Concordia University

Faculty of Engineering and Computer Science
ENGR 244 Mechanics of Materials Midterm (12.5%)
March 19th, 2013 1:15 to 2:30 PM
Drs. Ahmed (Sec. T), Bhowmick (Sec. X), El Baky (Sec. M)

Special Instructions:

Answer all 3 questions. All Questions carry equal marks. Total Marks (40 pts.) Show all your work. In order to qualify for partial grade you must present your work neatly with explanations. Provide complete answer in the space provided with units.

Name: Solution ID: _____ Section: _____

Formulas

$$\delta = \frac{PL}{AE} \qquad \tau = G\gamma \qquad \sigma = E\varepsilon \qquad \varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

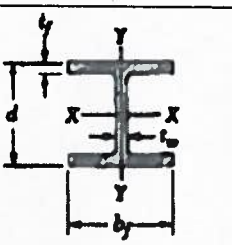
$$\tau_x = \frac{Ty}{\Sigma(x^2 + y^2)A} \qquad \phi = \frac{TL}{JG} \qquad G = \frac{E}{2(1+\nu)}$$

$$\tau_y = \frac{Tx}{\Sigma(x^2 + y^2)A} \qquad \sigma = -\frac{My}{I} \qquad \tau = \frac{Tc}{J} \qquad P = T\omega = 2\pi f T$$

$$M_P = SF_Y \qquad q = \frac{VQ}{I} \qquad \tau = \frac{VQ}{It} \qquad F_x = \frac{Ty}{\Sigma(x^2 + y^2)} \qquad F_y = \frac{Tx}{\Sigma(x^2 + y^2)}$$

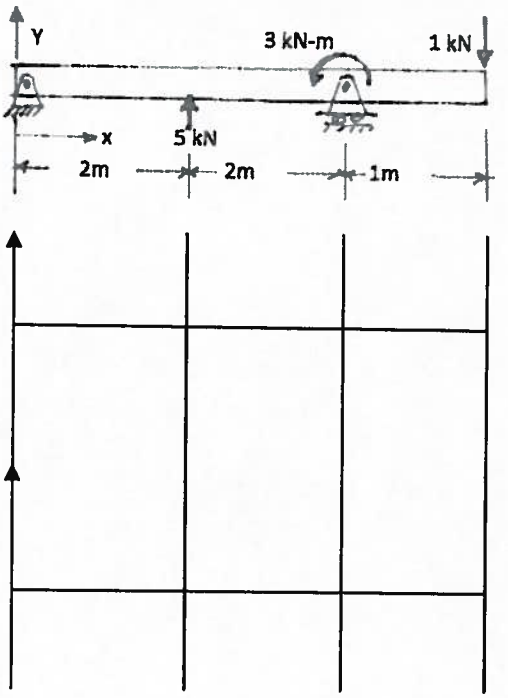
Material	Density (kg/m^3)	Ultimate Strength			Yield Strength		Modulus of Elasticity (GPa)	Modulus of Rigidity (GPa)
		Tension (MPa)	Compression (MPa)	Shear (MPa)	Tension (MPa)	Shear (MPa)		
Steel	7860	400	420	350	200	145	200	77.2
Aluminum	2719	100	110	70	95	55	70	26

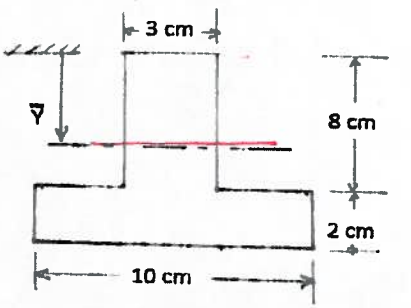
Beam W200	Area mm^2	Depth d, mm	Width b_f, mm	Thick t_f, mm	Web t_w, mm	I_x (10^6) mm^4	S_x (10^3) mm^3	I_y (10^6) mm^4	S_y (10^3) mm^3
X 86	11000	222	209	20.6	13	94.7	853	31.4	300
X 71	9100	216	206	17.4	10.2	76.6	709	25.4	274
X 59	7560	210	205	14.2	9.1	61.1	582	20.4	199



Question 1. (30 pts.) (Do your work on the adjacent page and provide only your final answers here for the given units or appropriate unit if not given).

a) For the given beam, its cross-section and loading as shown, draw a properly labeled shear force and Bending Moment diagrams in the space provided. (7 pts.)





BEAM CROSS-SECTION

Answer the following Questions by filling up the blanks: (2 pts.)

V_{max} = ____3____ kN

Its location is : ___2__ < x < ___4___ m.

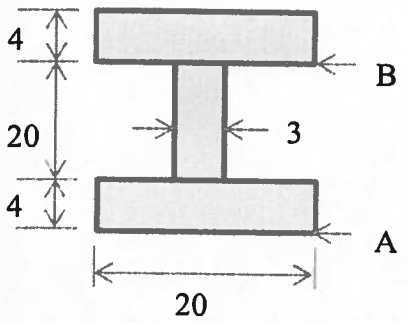
M_{max} = ____4____ kN-m

Its location is : x = ____2____ m.

For the given cross-section of the beam above, determine the location of the neutral axis from the top.

The neutral axis is at: $\bar{Y}$ = ____6.27 cm____ (3 pts.)

b) For the given cross section in the left, determine I_x about the neutral axis. Also determine the values of Q for shear stress at point A , at point B and for maximum stress. ***(All dimensions in cm).***



I_x = ____25253.33 cm^4____ (3 pts.)

Q_A = ____0__ (2 pts.)

Q_B = ____960 cm^3____ (2 pts.)

Q_{max} = ____1110 cm^3____ (2 pts.)

(Do your work on the adjacent page and provide only your final answers here for the given units or appropriate unit if not given).

c) A standard steel beam cross section W 200 X 71 is to be subjected to vertical loading. The Q_{max} is calculated to be 397.8×10^3 mm^3 and the maximum share force experienced by the beam is known to be 392.82 kN. Determine the maximum shear stress experience by the beam.

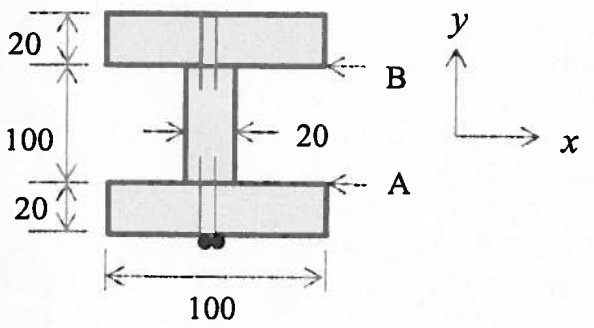
τ_{max} = _____200___MPa_ (3 pts.)

For the beam in part (c), what is the largest bending moment that can be applied if a factor of safety, FOS = ***1.418*** is desired.

M_{max} = _____200____KN-m_ (3 pts.)

d) The following wooden beam is made of three planks, *20 by 100 mm* in cross section. The beam is formed by nailing them using two nails at every *25 mm* along the length of the beam as shown. Determine the **shearing force** in each nail if the vertical shear in the beam is *1000 N*.

●●



(All dimensions in mm).

In order to save time, the followings are pre calculated:

Moment of Inertia about Neutral axis:

$I_x = 16.20 \times 10^{-6}\ m^4$, $I_y = 3.40 \times 10^{-6}\ m^4$

Cross sectional area, $A = 0.006\ m^2$

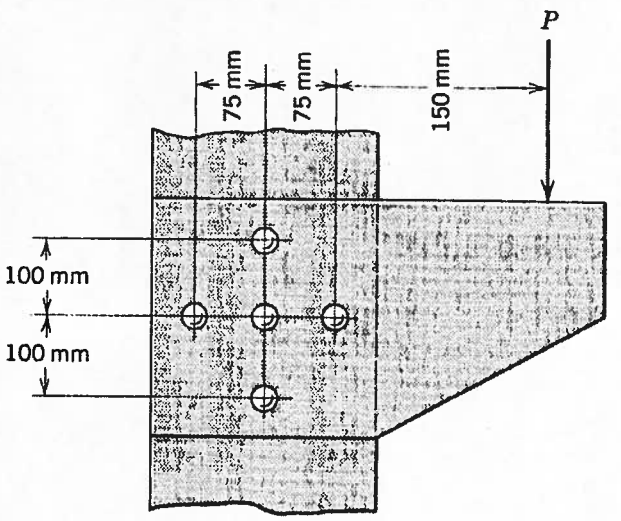
$Q_{max} = 145 \times 10^{-6}\ m^3$, $Q_A = Q_B = 120 \times 10^{-6}\ m^3$

The nail is made of steel (for property, see table)

The shear force in each nail is: **92.6 N** (4)

Question # 2 (10 pts.)

The 20 mm plate is to be bolted to the vertical support using only three bolts (not 5 as shown). The bolt material is aluminum. Determine the minimum diameter of the bolt required to support a vertical load P = 15 KN if factor of safety FOS = 2.5 is desired.



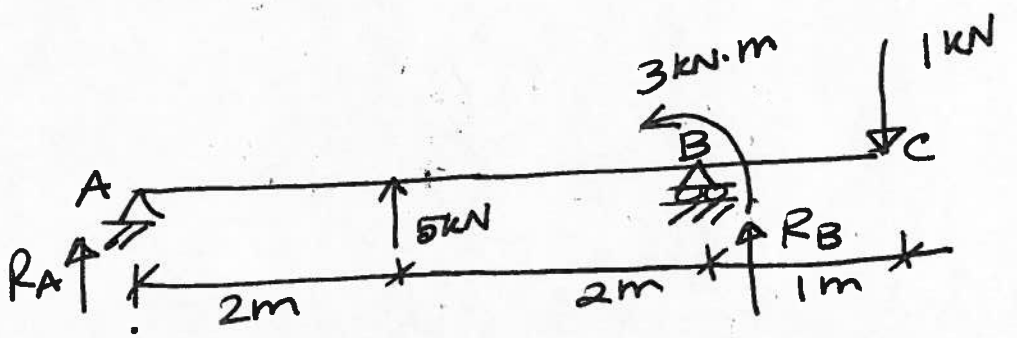
a) If only three vertical bolts are used.
b) If only three horizontal bolts are used.

ANSWERS:

a) Min Dia. of Bolt. **28.3 mm**

b) Min Dia. of Bolt. **35.36 mm**

Question 1.



$\Sigma M_A = 0$ ↻ (+)

$3 + 5*2 - 1*5 + R_B*4 = 0$

$13 - 5 + R_B*4 = 0$

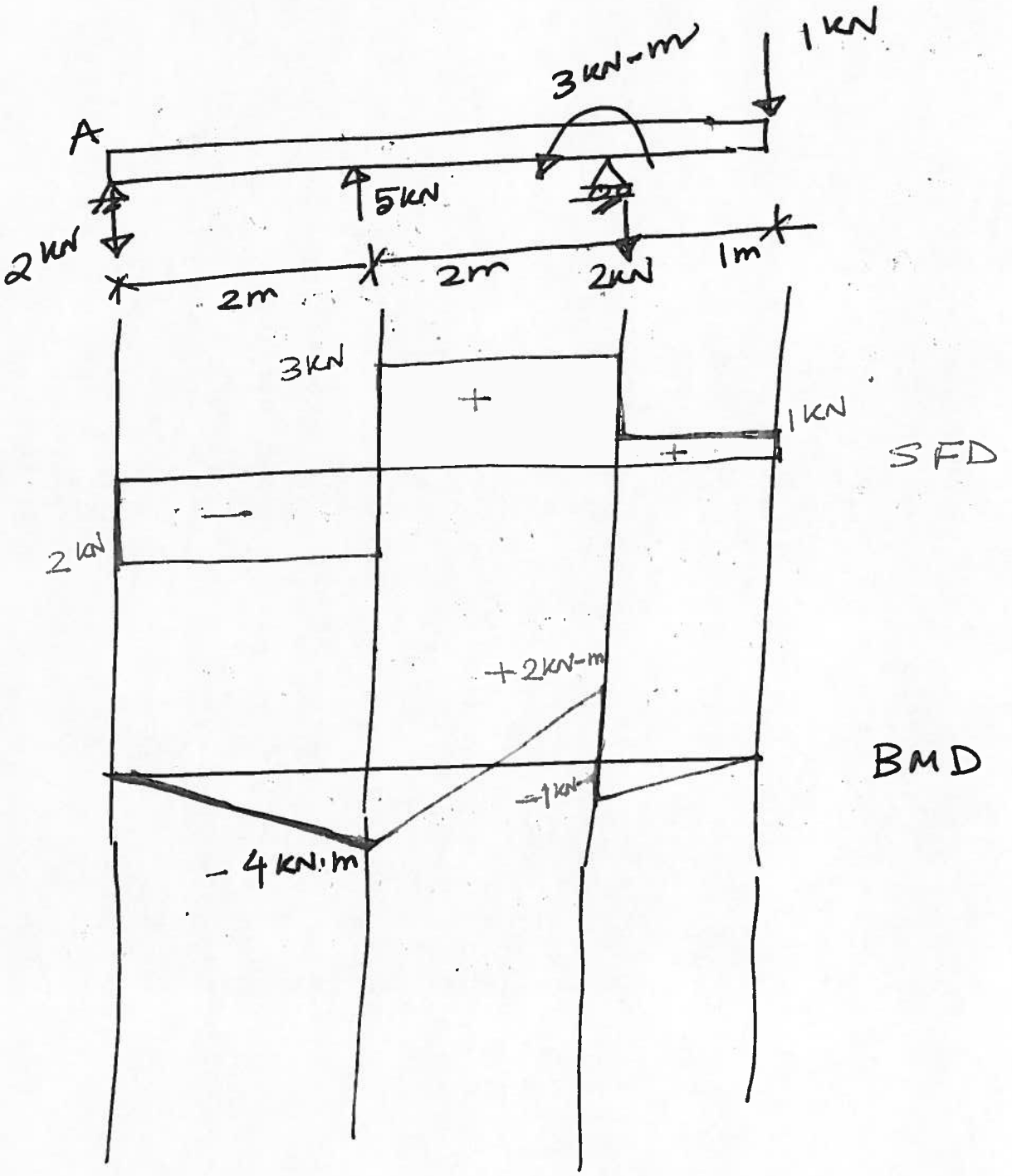
$R_B = -2$ kN (↓)

$\Sigma F_y = 0$,

$R_A + R_B + 5 - 1 = 0$

$R_A - 2 + 4 = 0$

$R_A = -2$ kN (↓)

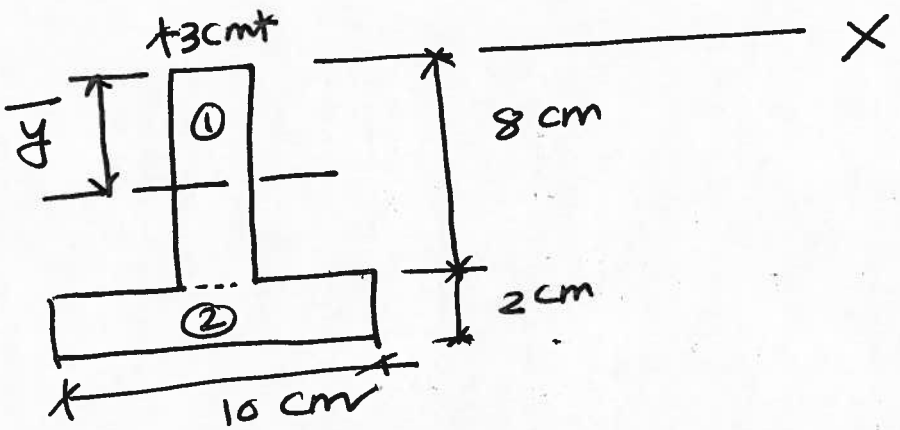


$V_{max} = 3$ KN

Its location is: $2m < x < 4m$

$M_{max} = 4$ KN.m

Its location is: $x = 2$ m



$A_1 = 8 \times 3 = 24\ cm^2$

$A_2 = 10\ cm \times 2cm = 20\ cm^2$

$\bar{y}_1 = 8/2 = 4\ cm$

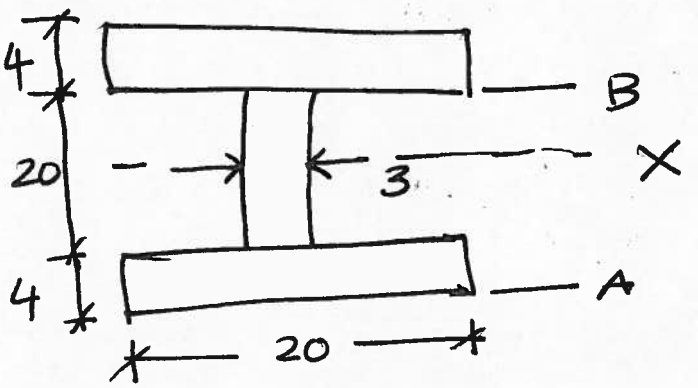
$\bar{y}_2 = \frac{2}{2} + 8 = 9\ cm$

$$\bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2}{A_1 + A_2}$$

$$= \frac{24*4 + 20*9}{24+20}$$

$= 6.27\ cm$ (from above)

(b)



$Q_{max} = Q$ about NA

$$Q_A = A \cdot \bar{y}$$
$$= 0 * y$$
$$= 0$$

$$I_x = \frac{3*20^3}{12} + 2\left[\frac{20*4^3}{12} + 80*(10+2)^2\right]$$

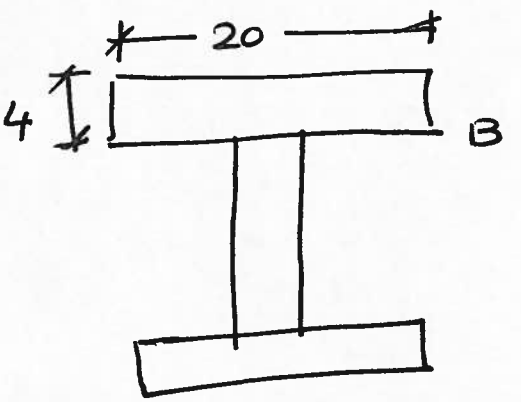
$$= 2000 + 2\left[11626.67\right]$$

$$= 25253.33 \text{ cm}^4$$

$$Q_B = A \cdot \bar{y}$$

$$= 80 * \left(\frac{20}{2} + \frac{4}{2}\right)$$

$$= 960 \text{ cm}^3$$

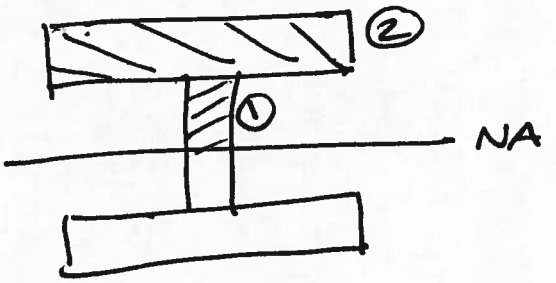


$$Q_{max} = A_1 Q_{NA}$$

$$= A_1 \bar{y}_1 + A_2 \bar{y}_2$$

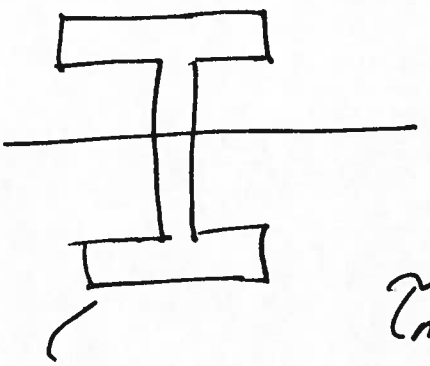
$$= (10*3)*5 + (20*4)*12$$

$$= 150 + 960 = 1110 \text{ cm}^3$$



(c) $Q_{max} = 397.8 * 10^2 \ mm^3$

$V_{max} = 392.82 \ kN$



$$\tau_{max} = \frac{VQ}{It}$$

$$\tau_{max} = \frac{392.82 * 10^3 * 397.8 * 10^2}{76.6 * 10^6 * 10.2}$$

$= 19.99$

$\simeq 20 \ MPa$

W200×71

$I_x = 76.6 * 10^6 \ mm^4$

$t = 10.2$

(d) $F.O.S = 1.418$

$$\sigma_{max}(t) = \frac{M * 216/2}{76.6 * 10^6}$$

$$\frac{400}{1.418} = \frac{M * 108}{76.6 * 10^6}$$

$M = 200 \ kN.m$ (for max tension)

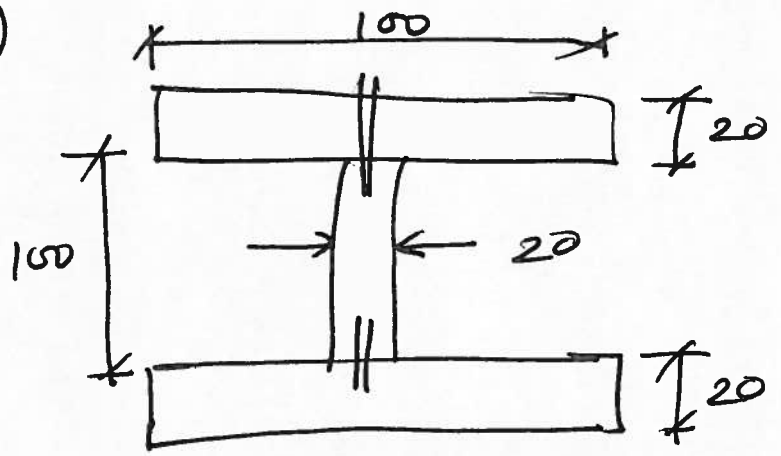
For max^m compression

$$\frac{420}{1.418} = \frac{M + 216/2}{766 * 10^6}$$

$$M = 210 \text{ kN.m}$$

$$M_{max} = 200 \text{ kN.m}$$

(d)



$$q = \frac{VQ}{I}$$

$$= \frac{1000 * 120 * 10^{-6} * (100)^3}{16.20 * 10^{-6} * (100)^4}$$

$$= 7.4 \text{ N/mm}$$

Shear force in each nail

Since we have two nail

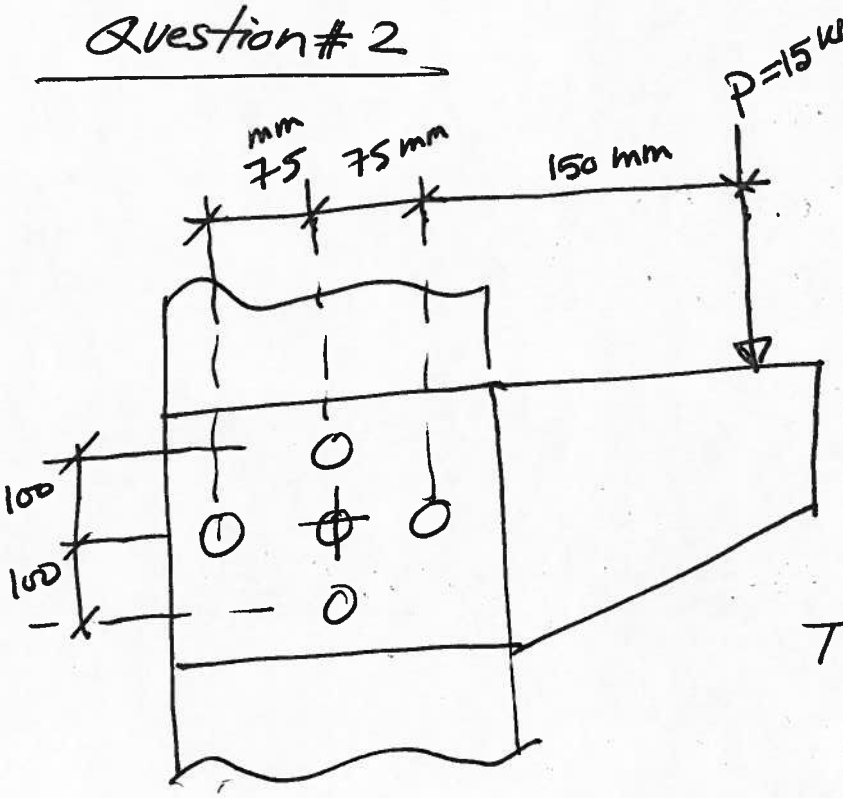
$$= \frac{7.4}{2} * 25 \text{ mm}$$

$$= 92.6 \text{ N}$$

Shear force = 92.6 N in each nail

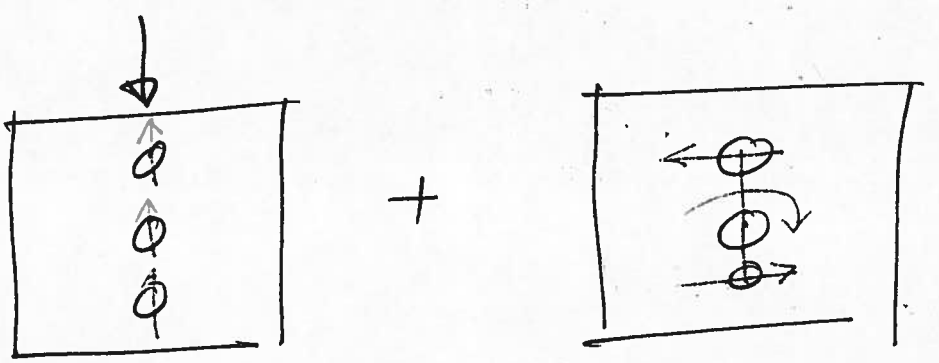
Question # 2

F.O.S = 2.5



$$T = 15 * 10^3 * (150 + 75)$$
$$= 3.375 * 10^6 \ N.mm$$

(a) When three vertical bolts are use



Direct shear

$$f_{dx} = 0$$

$$f_{dy} = \frac{15 * 10^3}{3A} = \frac{5000}{A} N$$

$$f_{ty} = \frac{T x_i}{\sum (x_i^2 + y_i^2) A}$$

$$= 0$$

since $x_i = 0$

$$f_{tx} = \frac{T y_i}{A \sum (x_i^2 + y_i^2)}$$

$$= \frac{3.375 \times 10^6 \times 100}{A\left(100^2 + 100^2\right)}$$

$$= \frac{16875}{A}$$

For most critical bolt

$$f_t = \sqrt{\left\{f_{dx_i} + f_{tx_i}\right\}^2 + \left\{\left(f_{dy_i} + f_{ty_i}\right)\right\}^2}$$

$$f_i = \sqrt{\left(\frac{16875}{A}\right)^2 + \left(\frac{5000}{A}\right)^2}$$

$$\frac{70\ MPa}{2.5} = \frac{1}{A}\sqrt{(16875)^2 + (5000)^2}$$

$$= \frac{17600.2}{A}$$

$$A = 628.577$$

$$\frac{\pi}{4} d^2 = 628.577$$

$$\boxed{d = 28.3\ mm}$$