

PHY1322 Winter 2015

LECTURE NOTES

PART II:

OSCILLATIONS AND WAVES

/FULL NOTES/

# PART II: OSCILLATIONS AND MECHANICAL WAVES

- M1 HARMONIC OSCILLATOR
- M2 MECHANICAL WAVES
- M3 SOUND WAVES

# OSCILLATIONS I

- Simple Harmonic Oscillator (SHO)



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# • Simple Harmonic Oscillator (SHO)

*While studying this Chapter we will learn the following new concepts:*

*Simple Harmonic Oscillator Equation*

*Oscillatory Motion*

*Motion of a particle under the Influence of elastic force*

*Mass on the spring*

*Ideal pendulum*

*Physical pendulum*

*Torsional pendulum*

**STUDENT SHOULD FOCUS ON RECOGNIZING OF THE SIMPLE HARMONIC OSCILLATOR EQUATION AND BEING ABLE TO READ THE ANGULAR FREQUENCY FROM IT.**

**ALSO STUDENTS SHOULD LEARN HOW TO READ THE AMPLITUDE, PERIOD AND PHASE FROM THE SIMPLE HARMONIC OSCILLATOR SOLUTION**

**READING ASSIGNMENT: Chapter 15**

# NOTE ABOUT "UNUSUAL" PEDAGOGY

- Typically the textbooks organize the material to lead reader from the singular observation ("concrete") to generalized cases ("abstract" mathematical model).
- It is supposed to work better for students, but in the case of Harmonic Oscillator it leads to many misconceptions.
- We will pursue the path in the opposite direction: we will start with the abstract mathematical model. We will discuss its solutions and limits, and later we will see how various physical systems are simple realizations of this model.

# Periodic Motion

- **Periodic motion** is motion of an object in which it that regularly repeats its motion parameters
  - The object returns to a given position after a fixed time interval

$$f(t + T) = f(t)$$

- A special kind of periodic motion occurs in mechanical systems when the force acting on the object is proportional to the position of the object relative to some equilibrium position
  - If the force is always directed toward the equilibrium position, *and proportional to spatial coordinate* the motion is called **simple harmonic motion**
  - Simple Examples of periodic functions: cosine, sine, tangent, cotangent

# Angular Frequency, Frequency and Period

- The **period**,  $T$ , is the time interval required for the particle to go through one full cycle of its motion
  - The values of  $x$  and  $v$  for the particle at time  $t$  equal the values of  $x$  and  $v$  at  $t + T$

$$T = \frac{2\pi}{\omega}$$

- The inverse of the period is called the **frequency**
- The frequency represents the number of oscillations that the particle undergoes per unit time interval

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

- Units are cycles per second = hertz (Hz)

# SIMPLE HARMONIC OSCILLATOR EQUATION (SHO)

$$\frac{d^2 X(t)}{dt^2} = -\omega^2 X(t)$$

Any system described by the above equation is a harmonic oscillator!  
Capital  $X(t)$  stands for any function - in the simplest cases it may be just linear or angular position.

There are many functions that obey this equation:  
 $\exp(i\omega t)$ ,  $\cos(\omega t)$ ,  $\sin(\omega t)$ , and their linear combinations

# SHO SOLUTION

- A function that satisfies the equation is needed

$$\frac{d^2 X(t)}{dt^2} = -\omega^2 X(t)$$

- Need a function  $x(t)$  whose second derivative is the same as the original function with a negative sign and multiplied by  $\omega^2$
- The sine and cosine functions meet these requirements

# HOMework:

- By direct substitution verify that following are correct solutions to the SHO equation
  - i)  $X(t) = A \cos(\omega t + \varphi_1) + A \cos(\omega t + \varphi_2)$
  - ii)  $X(t) = A_1 \sin(\omega t + \varphi_1) + A_2 \cos(\omega t + \varphi_1)$
  - iii)  $X(t) = A_1 \sin(\omega t + \varphi_1) + A_2 \sin(\omega t + \varphi_1)$
  - iv)  $X(t) = A_1 e^{i(\omega t + \varphi_1)} + A_2 e^{i(\omega t + \varphi_1)}$
- Note: any of these functions is equally good solution to SHO problem, which means that it has correct mathematical form and retains all of the physical attributes of the proper solution.

# Motion Equations for Simple Harmonic Motion

$$x(t) = A \cos(\omega t + \phi)$$

$$v = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi)$$

$$a = \frac{d^2x}{dt^2} = -\omega^2 A \cos(\omega t + \phi)$$

In our future discussions we will use the simplest form of the solution. Unless it is stipulated otherwise we will use a cosine function.

Remember, simple harmonic motion is not uniformly accelerated motion. Kinematic Equations are not valid for this motion

# Simple Harmonic Motion - Definitions

- $A$  is the amplitude of the motion
  - This is the maximum position of the particle in either the positive or negative direction
- $\omega$  is called the angular frequency units are rad/s
- $\phi$  is the phase constant or the initial phase angle
- $A$  and  $\phi$  are determined uniquely by the position and velocity of the particle at  $t = 0$
- If the particle is at  $x = A$  at  $t = 0$ , then  $\phi = 0$
- The **phase** of the motion is the quantity  $(\omega t + \phi)$
- $x(t)$  is periodic and its value is the same each time  $\omega t$  increases by  $2\pi$  radians

# Angular Frequency, Frequency and Period

- The **period**,  $T$ , is the time interval required for the particle to go through one full cycle of its motion
  - The values of  $x$  and  $v$  for the particle at time  $t$  equal the values of  $x$  and  $v$  at  $t + T$

$$T = \frac{2\pi}{\omega}$$

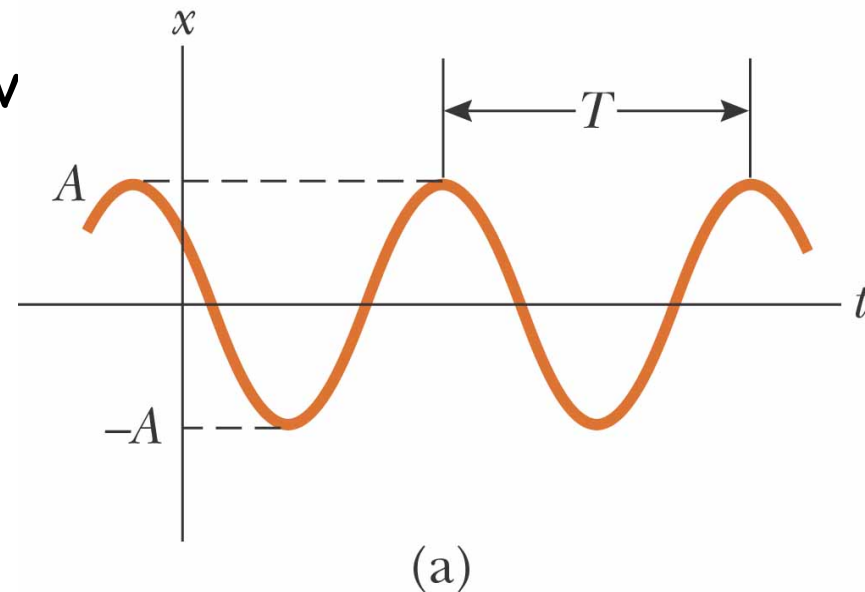
- The inverse of the period is called the **frequency**
- The frequency represents the number of oscillations that the particle undergoes per unit time interval

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

- Units are cycles per second = hertz (Hz)

# Simple Harmonic Motion - Graphical Representation

- A solution is  $x(t) = A \cos(\omega t + \phi)$
- $A, \omega, \phi$  are all constants
- A cosine curve can be used to give physical significance to these constants



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# Maximum Values of $v$ and $a$

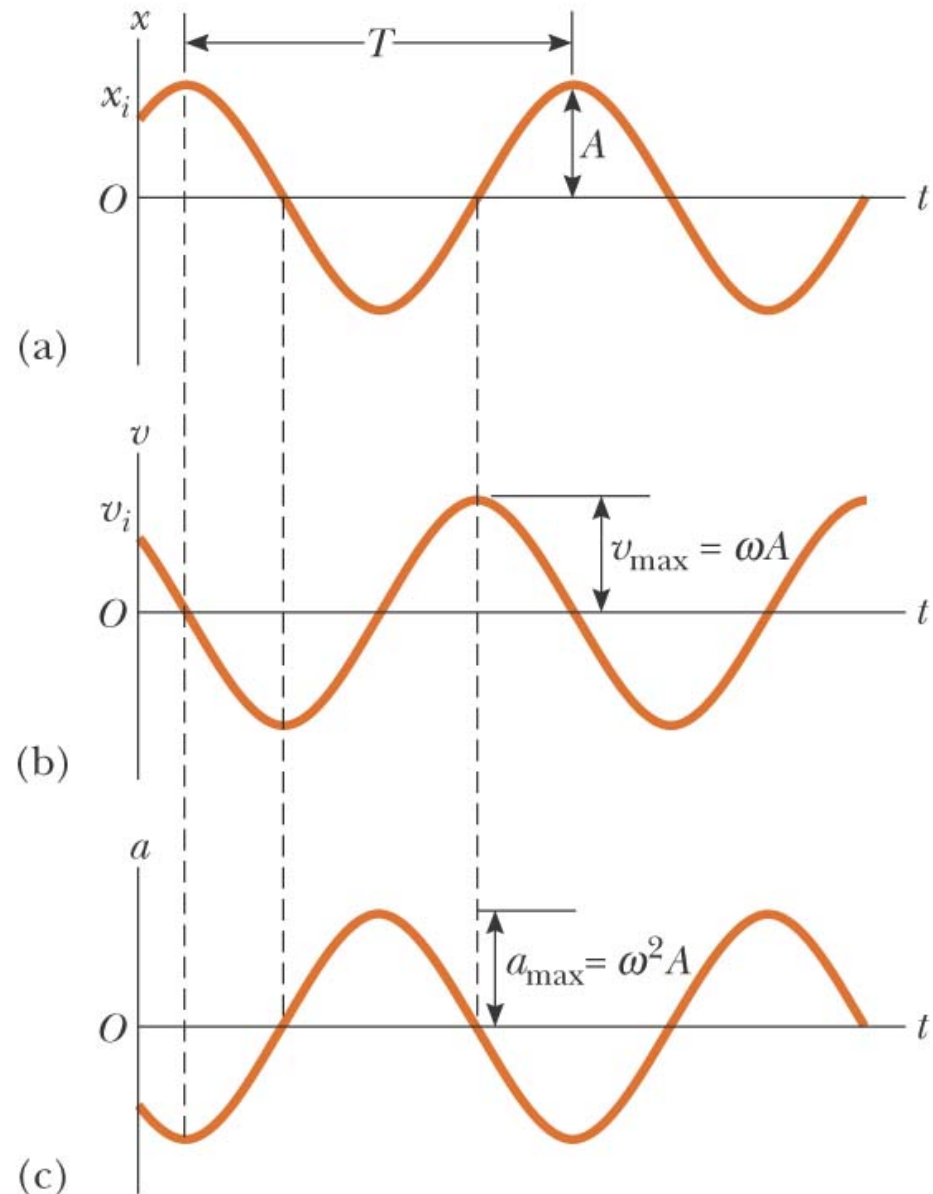
- Because the sine and cosine functions oscillate between  $\pm 1$ , we can easily find the maximum values of velocity and acceleration for an object in SHM

$$v_{\max} = \omega A = \sqrt{\frac{k}{m}} A$$

$$a_{\max} = \omega^2 A = \frac{k}{m} A$$

# Graphs

- The graphs show:
  - (a) displacement as a function of time
  - (b) velocity as a function of time
  - (c) acceleration as a function of time
- The velocity is  $90^\circ$  out of phase with the displacement and the acceleration is  $180^\circ$  out of phase with the displacement



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# Template for finding the solution in Various Oscillating Systems

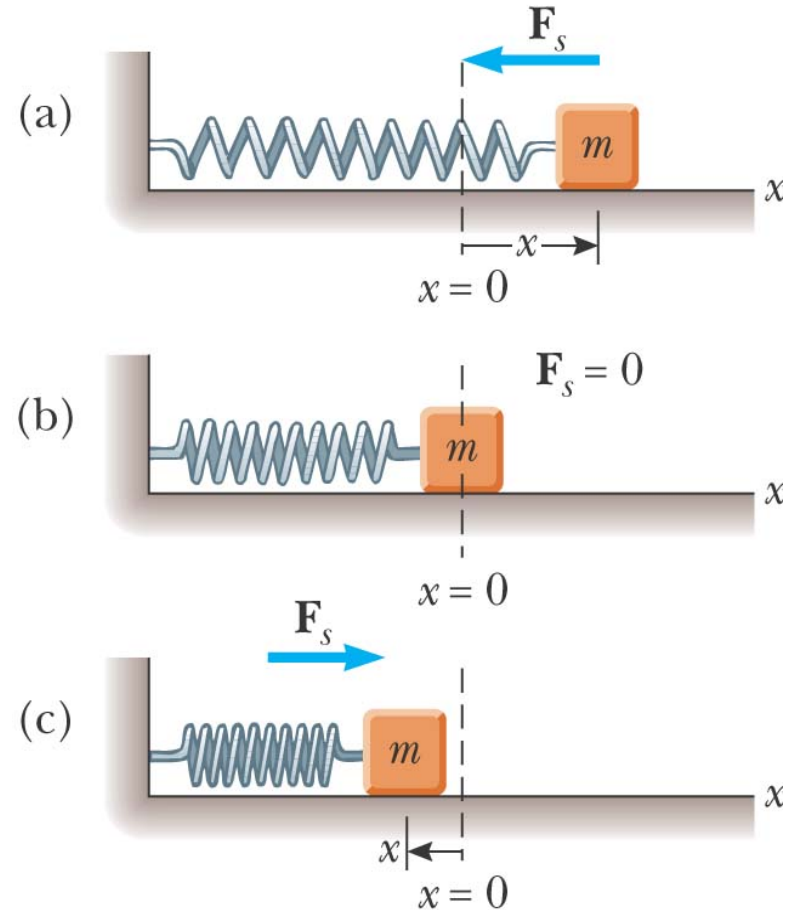
- We will discuss number of different systems as specific cases (“realizations”) of the Simple Harmonic Oscillator. We will follow the same reasoning in each of these cases:
  - 1 use Newton’s Second Law for the particular system to set up the Equation of Motion
  - 2 If necessary we will apply small angle approximation:  
 $\sin(\text{angle}) = (\text{angle})$
  - 3 Re-write the equation so it resembles the Equation for SHO. Identify the angular frequency ( $\omega$ )
  - 4 Using the known properties of general SHO equation write the solutions!

# Systems to be discussed

- Mass driven by the horizontal spring
- Ideal Pendulum ( point mass on the string)
- Physical Pendulum
- Torsional Pendulum
- Mass Driven by a Vertical Spring on Frictionless Surface

# Motion of a Spring-Mass System

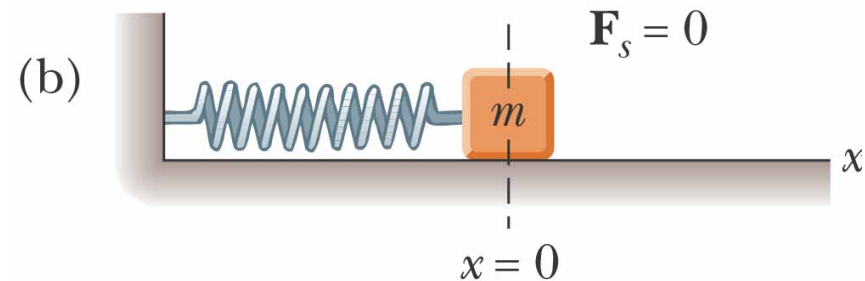
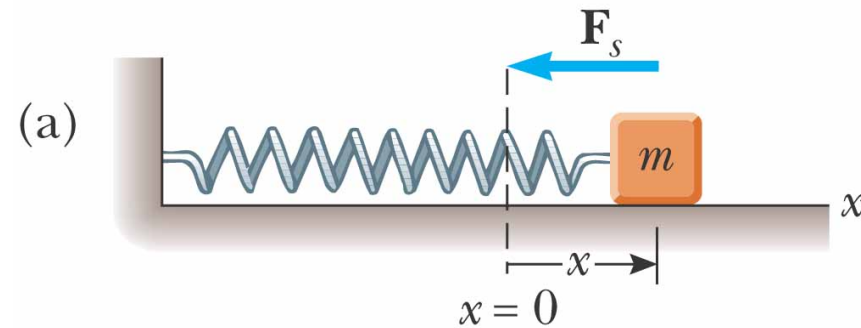
- A block of mass  $m$  is attached to a spring, the block is free to move on a frictionless horizontal surface
- When the spring is neither stretched nor compressed, the block is at the **equilibrium position**
  - $x = 0$
- Hooke's Law states  $F_s = -kx$ 
  - $F_s$  is the restoring force
    - It is always directed toward
    - the equilibrium position
    - Therefore, it is always opposite the displacement from equilibrium
  - $k$  is the force (spring) constant
  - $x$  is the displacement



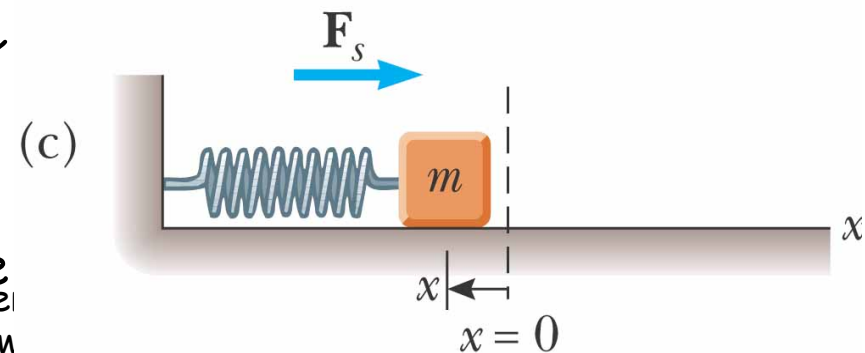
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# Mass on Spring: More About Restoring Force

- The block is displaced to the right of  $x = 0$ 
    - The position is positive
  - The restoring force is directed to the left
- 
- The block is at the equilibrium position
    - $x = 0$
  - The spring is neither stretched nor compressed
  - The force is 0



- The block is displaced to the left of  $x = 0$ 
  - The position is negative
- The restoring force is directed to the right



# Spring: Equation of Motion

- The force described by Hooke's Law is the net force in Newton's Second Law

$$F_{net} = F_{Hooke}$$

$$ma_x = -kx$$

$$m \frac{d^2 x}{dt^2} = -kx$$

$$\frac{d^2 x}{dt^2} = -\frac{k}{m} x$$

# Mass on the Spring: Representation of Harmonic Oscillator Motion

- Model the block as a particle
- Choose  $x$  as the axis along which the oscillation occurs

- Acceleration

$$\frac{d^2 x}{dt^2} = -\frac{k}{m} x$$

- We compare it with the General SHO Equation:

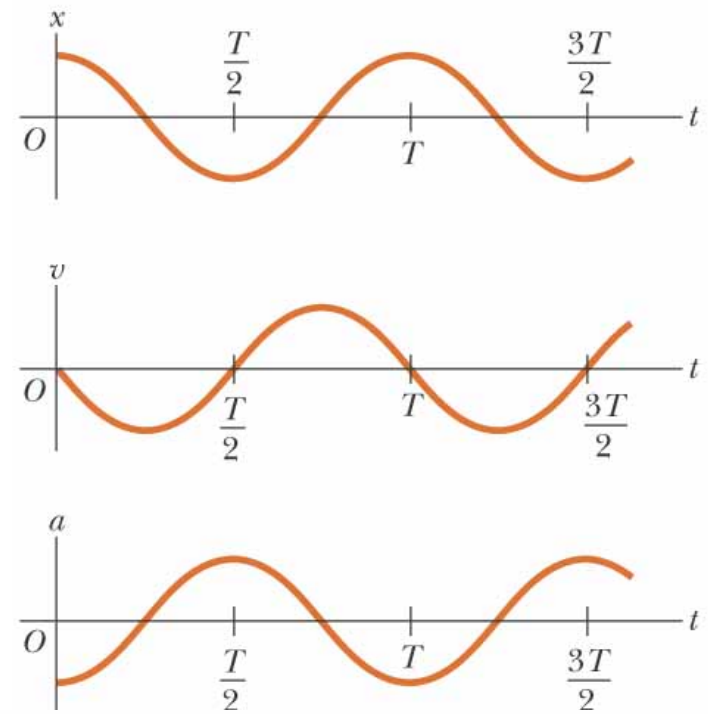
$$\frac{d^2 x}{dt^2} = -\omega^2 x$$

and we conclude that:

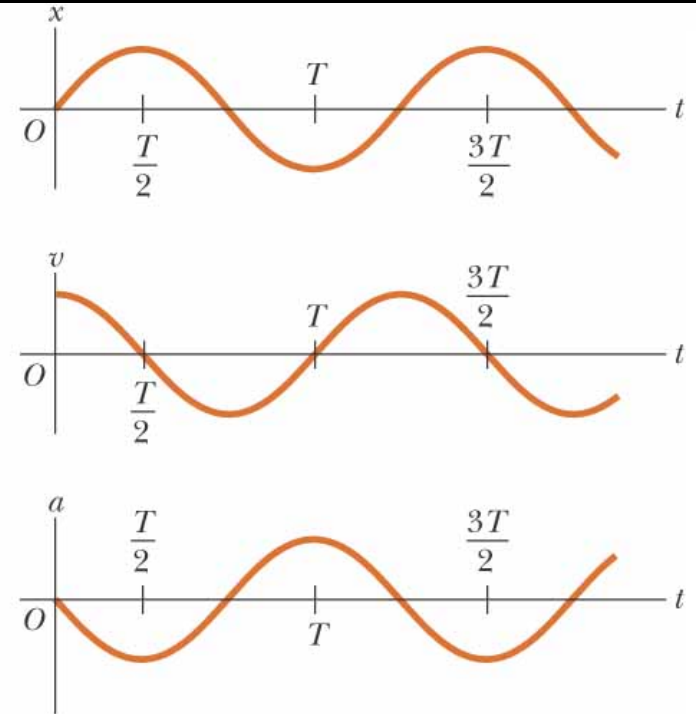
$$\omega^2 = \frac{k}{m}$$

# SHM Example

- Initial conditions at  $t = 0$  are
  - $x(0) = A$
  - $v(0) = 0$
- This means  $f = 0$
- The acceleration reaches extremes of  $\pm \omega^2 A$
- The velocity reaches extremes of  $\pm \omega A$

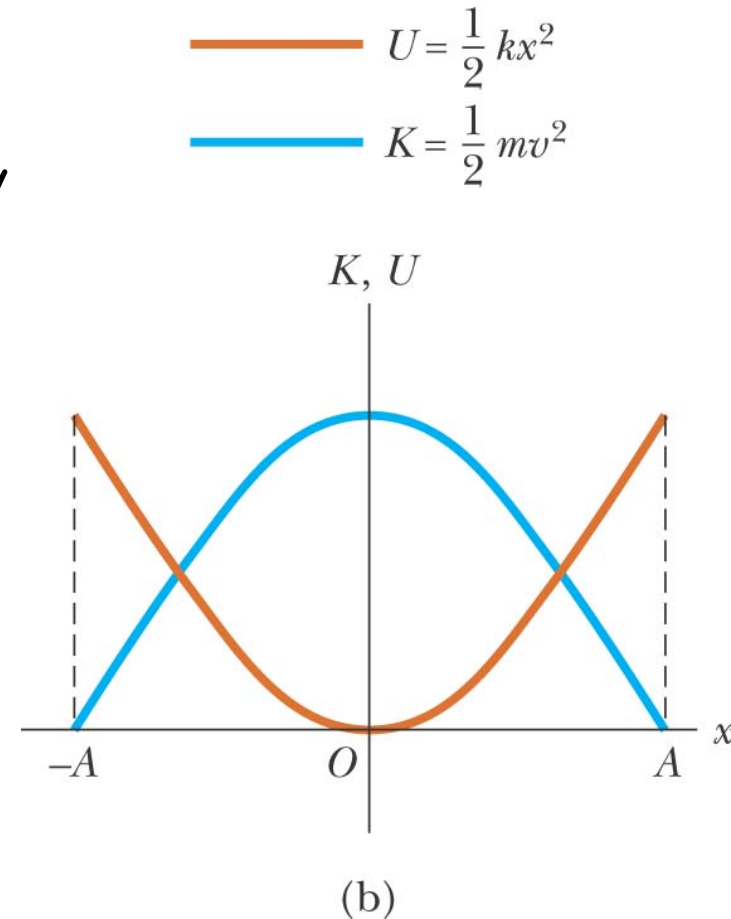


- Initial conditions at  $t = 0$  are
  - $x(0) = 0$
  - $v(0) = v_i$
- This means  $f = -\pi/2$
- The graph is shifted one-quarter cycle to the right compared to the graph of  $x(0) = A$



# Energy of the SHM Oscillator

- Assume a spring-mass system is moving on a frictionless surface
- This tells us the total energy is constant
- The kinetic energy can be found by
$$K = \frac{1}{2} mv^2 = \frac{1}{2} m\omega^2 A^2 \sin^2(\omega t + f)$$
- The elastic potential energy can be found by
  - $U = \frac{1}{2} kx^2 = \frac{1}{2} kA^2 \cos^2(\omega t + f)$
- The total energy is  $K + U = \frac{1}{2} kA^2$
  
- The total mechanical energy is constant
- The total mechanical energy is proportional to the square of the amplitude
- Energy is continuously being transferred between potential energy stored in the spring and the kinetic energy of the block

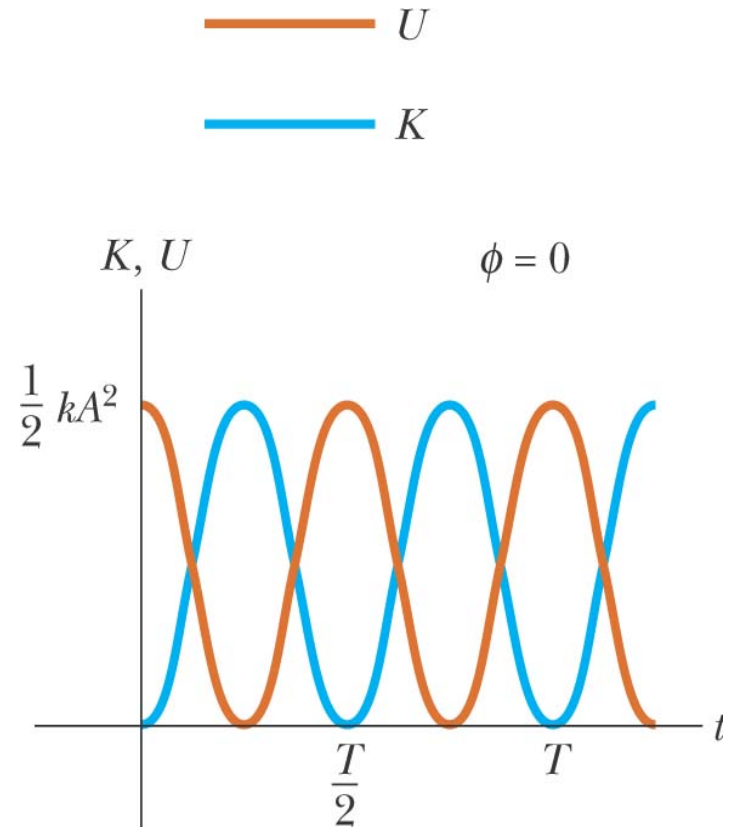


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# Energy of the SHM Oscillator, cont

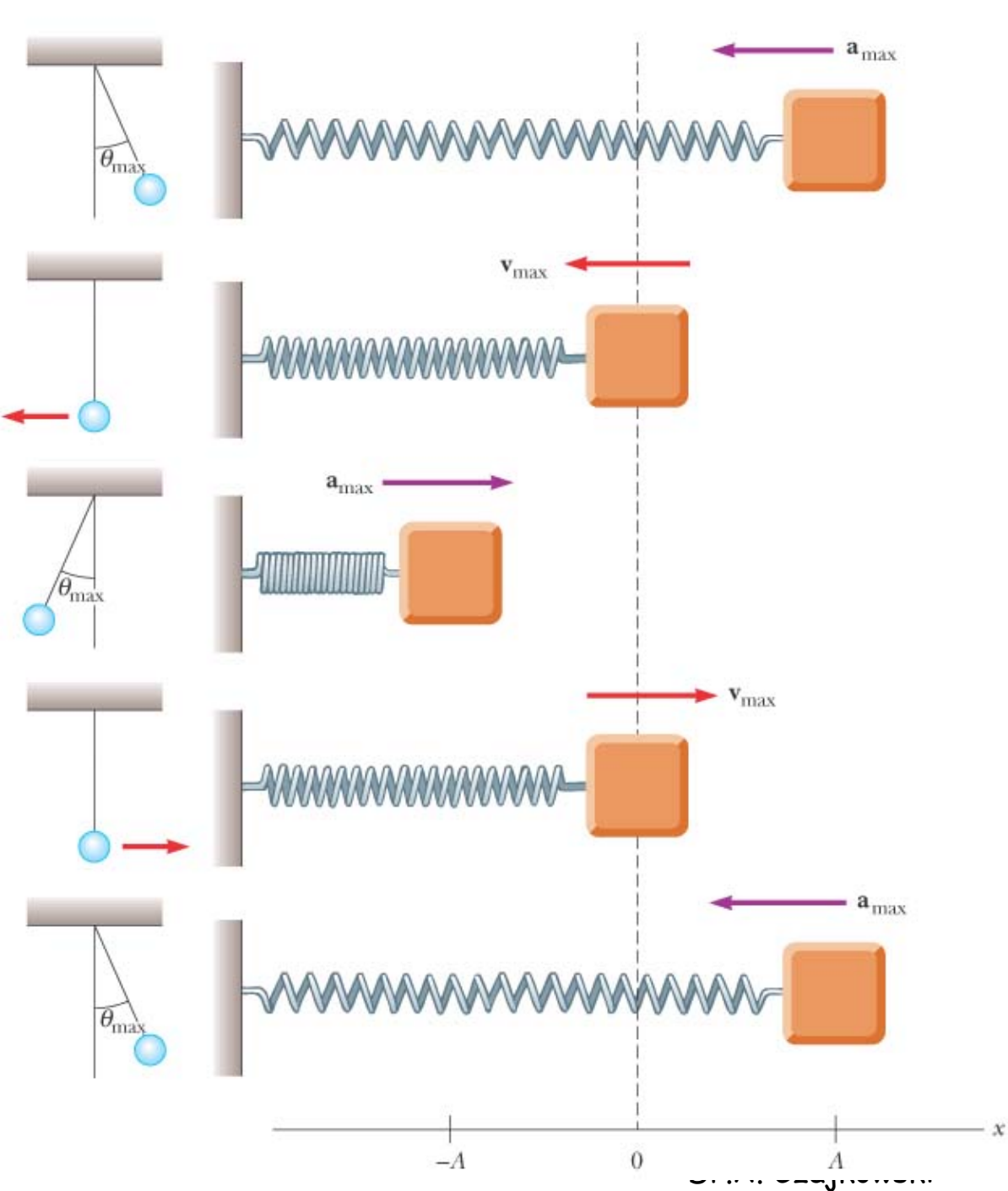
- As the motion continues, the exchange of energy also continues
- Energy can be used to find the velocity

$$v = \pm \sqrt{\frac{k}{m} (A^2 - x^2)}$$
$$= \pm \omega^2 \sqrt{A^2 - x^2}$$



(a)

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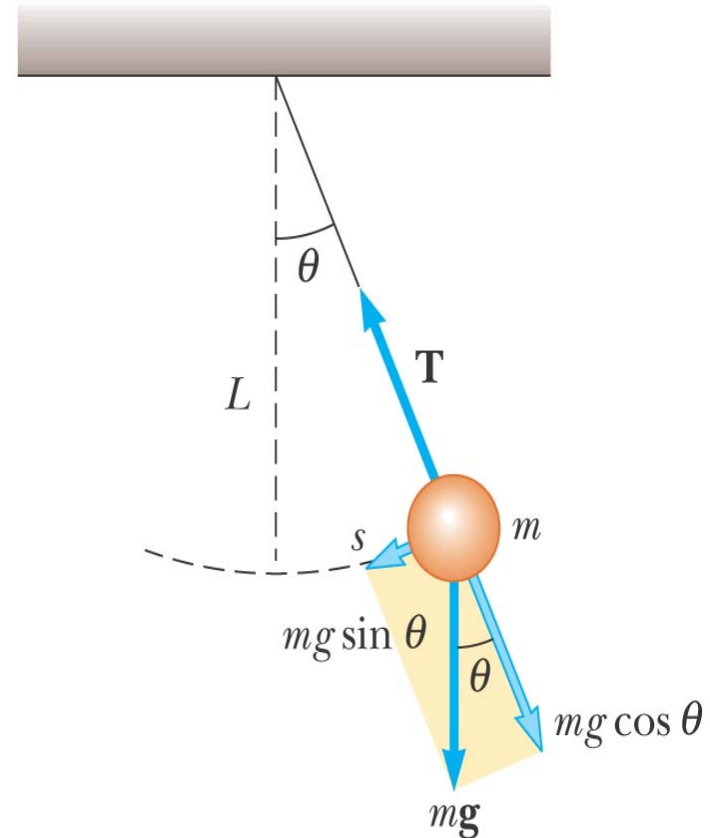


| $t$    | $x$  | $v$         | $a$           | $K$                 | $U$                 |
|--------|------|-------------|---------------|---------------------|---------------------|
| 0      | $A$  | 0           | $-\omega^2 A$ | 0                   | $\frac{1}{2} k A^2$ |
| $T/4$  | 0    | $-\omega A$ | 0             | $\frac{1}{2} k A^2$ | 0                   |
| $T/2$  | $-A$ | 0           | $\omega^2 A$  | 0                   | $\frac{1}{2} k A^2$ |
| $3T/4$ | 0    | $\omega A$  | 0             | $\frac{1}{2} k A^2$ | 0                   |
| $T$    | $A$  | 0           | $-\omega^2 A$ | 0                   | $\frac{1}{2} k A^2$ |

# Simple Pendulum

- A simple pendulum also exhibits periodic motion
- The motion occurs in the vertical plane and is driven by gravitational force
- The motion is very close to that of the SHM oscillator If the angle is  $< 10^\circ$
- The forces acting on the bob are  $T$  and  $mg$ 
  - $T$  is the force exerted on the bob by the string
  - $mg$  is the gravitational force
- The tangential component of the gravitational force is a restoring force
- In the tangential direction,

$$F_t = -mg \sin \theta = m \frac{d^2 s}{dt^2}$$



The length,  $L$ , of the pendulum is constant, and for small values of  $\theta$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta = -\frac{g}{L}\theta$$

• This confirms the form of the motion is SHM

The function  $q$  can be written as  $q = q_{\max} \cos(\omega t + f)$

The angular frequency is

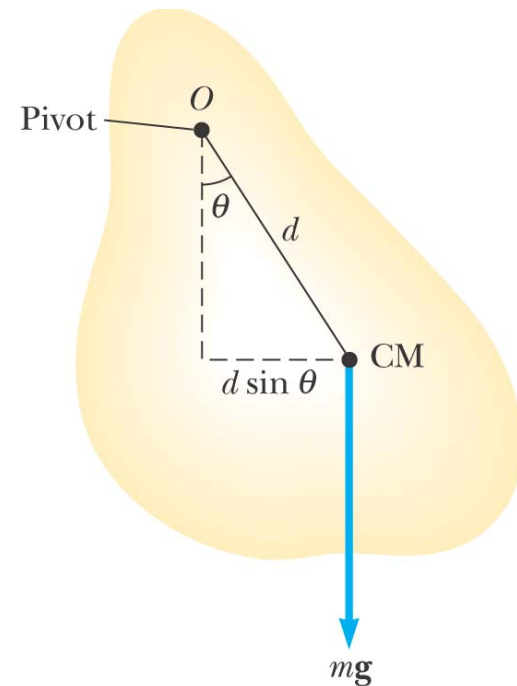
$$\omega = \sqrt{\frac{g}{L}}$$

The period is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

- The period and frequency of a simple pendulum depend only on the length of the string and the acceleration due to gravity.
- The period is independent of the mass.
- All simple pendula that are of equal length and are at the same location oscillate with the same period.

# Physical Pendulum



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- If a hanging object oscillates about a fixed axis that does not pass through the center of mass and the object cannot be approximated as a particle, the system is called a **physical pendulum**
  - It cannot be treated as a simple pendulum

- From Newton's Second Law,

$$-mgd \sin \theta = I \frac{d^2 \theta}{dt^2}$$

- The gravitational force provides a torque about an axis through  $O$
- The magnitude of the torque is  $mgd \sin \theta$
- $I$  is the moment of inertia about the axis through  $O$

The gravitational force produces a restoring force  
Assuming  $\theta$  is small, this becomes

$$\frac{d^2 \theta}{dt^2} = - \left( \frac{mgd}{I} \right) \theta = -\omega^2 \theta$$

# Physical Pendulum, cont.

This equation is in the form of an object in simple harmonic motion  
The angular frequency is

$$\omega = \sqrt{\frac{mgd}{I}}$$

The period is

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{I}{mgd}}$$

## NOTE:

Any physical pendulum can be used to measure the moment of inertia of a flat rigid object

- If you know  $d$ , you can find  $I$  by measuring the period

- If  $I = md^2$  then the physical pendulum is the same as a simple pendulum

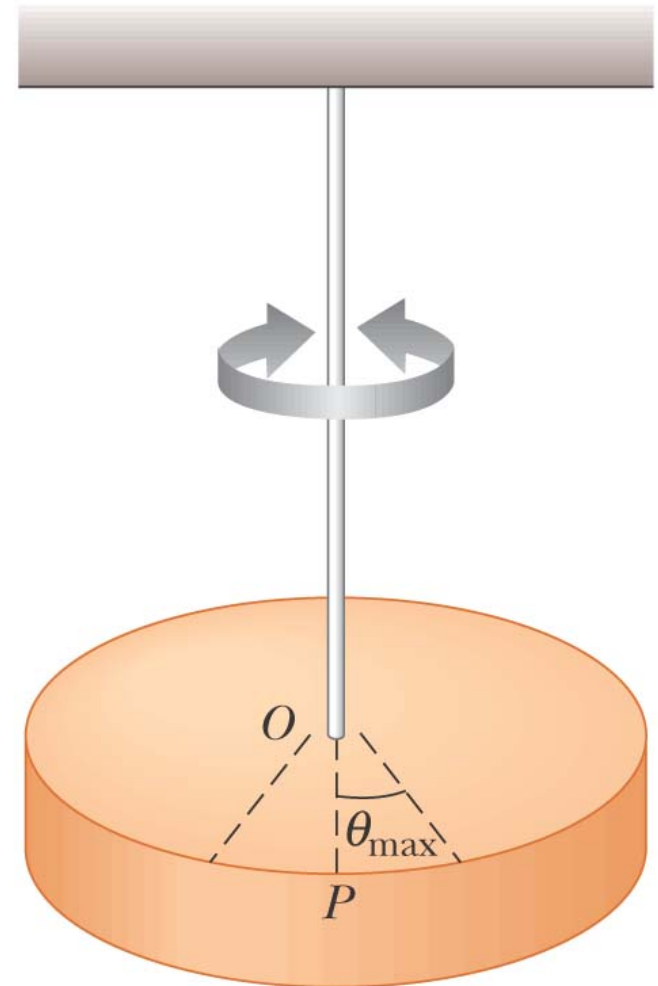
- The mass is all concentrated at the center of mass

# Torsional Pendulum

- Assume a rigid object is suspended from a wire attached at its top to a fixed support
- The twisted wire exerts a restoring torque on the object that is proportional to its angular position
- The restoring torque is  $\tau = -k\theta$ ;  $k$  is the *torsion constant* of the support wire
- Newton's Second Law gives

$$\tau = -k\theta = I \frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} = -\frac{k}{I}\theta$$



# Torsional Pendulum Period

- The torque equation produces a motion equation for simple harmonic motion

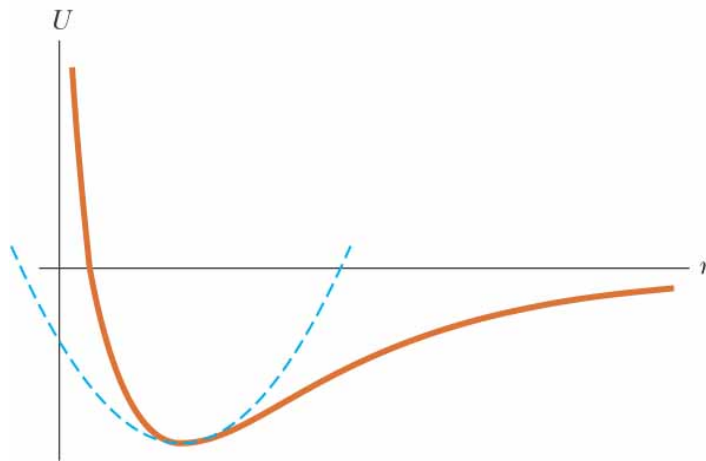
- The angular frequency is  $\omega = \sqrt{\frac{\kappa}{I}}$

- The period is  $T = 2\pi\sqrt{\frac{I}{\kappa}}$

- No small-angle restriction is necessary
- Assumes the elastic limit of the wire is not exceeded

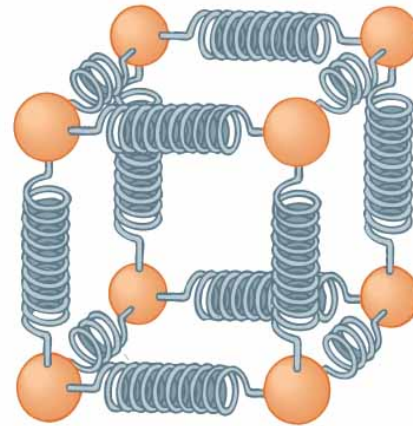
# Molecular Model of SHO

- If the atoms in the molecule do not move too far, the force between them can be modeled as if there were springs between the atoms
- The potential energy acts similar to that of the SHM oscillator



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(a)



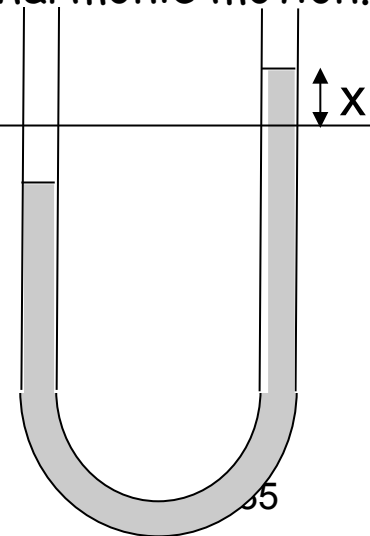
(b)

# Importance of Simple Oscillator Model

- It was the first quantitative model of matter
- It is an excellent starting point for any bound system:
  - Near minimum any potential resembles the parabola
  - There is well known Quantum Mechanical Model for Harmonic Oscillator

# Follow-up problems

- 1 Show that total energy of the spring is indeed given by  $E=kA^2$
- 2 Show that the relation  $\omega=2\pi/T$  holds for any harmonic motion
- 3 The periodic motion is given in the form:  $f(t)= A_m \cos (\omega t+ \varphi)$ .  
What is the amplitude and phase constant for the harmonic oscillator when
  - a)  $f(t)$  represents position function  $q(t)$
  - b)  $f(t)$  represents velocity function  $v(t)$
  - c)  $f(t)$  represents acceleration function  $a(t)$
- 4 Physical pendulum consists of a uniform solid disk ( of mass  $M$  and radius  $R$ ) supported in a vertical plane by a pivot located a distance  $d$  from the centre of the disk. The disk is displaced by a small angle and released.  
Find the expression for the period of the resulting simple harmonic motion.
- 5 Water fills a length  $l$  of a U tube, as shown in the figure.  
The water is slightly displaced from its equilibrium and then allowed to move freely:
  - (A) show that the liquid executes simple harmonic motion
  - (B) what is the period?

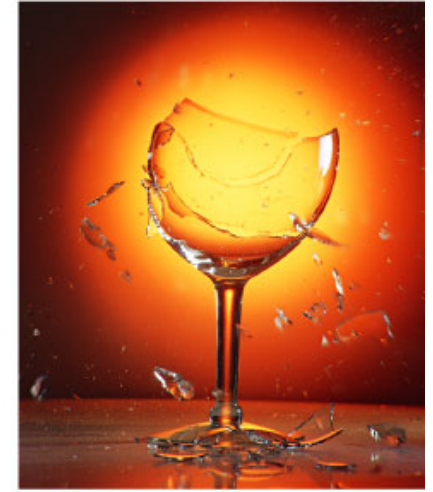


# READING HOMEWORK

- Read carefully chapter 15.6 and 15.7
- Prepare detailed note on damped oscillations and resonance
- Give real life examples of resonances. (draw from your everyday experience as well as from the field of engineering, music, chemistry and telecommunications).



(a)

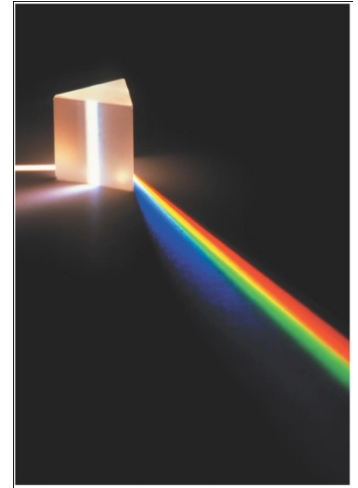
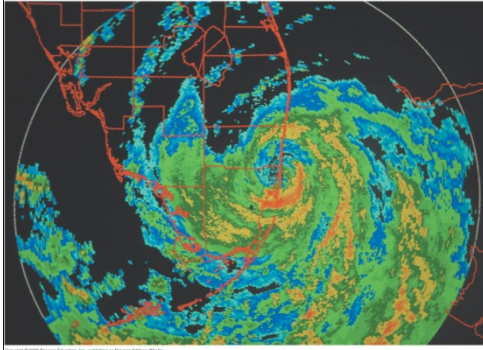


(b)

Please note that this material will be discussed only briefly in class. Nevertheless students are expected to master it, and will be tested on it.

# OSCILLATIONS II

## Mechanical Waves



# Mechanical Waves

*While studying this Chapter we will learn the following new concepts:*

*What is Wave ?  
What Types of Waves do we know?*

*The Traveling Wave  
Properties of Waves Propagating on Strings  
Wave Equation*

**FOCUS ON RECOGNIZING OF THE SIMPLE SINUSOIDAL WAVE  
MODEL**

**AFTER THIS SECTION YOU SHOULD BE ABLE TO PROPERLY USE  
THE TERMINOLOGY DESCRIBING WAVES: AMPLITUDE,  
FREQUENCY, PERIOD, WAVELENGTH AND PHASE**

**READING ASSIGNMENT:  
Chapter 17**

# Waves: General Features

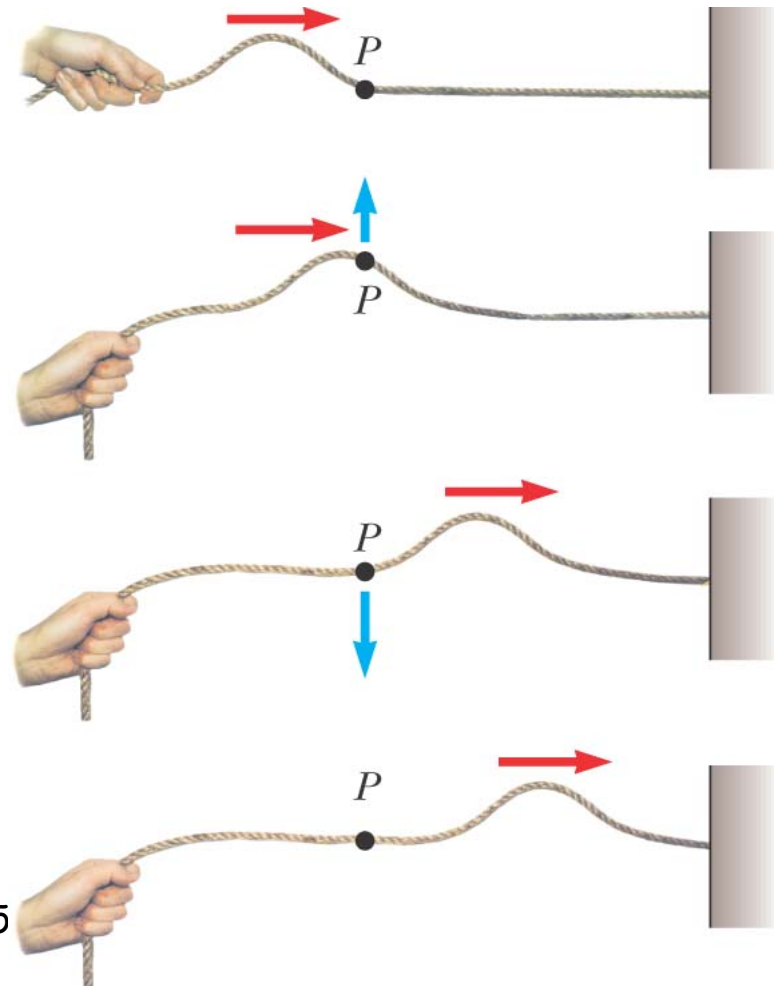
- The propagating disturbance is called a wave
- There are two main types of waves
  - Mechanical waves
    - Some physical medium is being disturbed
    - The wave is the propagation of a disturbance through a medium
  - Electromagnetic waves
    - No medium required
    - Examples are light, radio waves, x-rays
- In wave motion, energy is transferred over a distance
- Matter is not transferred over a distance
- All waves carry energy
  - The amount of energy and the mechanism responsible for the transport of the energy differ

# Mechanical Wave Requirements

- Some source of disturbance
- A medium that can be disturbed
- Some physical mechanism through which elements of the medium can influence each other

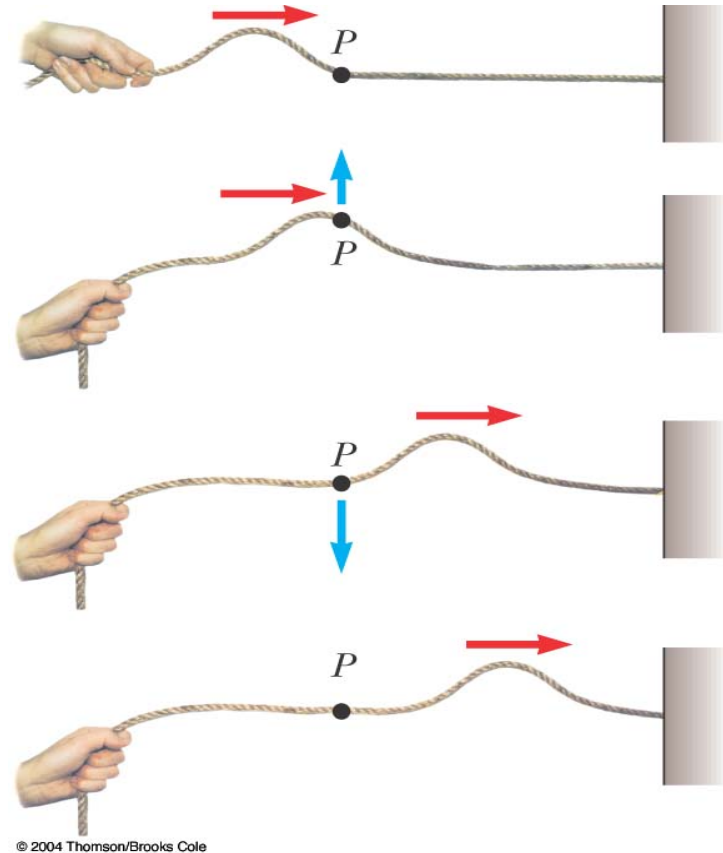
## Pulse on a Rope

- The wave is generated by a flick on one end of the rope
- The rope is under tension
- A single bump is formed and travels along the rope
  - The bump is called a **pulse**
- The pulse has a definite height and a definite speed of propagation along the medium
- A continuous flicking of the rope would produce a periodic disturbance which would form a wave.

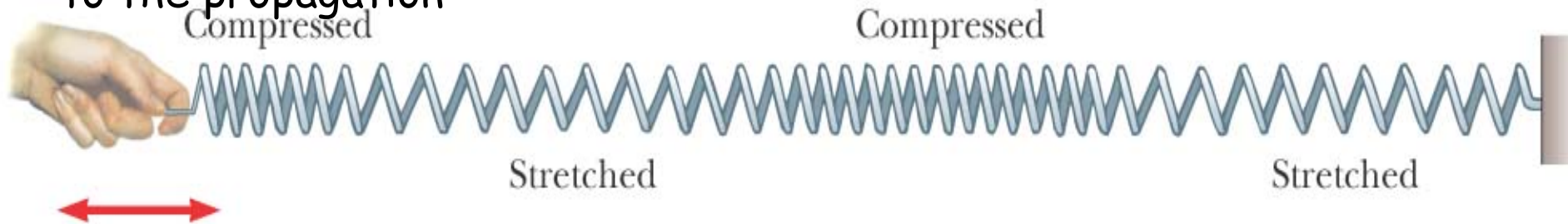


# Transverse and Longitudinal Waves

- A traveling wave or pulse that causes the elements of the disturbed medium to move perpendicular to the direction of propagation is called a **transverse wave**
- The particle motion is shown by the blue arrow
- The direction of propagation is shown by the red arrow

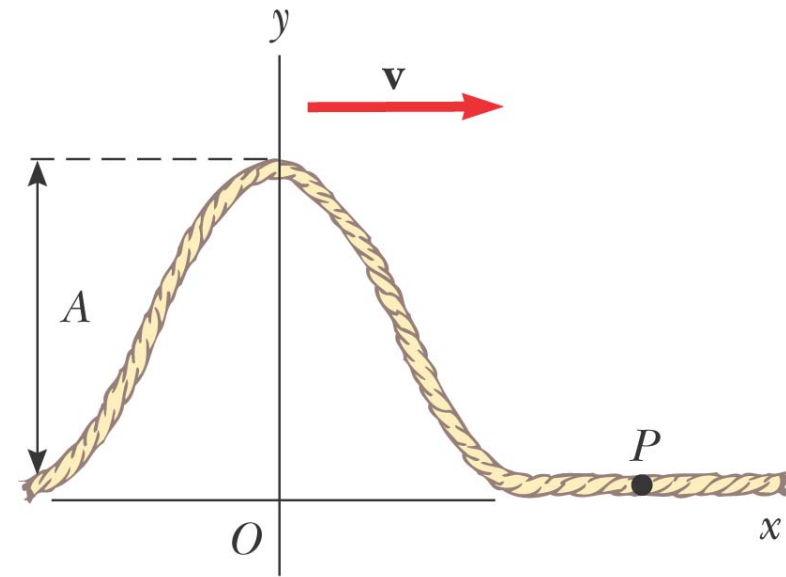


- A traveling wave or pulse that causes the elements of the disturbed medium to move parallel to the direction of propagation is called a **longitudinal wave**
- The displacement of the coils is parallel to the propagation



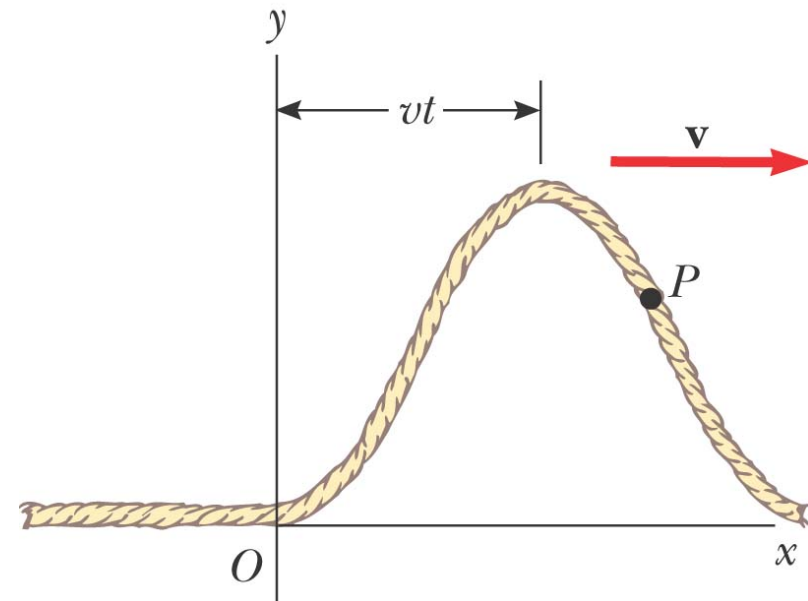
# Traveling Pulse

- The shape of the pulse at  $t = 0$  is shown
- The shape can be represented by  $y(x,0) = f(x)$ 
  - This describes the transverse position  $y$  of the element of the string located at each value of  $x$  at  $t = 0$



(a) Pulse at  $t = 0$

- The speed of the pulse is  $v$
- At some time,  $t$ , the pulse has traveled a distance  $vt$
- The shape of the pulse does not change
- Its position is now  $y = f(x - vt)$



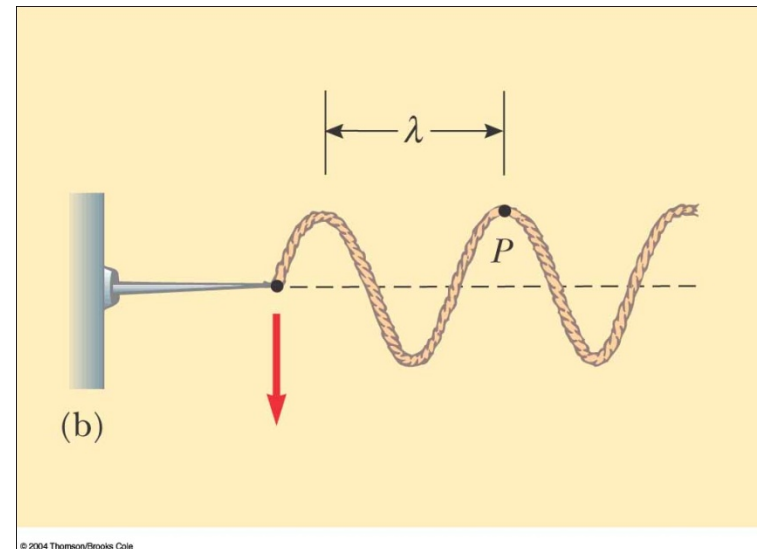
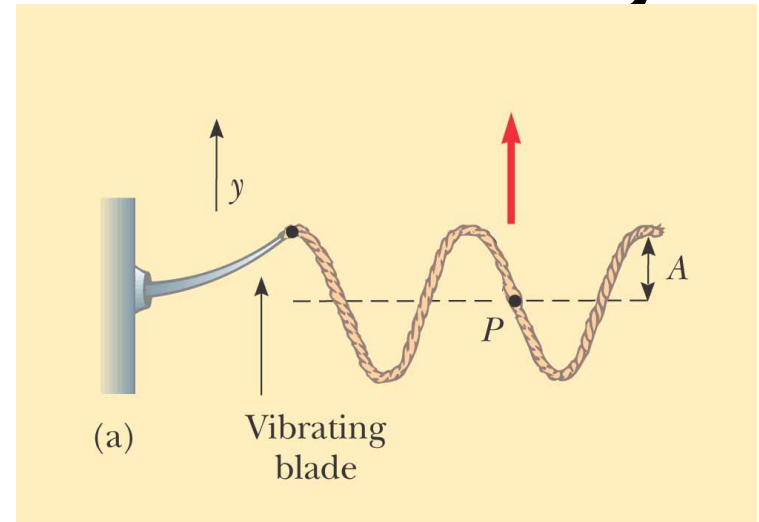
(b) Pulse at time  $t$

# Traveling Pulse, 2

- For a pulse traveling to the right
  - $y(x, t) = f(x - vt)$
- For a pulse traveling to the left
  - $y(x, t) = f(x + vt)$
- The function  $y$  is also called the **wave function**:  $y(x, t)$
- The wave function represents the  $y$  coordinate of any element located at position  $x$  at any time  $t$ 
  - The  $y$  coordinate is the transverse position
- If  $t$  is fixed then the wave function is called the **waveform**
  - It defines a curve representing the actual geometric shape of the pulse at that time

# Sinusoidal Wave on a String

- To create a series of pulses, the string can be attached to an oscillating blade
- The wave consists of a series of identical waveforms
- The relationships between speed, velocity, and period hold
- Each element of the string oscillates vertically with simple harmonic motion
  - For example, point  $P$
- Every element of the string can be treated as a simple harmonic oscillator vibrating with a frequency equal to the frequency of the oscillation of the blade

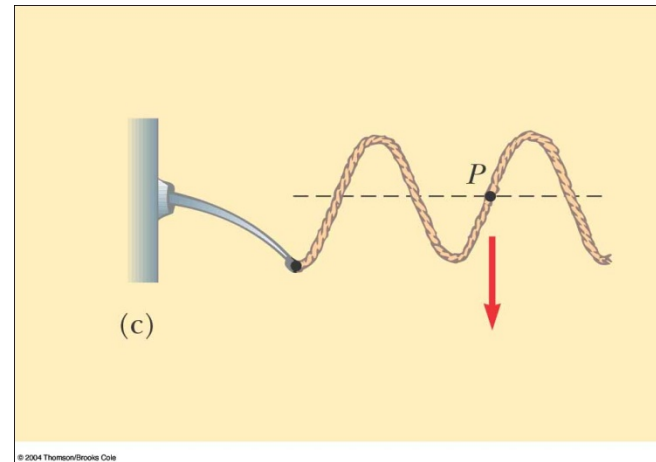


# Sinusoidal Wave on a String

- The transverse speed of the element is

$$v_y = \left. \frac{dy}{dt} \right]_{x=\text{constant}}$$

- or  $v_y = -\omega A \cos(kx - \omega t)$
- This is different than the speed of the wave itself!!



- The transverse acceleration of the element is

$$a_y = \left. \frac{dv_y}{dt} \right]_{x=\text{constant}}$$

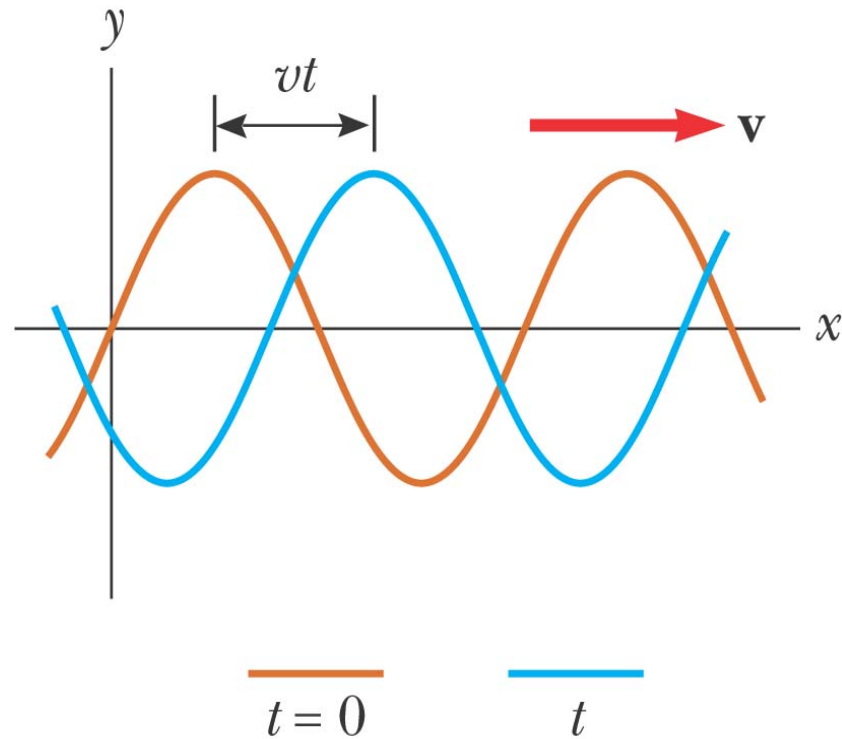
- or  $a_y = -\omega^2 A \sin(kx - \omega t)$

# Sinusoidal Wave on a String

- The maximum values of the transverse speed and transverse acceleration are
  - $v_{y, \max} = \omega A$
  - $a_{y, \max} = \omega^2 A$
- The transverse speed and acceleration do not reach their maximum values simultaneously
  - $v$  is a maximum at  $y = 0$
  - $a$  is a maximum at  $y = \pm A$

# Sinusoidal Waves

- The wave represented by the curve shown is a **sinusoidal wave**
- It is the same curve as  $\sin q$  plotted against  $q$
- This is the simplest example of a periodic continuous wave
  - It can be used to build more complex waves



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The wave moves toward the right

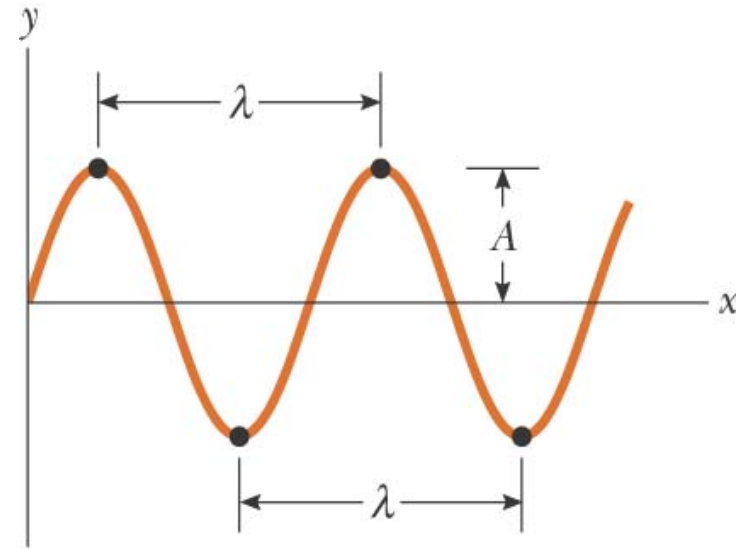
- In the previous example, the brown wave represents the initial position
- As the wave moves toward the right, it will eventually be at the position of the blue curve

Each element moves up and down in simple harmonic motion

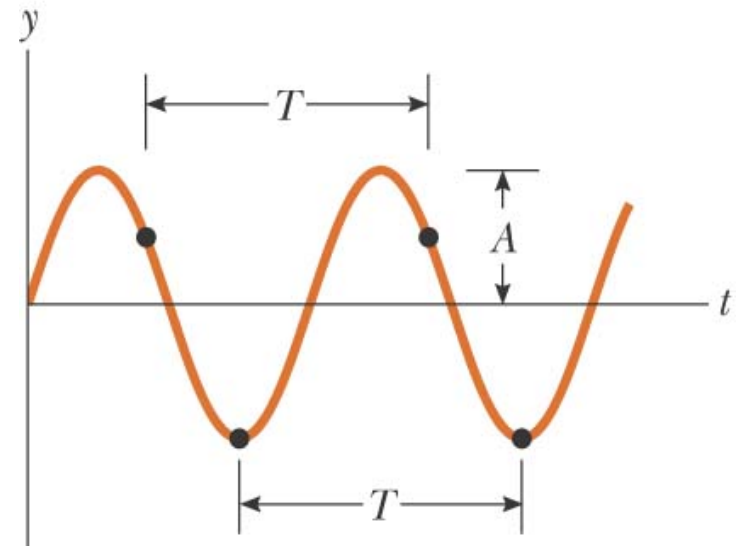
Distinguish between the motion of the wave and the motion of the particles of the medium

# Terminology: Amplitude, Wavelength, and

- The **crest** of the wave is the location of the maximum displacement of the element from its normal position
  - This distance is called the **amplitude,  $A$**
- The **wavelength,  $\lambda$** , is the distance from one crest to the next
- More generally, the **wavelength** is the minimum distance between any two identical points on adjacent waves
- The **period,  $T$** , is the time interval required for two identical points of adjacent waves to pass by a point
  - The period of the wave is the same as the period of the simple harmonic oscillation of one element of the medium



(a)



(b)

# Terminology: Frequency

- The **frequency**,  $f$ , is the number of crests (or any point on the wave) that pass a given point in a unit time interval
  - The time interval is most commonly the second
  - The frequency of the wave is the same as the frequency of the simple harmonic motion of one element of the medium

- The frequency and the period are related

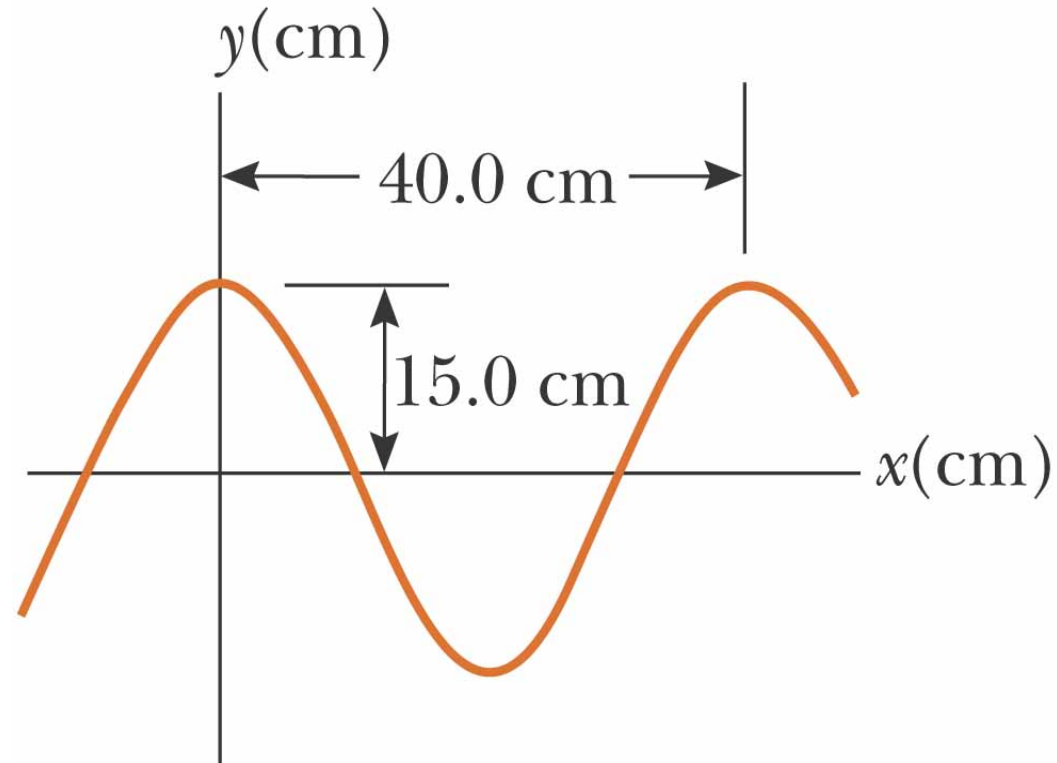
$$f = \frac{1}{T}$$

- When the time interval is the second, the units of frequency are
- $s^{-1} = \text{Hz}$  (Hz is a hertz)

# Terminology, Example

- The wavelength,  $\lambda$ , is 40.0 cm
- The amplitude,  $A$ , is 15.0 cm
- The wave function can be written in the form

$$y = A \cos(kx - \omega t)$$



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# Speed of Waves

- Waves travel with a specific speed
  - The speed depends on the properties of the medium being disturbed
- The wave function is given by

$$y(x, t) = A \sin \left[ \frac{2\pi}{\lambda} (x - vt) \right]$$

- This is for a wave moving to the right
- For a wave moving to the left, replace  $x - vt$  with  $x + vt$

# Speed of a Wave on a String

- The speed of the wave depends on the physical characteristics of the string and the tension to which the string is subjected

$$v = \sqrt{\frac{\text{tension}}{\text{mass/length}}} = \sqrt{\frac{T}{\mu}}$$

- This assumes that the tension is not affected by the pulse
- This does not assume any particular shape for the pulse

# Wave Function, Another Form

- Since speed is distance divided by time,  $v = \lambda / T$
- The wave function can then be expressed as

$$y(x, t) = A \sin \left[ 2\pi \left( \frac{x}{\lambda} - \frac{t}{T} \right) \right]$$

- This form shows the *periodic* nature of  $y$

# Wave Equations

- We can also define the angular wave number (or just wave number),  $k$

$$k = \frac{2\pi}{\lambda}$$

- The angular frequency can also be defined

$$\omega = \frac{2\pi}{T}$$

# Wave Equations, cont

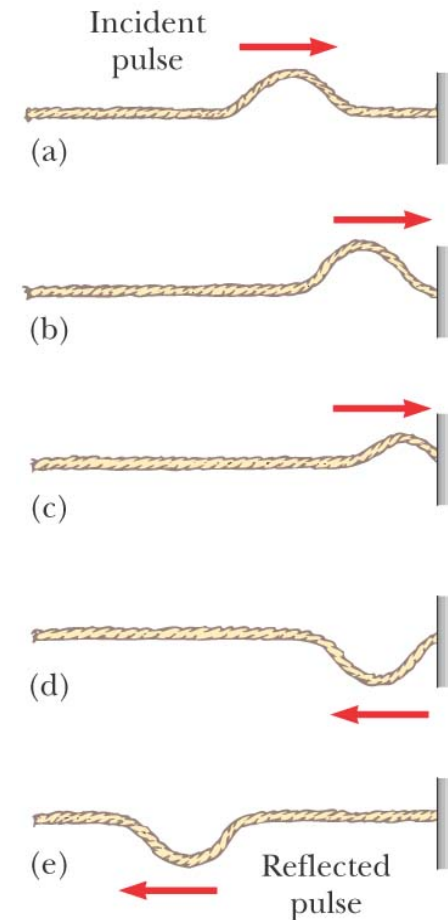
- The wave function can be expressed as  
 $y = A \sin(kx - \omega t)$
- The speed of the wave becomes  $v = \lambda f$
- If  $x \neq 0$  at  $t = 0$ , the wave function can be generalized to

$$y = A \sin(kx - \omega t + \phi)$$

where  $\phi$  is called the phase constant

# Reflection of a Wave, Fixed End

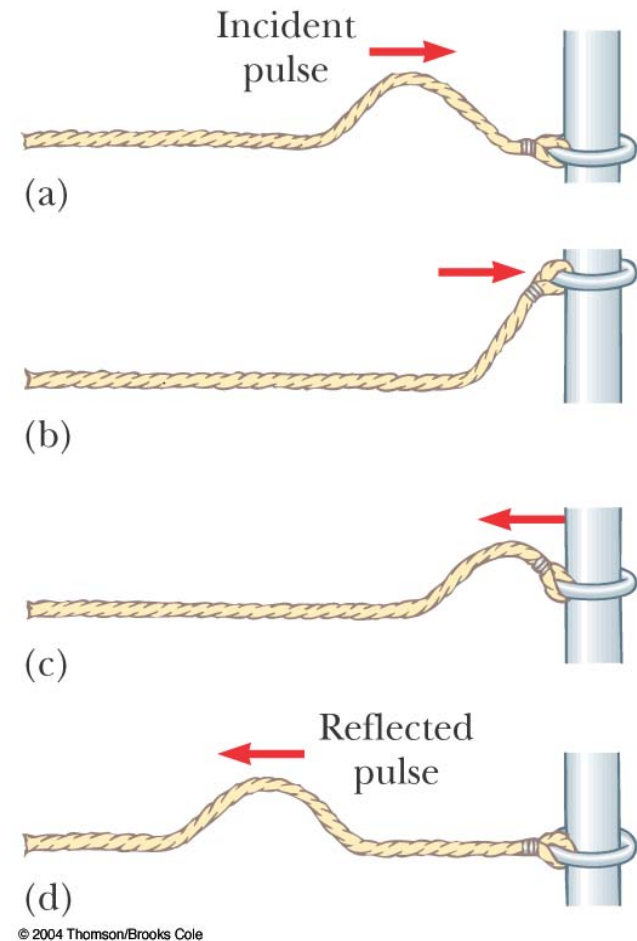
- When the pulse reaches the support, the pulse moves back along the string in the opposite direction
- This is the **reflection** of the pulse
- The pulse is inverted



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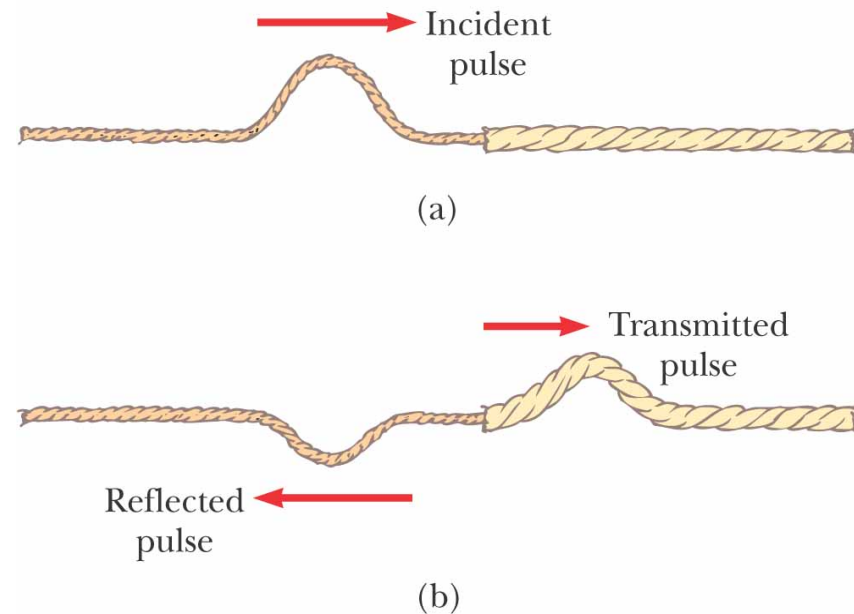
# Reflection of a Wave, Free End

- With a free end, the string is free to move vertically
- The pulse is reflected
- The pulse is not inverted



# Transmission of a Wave

- When the boundary is intermediate between the last two extremes
  - Part of the energy in the incident pulse is reflected and part undergoes **transmission**
    - Some energy passes through the boundary

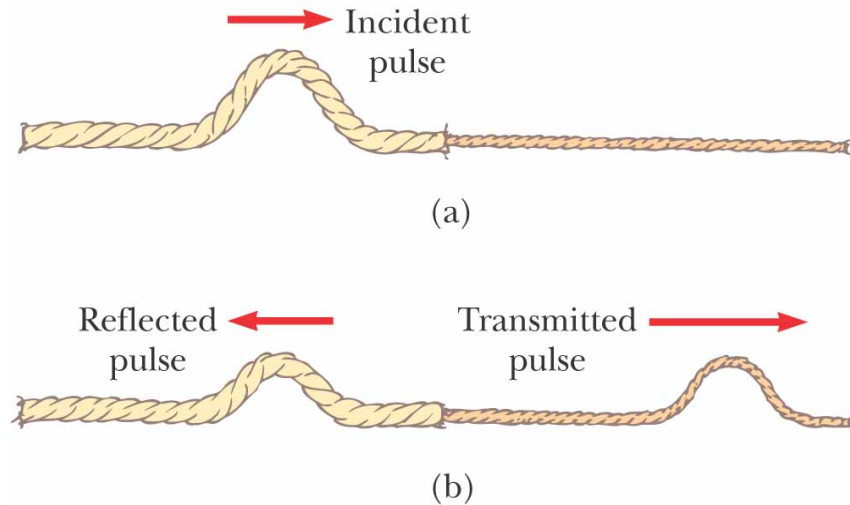


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- Assume a light string is attached to a heavier string
- The pulse travels through the light string and reaches the boundary
- The part of the pulse that is reflected is inverted
- The reflected pulse has a smaller amplitude

# Transmission of a Wave, 2

- Assume a heavier string is attached to a light string
- Part of the pulse is reflected and part is transmitted
- The reflected part is not inverted



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- Conservation of energy governs the pulse
- When a pulse is broken up into reflected and transmitted parts at a boundary, the sum of the energies of the two pulses must equal the energy of the original pulse

# Energy in Waves in a String

- Waves transport energy when they propagate through a medium
- We can model each element of a string as a simple harmonic oscillator
  - The oscillation will be in the  $y$ -direction
- Every element has the same total energy
- Each element can be considered to have a mass of  $\Delta m$
- Its kinetic energy is  $\Delta K = \frac{1}{2} (\Delta m) v_y^2$
- The mass  $\Delta m$  is also equal to  $m \Delta x$
- As the length of the element of the string shrinks to zero, and  $\Delta K = \frac{1}{2} (m\Delta x) v_y^2$
- Integrating over all the elements, the total kinetic energy in one wavelength is  $K_\lambda = \frac{1}{4}\mu\omega^2 A^2 \lambda$
- The total potential energy in one wavelength is  $U_\lambda = \frac{1}{4}\mu\omega^2 A^2 \lambda$
- This gives a total energy of
  - $E_\lambda = K_\lambda + U_\lambda = \frac{1}{2}\mu\omega^2 A^2 \lambda$

# Power Associated with a Wave

- The power is the rate at which the energy is being transferred:

$$\mathcal{P} = \frac{\Delta E}{\Delta t} = \frac{\frac{1}{2} \mu \omega^2 A^2 \lambda}{T} = \frac{1}{2} \mu \omega^2 A^2 v$$

- The power transfer by a sinusoidal wave on a string is proportional to the
  - Frequency squared
  - Square of the amplitude
  - Wave speed

# Linear Wave Equation, General

- The equation can be written as 
$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$
- This applies in general to various types of traveling waves
  - $y$  represents various positions
    - For a string, it is the vertical displacement of the elements of the string
    - For a sound wave, it is the longitudinal position of the elements from the equilibrium position
    - For em waves, it is the electric or magnetic field components
- The linear wave equation is satisfied by any wave function having the form 
$$y = f(x \pm vt)$$
- The linear wave equation is also a direct consequence of Newton's Second Law applied to any element of a string carrying a traveling wave

# The Linear Wave Equation

- The wave functions  $y(x, t)$  represent solutions of an equation called the **linear wave equation**
- This equation gives a complete description of the wave motion
- From it you can determine the wave speed
- The linear wave equation is basic to many forms of wave motion

## FOLLOW UP PROBLEMS

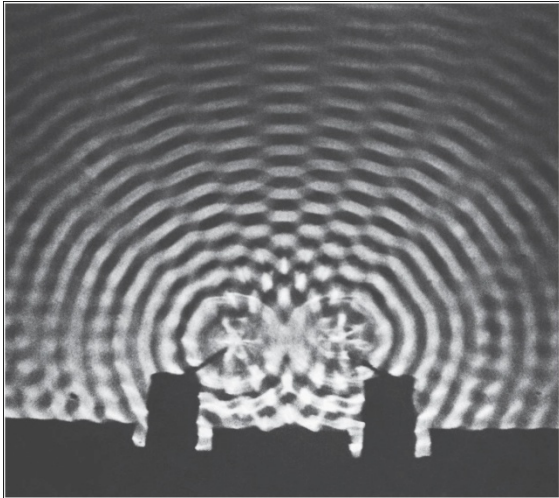
- Transverse pulses travel with a speed of 200 m/s along a taut copper wire whose diameter is 1.50 mm. What is the tension in the wire? (The density of copper is 8.92 g/cm<sup>3</sup>.)
- A taut rope has a mass of 0.180 kg and a length of 3.60 m. What power must be supplied to the rope in order to generate sinusoidal waves having an amplitude of 0.100 m and a wavelength of 0.500 m and traveling with a speed of 30.0 m/s?





# OSCILLATIONS III

## Sound Waves



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# SOUND WAVES

*While studying this Chapter we will learn the following new concepts:*

*What is Sound?*

*Intensity of sound and the definition of decibel*

*The Doppler Effect*

*The Doppler Effect for Light Waves*

*Principle of Superposition*

*Interference of Mechanical Waves*

*Vibrating Strings and Air Columns: Standing Waves*

**READING ASSIGNMENT:**

**Chapter 18**

**Chapter 19**

# Introduction to Sound Waves

- Sound waves are longitudinal waves
- They travel through any material medium
- The speed of the wave depends on the properties of the medium
- The mathematical description of sinusoidal sound waves is very similar to sinusoidal waves on a string

# Categories of Sound Waves

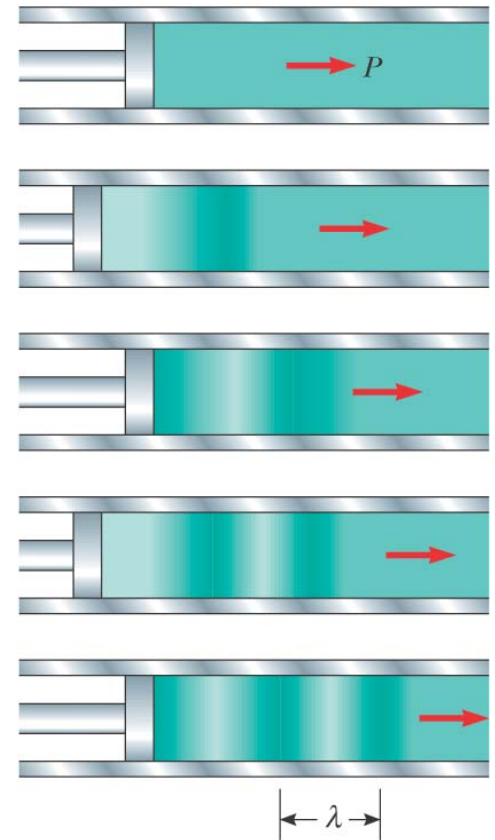
- The categories cover different frequency ranges
- **Audible waves** are within the sensitivity of the human ear
  - Range is approximately 20 Hz to 20 kHz
- **Infrasonic waves** have frequencies below the audible range
- **Ultrasonic waves** have frequencies above the audible range

# Periodic Sound Waves

- A compression moves through a material as a pulse, continuously compressing the material just in front of it
- The areas of compression alternate with areas of lower pressure and density called **rarefactions**
- These two regions move with the speed equal to the speed of sound in the medium

# Periodic Sound Waves, Example

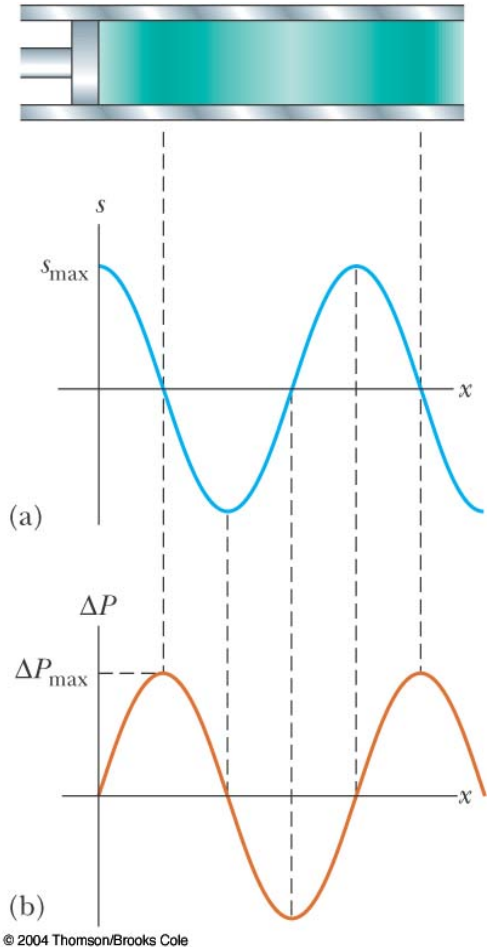
- A longitudinal wave is propagating through a gas-filled tube
- The source of the wave is an oscillating piston
- The distance between two successive compressions (or rarefactions) is the wavelength
- As the regions travel through the tube, any small element of the medium moves with simple harmonic motion parallel to the direction of the wave
- The harmonic position function is
$$s(x, t) = s_{\max} \cos(kx - \omega t)$$
  - $s_{\max}$  is the maximum position from the equilibrium position
  - This is also called the **displacement amplitude** of the wave



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# Periodic Sound Waves, Pressure

- The variation in gas pressure,  $\Delta P$ , is also periodic
- $\Delta P = \Delta P_{\max} \sin(kx - \omega t)$
- $\Delta P_{\max}$  is the pressure amplitude
- It is also given by  $\Delta P_{\max} = \rho v \omega s_{\max}$
- $k$  is the wave number (in both equations)
- $\omega$  is the angular frequency (in both equations)
- A sound wave may be considered either a displacement wave or a pressure wave
- The pressure wave is  $90^\circ$  out of phase with the displacement wave
  - The pressure is a maximum when the displacement is zero, etc.



# Intensity of Sound

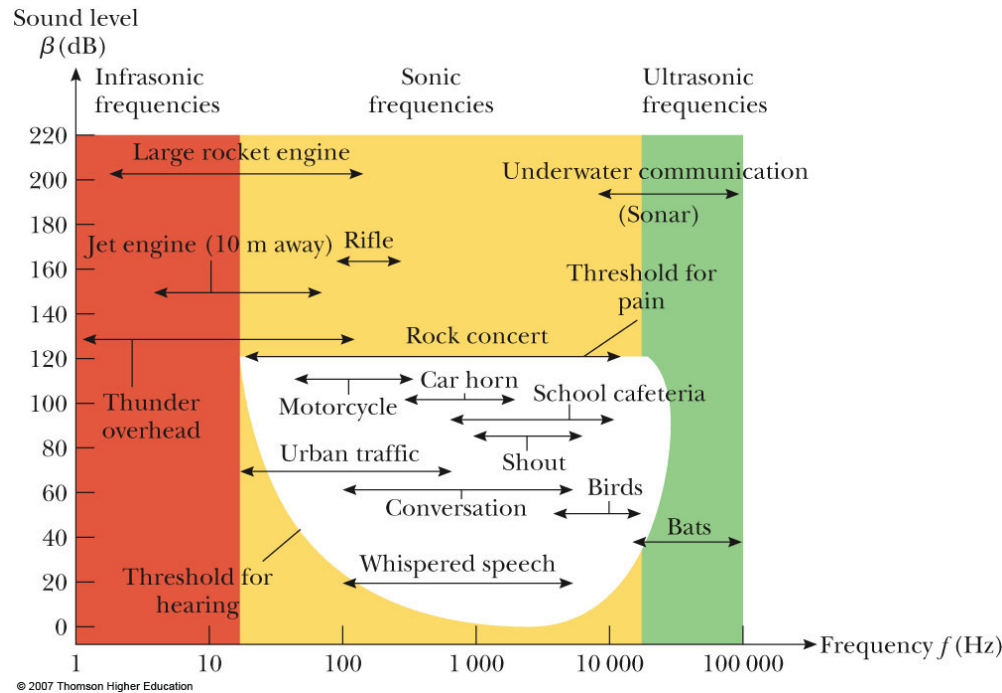
Intensity ( $I$ ) of uniform spherical source emitting power  $P$  at distance  $r$  from the source

$$I = \frac{P}{4\pi r^2}$$

Sound intensity level expressed in decibel (dB) is given by:

$$\beta = (10\text{dB}) \log_{10} \frac{I}{I_0}$$

where  $I_0 = 10^{-12} \text{ W/m}^2$



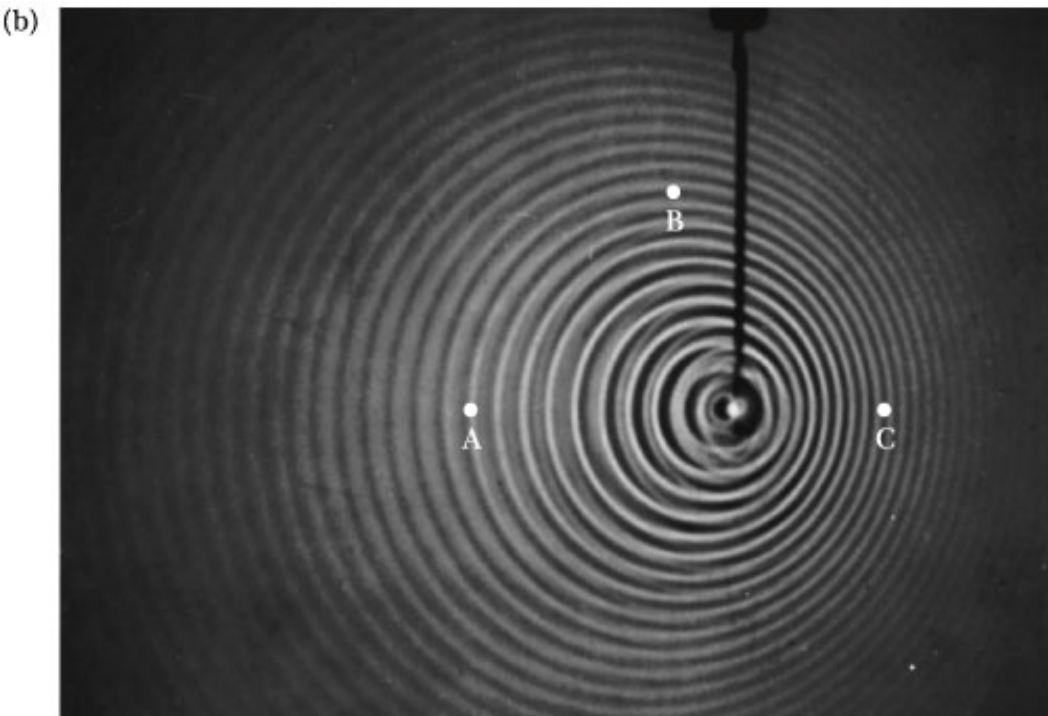
The white area shows average human response to sound

The lower curve of the white area shows the threshold of hearing

The upper curve shows the threshold of pain

# The Doppler Effect

- The Doppler effect is the apparent change in frequency (or wavelength) that occurs because of motion of the source or observer of a wave



When the relative speed of the source and observer is higher than the speed of the wave, the frequency appears to increase

When the relative speed of the source and observer is lower than the speed of the wave, the frequency appears to decrease

# Doppler Effect, General

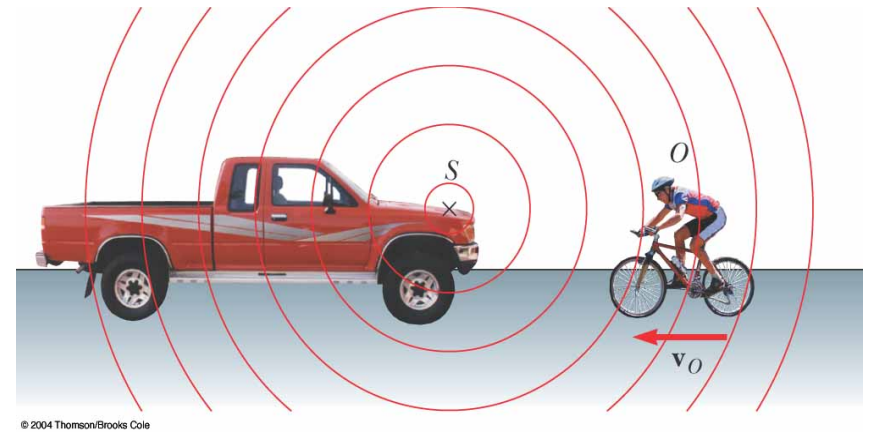
- Combining the motions of the observer and the source

$$f' = \left( \frac{v \pm v_o}{v \pm v_s} \right) f$$

- The signs may be established based on the following convenient rule for signs:
  - The word *toward* is associated with an *increase* in the observed frequency
  - The words *away from* are associated with a *decrease* in the observed frequency
- The Doppler effect is common to all waves
- The Doppler effect does not depend on distance

# Doppler Effect, Observer Moving

- The observer moves with a speed of  $v_o$
- Assume a point source that remains stationary relative to the air
- It is convenient to represent the waves with a series of circular arcs concentric to the source
  - These surfaces are called a *wave front*



# Doppler Effect, Observer Moving, cont

- The distance between adjacent wave fronts is the wavelength
- The speed of the sound is  $v$ , the frequency is  $f$ , and the wavelength is  $\lambda$
- When the observer moves toward the source, the speed of the waves relative to the observer is  $v' = v + v_o$ 
  - The wavelength is unchanged
- The frequency heard by the observer,  $f'$ , appears higher when the observer approaches the source

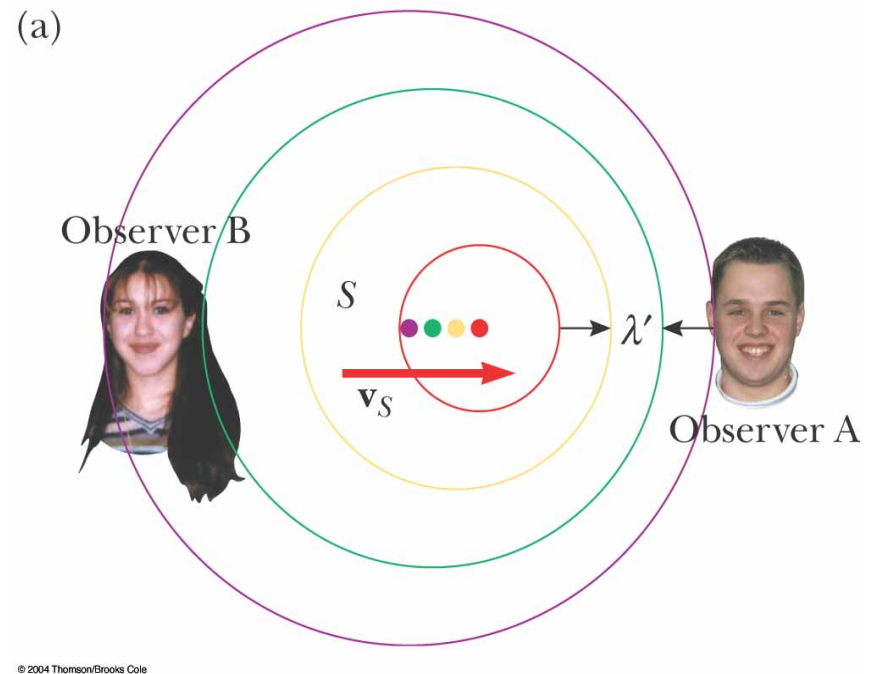
$$f' = \left( \frac{v + v_o}{v} \right) f$$

- The frequency heard by the observer,  $f'$ , appears lower when the observer moves away from the source

$$f' = \left( \frac{v - v_o}{v} \right) f$$

# Doppler Effect, Source Moving

- Consider the source being in motion while the observer is at rest
- As the source moves toward the observer, the wavelength appears shorter
- As the source moves away, the wavelength appears longer



# Doppler Effect, Source Moving, cont

- When the source is moving toward the observer, the apparent frequency is higher

$$f' = \left( \frac{v}{v - v_s} \right) f$$

- When the source is moving away from the observer, the apparent frequency is lower

$$f' = \left( \frac{v}{v + v_s} \right) f$$

# Doppler Effect for EM Waves

There is similar effect for Electromagnetic Waves.

If the source and observer are in relative motion with respect to each other, this leads to longitudinal Doppler Effect for light. The observed wavelength ( $\lambda'$ ) is related to the emitted wavelength ( $\lambda$ ) of the moving source by the following formula

$$\lambda' = \sqrt{\left( \frac{1 \pm \frac{v_s}{c}}{1 \mp \frac{v_s}{c}} \right)} \lambda$$

Upper signs are taken for the receding source (light is red shifted).

Lower signs are taken for the approaching source (light is blue shifted).

Observation of red shift of majority of the galactic sources discovered by Edwin Hubble led Physicists to accept the Expanding Universe Model.

# Superposition Principle

- If two or more traveling waves are moving through a medium, the resultant value of the wave function at any point is the algebraic sum of the values of the wave functions of the individual waves
- Waves that obey the superposition principle are **linear waves**
  - For mechanical waves, linear waves have amplitudes much smaller than their wavelengths

## Superposition and Interference

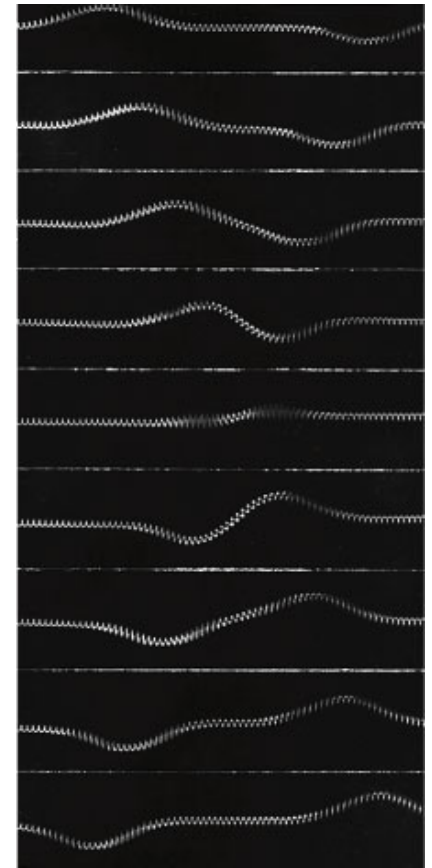
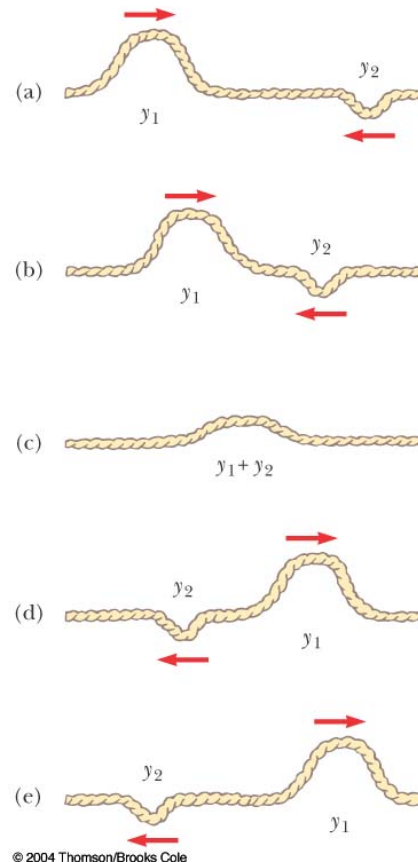
- Two traveling waves can pass through each other without being destroyed or altered
  - A consequence of the superposition principle
- The combination of separate waves in the same region of space to produce a resultant wave is called **interference**

# Types of Interference

- **Constructive interference** occurs when the displacements caused by the two pulses are of the same sign
  - The amplitude of the resultant pulse is greater than either individual pulse
- **Destructive interference** occurs when the displacements caused by the two pulses are of opposite signs
  - The amplitude of the resultant pulse is less than either individual pulse

# Destructive Interference Example

- Two pulses traveling in opposite directions
- Their displacements are inverted with respect to each other
- When they overlap, their displacements partially cancel each other



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# Superposition of Sinusoidal Waves

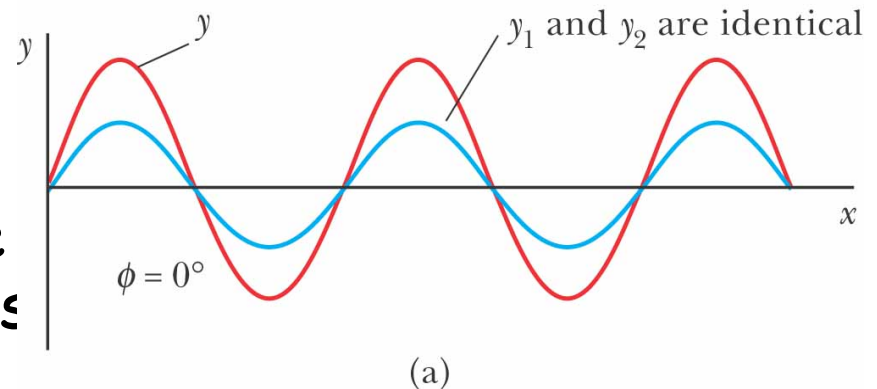
- Assume two waves are traveling in the same direction, with the same frequency, wavelength and amplitude
- The waves differ in phase
- $y_1 = A \sin(kx - \omega t)$
- $y_2 = A \sin(kx - \omega t + \phi)$
- $y = y_1 + y_2$   
 $= 2A \cos(\phi/2) \sin(kx - \omega t + \phi/2)$

# Superposition of Sinusoidal Waves, cont

- The resultant wave function,  $y$ , is also sinusoidal
- The resultant wave has the same frequency and wavelength as the original waves
- The amplitude of the resultant wave is  $2A \cos(\phi/2)$
- The phase of the resultant wave is  $\phi/2$

# Sinusoidal Waves with Constructive Interference

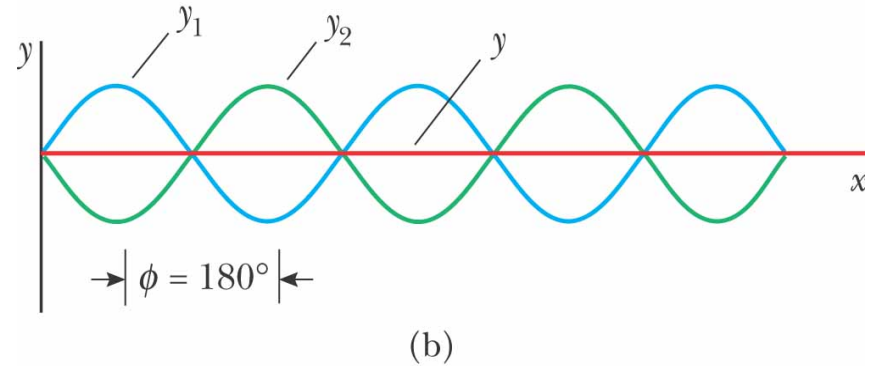
- When  $\phi = 0$ , then  $\cos(\phi/2) = 1$
- The amplitude of the resultant wave is  $2A$ 
  - The crests of one wave coincide with the crests of the other wave
- The waves are everywhere in phase
- The waves interfere constructively



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# Sinusoidal Waves with Destructive Interference

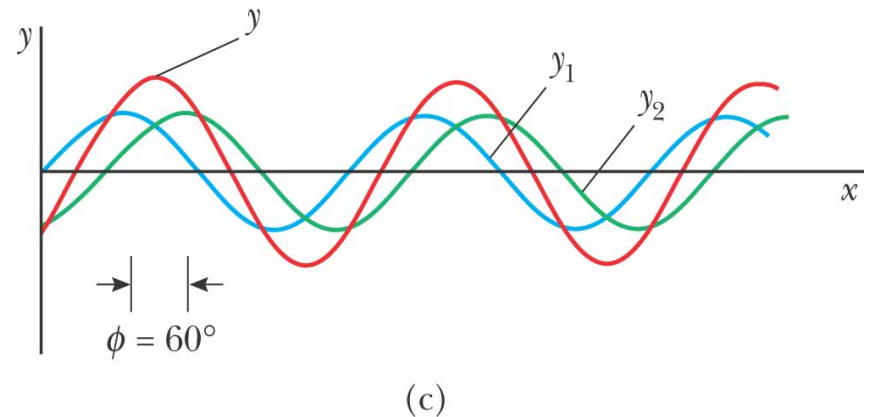
- When  $\phi = \pi$ , then  $\cos(\phi/2) = 0$
- Also any odd multiple of  $\pi$
- The amplitude of the resultant wave is 0
  - Crests of one wave coincide with troughs of the other wave
- The waves interfere destructively



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# Sinusoidal Waves, General Interference

- When  $\phi$  is other than 0 or an even multiple of  $\pi$ , the amplitude of the resultant is between 0 and  $2A$
- The wave functions still add



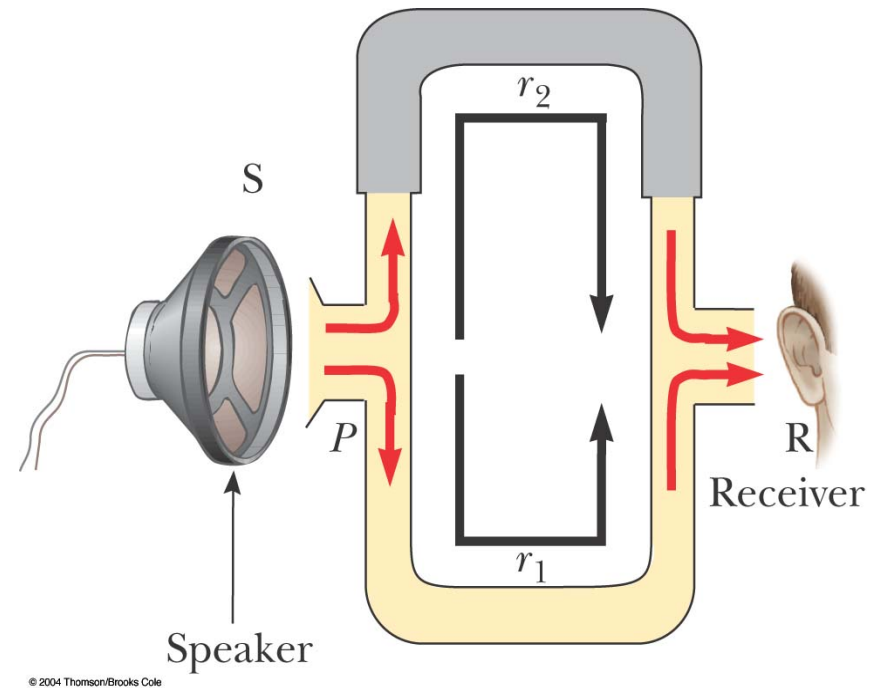
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# Sinusoidal Waves, Summary of Interference

- Constructive interference occurs when  $\phi = 0$ 
  - Amplitude of the resultant is  $2A$
- Destructive interference occurs when  $\phi = n\pi$  where  $n$  is an odd integer
  - Amplitude is  $0$
- General interference occurs when  $0 < \phi < n\pi$ 
  - Amplitude is  $0 < A_{\text{resultant}} < 2A$

# Interference in Sound Waves

- Sound from S can reach R by two different paths
- The upper path can be varied
- Whenever  $\Delta r = |r_2 - r_1| = n\lambda$  ( $n = 0, 1, \dots$ ), constructive interference occurs



# Standing Waves

- Assume two waves with the same amplitude, frequency and wavelength, traveling in opposite directions in a medium
- $y_1 = A \sin(kx - \omega t)$  and  $y_2 = A \sin(kx + \omega t)$
- They interfere according to the superposition principle

The resultant wave will be

$$y = (2A \sin kx) \cos \omega t$$

This is the wave function of a standing wave

- There is no  $kx - \omega t$  term, and therefore it is not a traveling wave

In observing a standing wave, there is no sense of motion in the direction of propagation of either of the original waves

PHY1322 Winter2015  
Dr.A. Czajkowski



# Note on Amplitudes

- There are three types of amplitudes used in describing waves
  - The amplitude of the individual waves,  $A$
  - The amplitude of the simple harmonic motion of the elements in the medium,  
 $2A \sin kx$
  - The amplitude of the standing wave,  $2A$ 
    - A given element in a standing wave vibrates within the constraints of the envelope function  $2A \sin kx$ , where  $x$  is the position of the element in the medium

## Standing Waves, Particle Motion

- Every element in the medium oscillates in simple harmonic motion with the same frequency,  $\omega$
- However, the amplitude of the simple harmonic motion depends on the location of the element within the medium

# Standing Waves, Definitions

- A **node** occurs at a point of zero amplitude
  - These correspond to positions of  $x$  where

$$x = \frac{n\lambda}{2} \quad n = 0, 1, 2, \dots$$

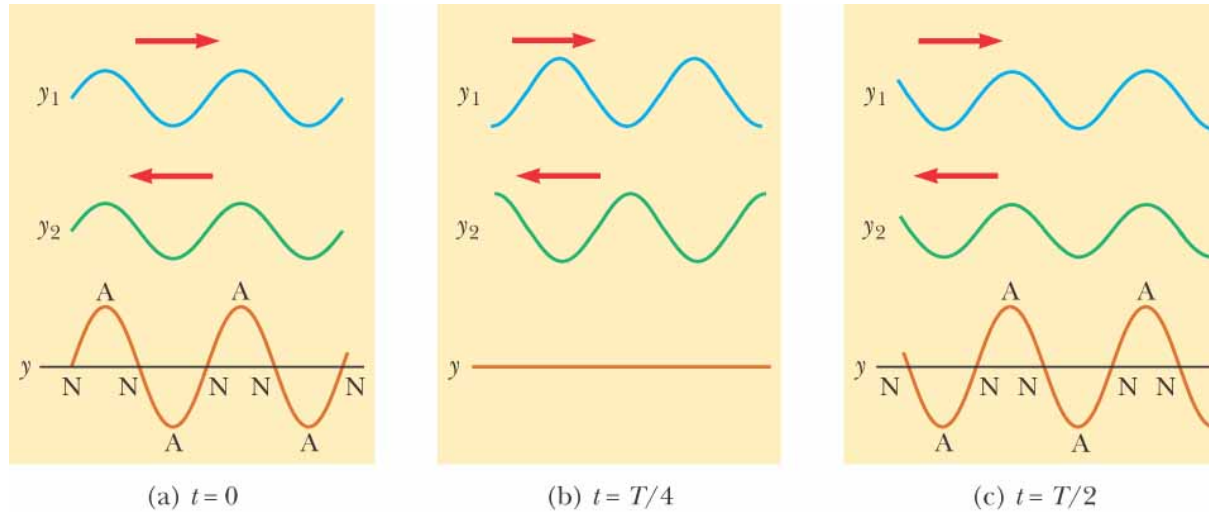
- An **antinode** occurs at a point of maximum displacement,  $2A$ 
  - These correspond to positions of  $x$  where

$$x = \frac{n\lambda}{4} \quad n = 1, 3, 5, \dots$$

# Features of Nodes and Antinodes

- The distance between adjacent antinodes is  $\lambda/2$
- The distance between adjacent nodes is  $\lambda/2$
- The distance between a node and an adjacent antinode is  $\lambda/4$

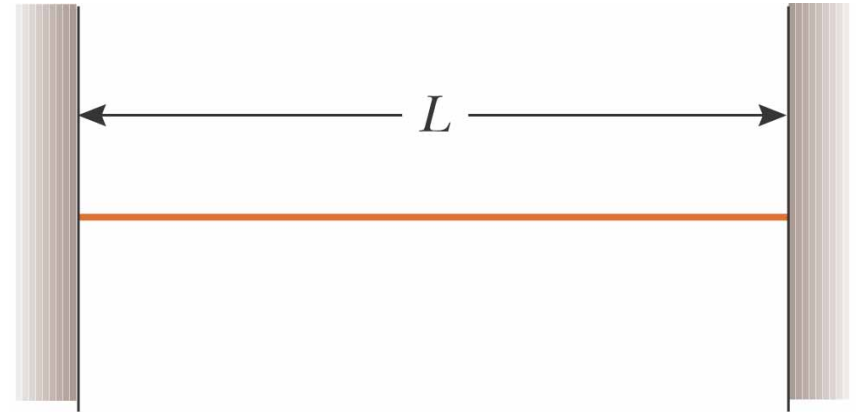
# Nodes and Antinodes, cont



- The diagrams above show standing-wave patterns produced at various times by two waves of equal amplitude traveling in opposite directions
- In a standing wave, the elements of the medium alternate between the extremes shown in (a) and (c)

# Standing Waves in a String

- Consider a string fixed at both ends
- The string has length  $L$
- Standing waves are set up by a continuous superposition of waves incident on and reflected from the ends
- There is a boundary condition on the waves



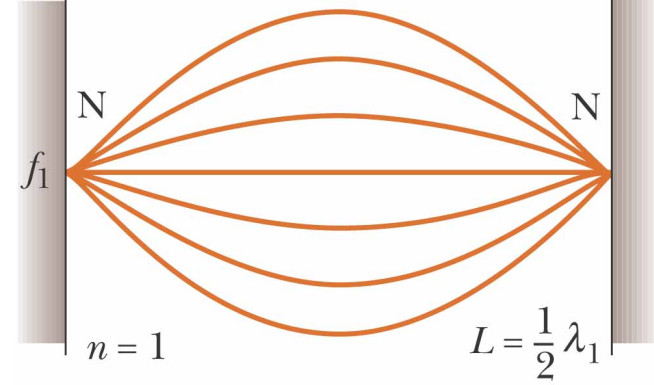
(a)

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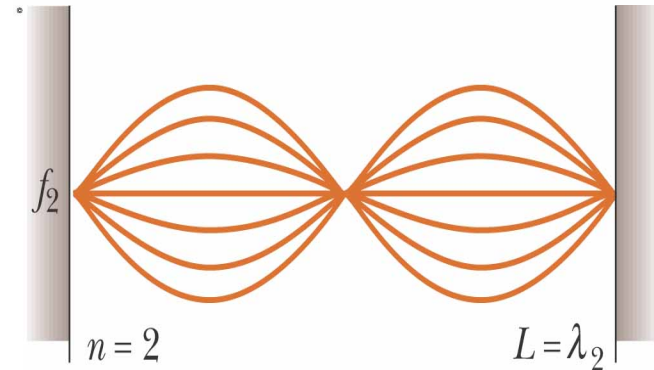
- The ends of the strings must necessarily be nodes
  - They are fixed and therefore must have zero displacement
- The boundary condition results in the string having a set of **normal modes** of vibration
  - Each mode has a characteristic frequency
  - The normal modes of oscillation for the string can be described by imposing the requirements that the ends be nodes and that the nodes and antinodes are separated by  $\lambda/4$

# Standing Waves in a String, cont.

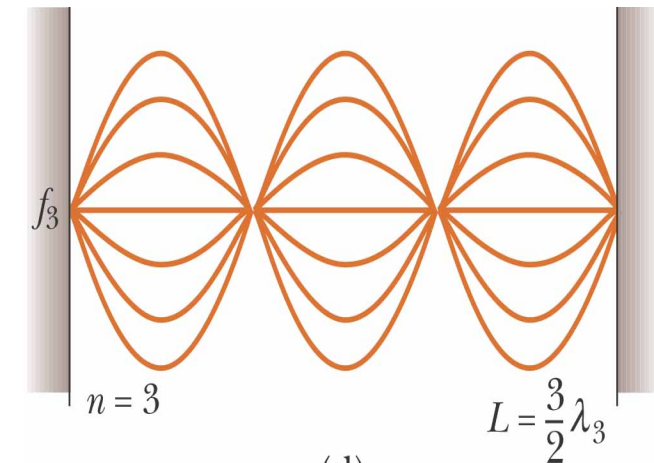
- This is the first normal mode that is consistent with the boundary conditions
- There are nodes at both ends
- There is one antinode in the middle
- This is the longest wavelength mode
  - $\frac{1}{2}\lambda = L$  so  $\lambda = 2L$
- Consecutive normal modes add an antinode at each step
- The second mode (c) corresponds to  $l = L$
- The third mode (d) corresponds to  $l = 2L/3$



(b)



(c)



(d)

# Standing Waves on a String, Summary

- The wavelengths of the normal modes for a string of length  $L$  fixed at both ends are  $\lambda_n = 2L / n$   $n = 1, 2, 3, \dots$ 
  - $n$  is the  $n^{\text{th}}$  normal mode of oscillation
  - These are the possible modes for the string
- The natural frequencies are 
$$f_n = n \frac{v}{2L} = \frac{n}{2L} \sqrt{\frac{T}{\mu}}$$
- The **fundamental frequency** corresponds to  $n = 1$ 
  - It is the lowest frequency,  $f_1$
- The frequencies of the remaining natural modes are integer multiples of the fundamental frequency
  - $f_n = n f_1$
- Frequencies of normal modes that exhibit this relationship form a **harmonic series**
- The normal modes are called **harmonics**
- This situation in which only certain frequencies of oscillation are allowed is called **quantization**
- Quantization is a common occurrence when waves are subject to boundary conditions

# Standing Waves in Air Columns

- Standing waves can be set up in air columns as the result of interference between longitudinal sound waves traveling in opposite directions
- The phase relationship between the incident and reflected waves depends upon whether the end of the pipe is opened or closed

## Standing Waves in Air Columns, Closed End

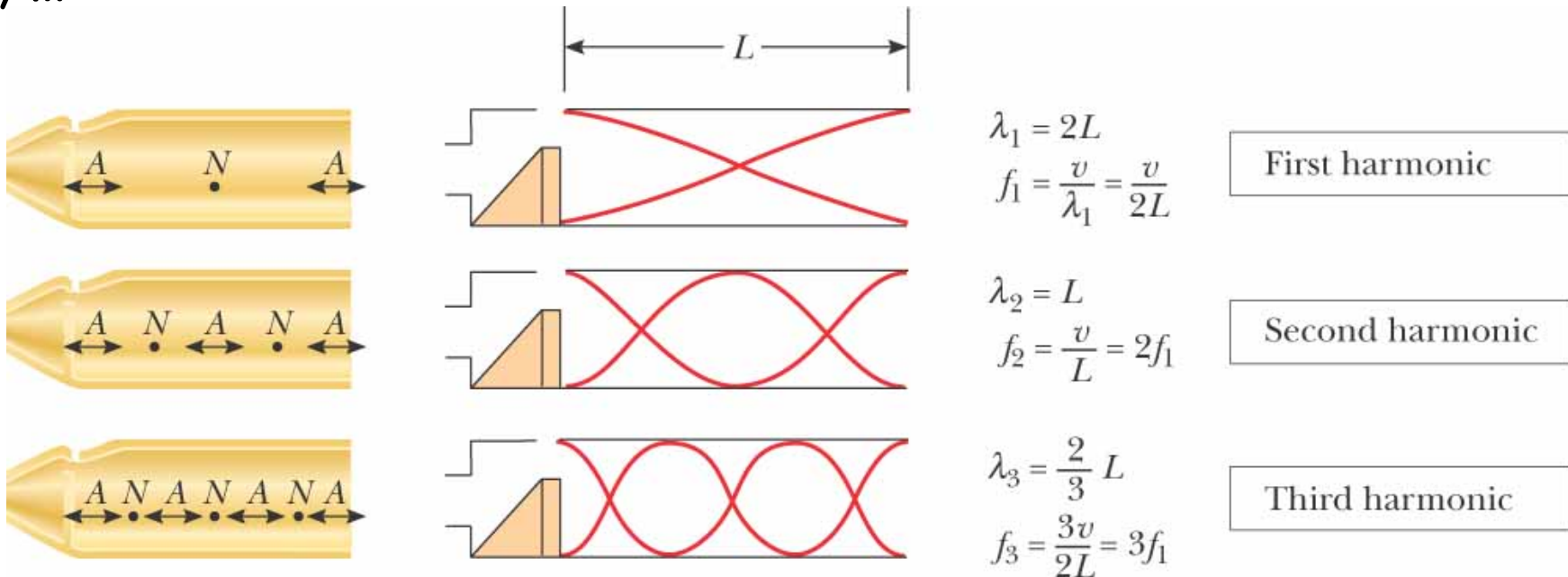
- A closed end of a pipe is a displacement node in the standing wave
  - The wall at this end will not allow longitudinal motion in the air
  - The reflected wave is  $180^\circ$  out of phase with the incident wave
- The closed end corresponds with a pressure antinode
  - It is a point of maximum pressure variations

## Standing Waves in Air Columns, Open End

- The open end of a pipe is a displacement antinode in the standing wave
  - As the compression region of the wave exits the open end of the pipe, the constraint of the pipe is removed and the compressed air is free to expand into the atmosphere
- The open end corresponds to a pressure node
  - It is a point of no pressure variation

# Standing Waves in an Open Tube

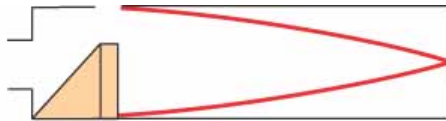
- Both ends are displacement antinodes
- The fundamental frequency is  $v/2L$ 
  - This corresponds to the first diagram
- The higher harmonics are  $f_n = nf_1 = n(v/2L)$  where  $n = 1, 2, 3, \dots$



(a) Open at both ends

# Standing Waves in a Tube Closed at One End

- The closed end is a displacement node
- The open end is a displacement antinode
- The fundamental corresponds to  $\frac{1}{4}\lambda$
- The frequencies are  $f_n = nf = n(v/4L)$  where  $n = 1, 3, 5, \dots$



$$\lambda_1 = 4L$$

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$$

First harmonic



$$\lambda_3 = \frac{4}{3}L$$

$$f_3 = \frac{3v}{4L} = 3f_1$$

Third harmonic



$$\lambda_5 = \frac{4}{5}L$$

$$f_5 = \frac{5v}{4L} = 5f_1$$

Fifth harmonic

(b) Closed at one end, open at the other

# Standing Waves in Air Columns, Summary

- In a pipe open at both ends, the natural frequencies of oscillation form a harmonic series that includes all integral multiples of the fundamental frequency
- In a pipe closed at one end, the natural frequencies of oscillations form a harmonic series that includes only odd integral multiples of the fundamental frequency

# Follow-up problems

- 1 Sub A (source) travels at 8.00 m/s emitting at a frequency of 1400 Hz  
Sub B (observer) travels at 9.00 m/s. What is the apparent frequency heard by the observer as the subs approach each other? Then as they recede from each other?  
The speed of sound is 1533 m/s
- 2 What is the length of a closed organ pipe with a fundamental frequency of 24Hz?  
What is the length of open organ pipe that has a fifth harmonic frequency of 500Hz?  
What is the length of the string with a second harmonic frequency of 1200Hz?
- 3 A tuning fork vibrating at 512 Hz falls from rest and accelerates at 9.80 m/s<sup>2</sup>. How far below the point of release is the tuning fork when waves of frequency 485 Hz reach the release point? Take the speed of sound in air to be 340 m/s.
- 4 A police car whose siren has a natural frequency of 500Hz approaches a large wall at 30km/h. A stationary observer detects the direct and reflected waves. What is the beat frequency?
- 5 Two small speakers emit sound waves of different frequencies. Speaker A has an output of 1.00 mW and speaker B has an output of 1.50 mW. Determine the sound level (in dB) at point C if (a) only speaker A emits sound, (b) only speaker B emits sound, (c) both speakers emit sound.

