] = 50 + 30y - 18y^2 + 3y^3$.

- $c_v(y) = 30y - 18y^2 + 3y^3$
- $F = 50$
- $AVC(y) = \dfrac{c_v(y)}{y} \Rightarrow AVC(y) = \dfrac{30y - 18y^2 + 3y^3}{y} = 30 - 18y + 3y^2$
- $AFC(y) = \dfrac{F}{y} \Rightarrow AFC(y) = \dfrac{50}{y}$
- $ATC(y) = \dfrac{c(y)}{y} \Rightarrow ATC(y) = \dfrac{50 + 30y - 18y^2 + 3y^3}{y} = \dfrac{50}{y} + 30 - 18y + 3y^2$, or just $ATC(y) = AFC(y) + AVC(y) \Rightarrow ATC(y) = \dfrac{50}{y} + 30 - 18y + 3y^2$
- $MC(y) = \dfrac{\partial c(y)}{\partial y} \Rightarrow MC(y) = \dfrac{\partial}{\partial y}\left(50 + 30y - 18y^2 + 3y^3\right) = 9y^2 - 36y + 30$

(b) The shut-down price is given by the minimum value of the average variable cost (*i.e.*, $AVC_{MINIMUM}$). To find the minimum there are two ways: first, straight minimization of the AVC function; second, by setting the AVC equal to MC.

AVC minimization:

$$\min_y \left\{AVC(y) = 30 - 18y + 3y^2\right\}$$

The FOC is

$$\frac{\partial AVC(y)}{\partial y} = 0 \Rightarrow \frac{\partial}{\partial y}\left(30 - 18y + 3y^2\right) = 0 \Rightarrow y = 3$$

When $y = 3$ we get

$$AVC_{MINIMUM} = AVC[3] \Rightarrow AVC_{MINIMUM} = 30 - 18(3) + 3(3)^2 = \$3.$$

AVC equal to MC:

$$MC\left(y\right)=AVC\left(y\right)\Rightarrow 9y^{2}-36y+30=30-18y+3y^{2}\Rightarrow 6y^{2}=18y\Rightarrow ...\Rightarrow y=3$$

When $y=3$ we get

$$AVC_{MINIMUM}=AVC\left[3\right]=...=\$3.$$

Therefore, the shut-down price is \$3.

(c) The supply is the part of MC that lies above the $AVCminimum$. Specifically, we have

$$MC\left(y\right)=p\Rightarrow 9y^{2}-36y+30=p\Rightarrow ...\Rightarrow y=\frac{1}{3}\sqrt{p+6}+2$$

Therefore, the supply of the firm is $y=\frac{1}{3}\sqrt{p+6}+2$, provided that $p\geq \$3$.