MAT 2377, Probability and Statistics for Engineers

Assignment 1
Professor: Termeh Kousha
Deadline: **Friday, Jan 29, 2016 3pm (Math department (585 KED) drop boxes)**

Late assignments will NOT be accepted; nor will unstapled assignments. Professors in the math department will not lend you a stapler.

Student Name ____________________________
Student Number ____________________________
By signing below, you declare that this work was your own and that you have not copied from any other individual or other source.

Signature ____________________________

Total=50

1. A printed circuit board has eight different locations in which a component can be replaced. If five identical components are to be placed on the board, how many different designs are possible?

   Solution: Each design is a subset of the eight locations that are to contain the component. Thus, the number of possible designs is
$$\frac{8!}{5!3!}$$

   3 points.

2. In a group of 16 candidates for laboratory research positions, 7 are chemists and 9 are physicians. In how many ways can we choose 2 chemists and 3 physicians?

   Solution
$$\binom{7}{2} * \binom{9}{3} = 21 * 84 = 1764$$

   2 point for use of binomial coefficients. 1 point for the correct answer. Total - 3 points.

3. A bin of 50 manufactured parts contains 3 defective parts and 47 non-defective parts. A sample of size 6 parts is selected from 50 parts. Selected parts are not replaced. How many different samples are there of size six that contain exactly 2 defective parts? What is the probability that a sample contains exactly 2 defective parts?

   Solution
$$\binom{3}{2} * \binom{47}{4} = 3 * 178365 = 535095$$

The total number of different subsets of size 6 is :

$$\binom{50}{6} = 15890700$$

Therefore, the probability that a sample contains exactly 2 defective parts is $\frac{535095}{15890700} = 0.034$

3 points for the first part. 2 points for finding the total and 2 points for calculating the probability. Total 7 points.

4. Pieces of aluminium are classified according to the finishing of the surface and according to the finishing of edge. The results from 85 samples are summarized as follows:

	Edge	
Surface	excellent	good
excellent	60	5
good	16	4

Let A denote the event that a selected piece has "excellent" surface, and let B denote the event that a selected piece has "excellent" edge. If samples are selected randomly, determine the following probabilities:

(a) $P(A)$

(b) $P(B)$

(c) $P(A^c)$

(d) $P(A \cap B)$

(e) $P(A \cup B)$

(f) $P(A^c \cup B)$

(g) If the selected piece has excellent edge finishing, what is the probability that it has excellent surface finishing?

(h) If the selected piece has good surface finishing, what is the probability that it has excellent edge finishing?

(i) Are A and B independent?

solution

(a) $P(A) = 65/85$

(b) $P(B) = 76/85$

(c) $P(A^c) = 1 - 65/85 = 20/85$

(d) $P(A \cap B) = 60/85$

(e) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 65/85 + 76/85 - 60/85 = 81/85$

(f) $P(A^c \cup B) = P(A^c) + P(B) - P(A^c \cap B) = 20/85 + 76/85 - 16/85 = 80/85$

(g) $P(A|B) = P(A \cap B)/P(B) = 60/76$

(h) $P(B|A^c) = P(B \cap A^c)/P(A^c) = 16/20$

(i) $P(A|B) \neq P(A)$ - events are not independent.

1 point for each correct answer. Total - 9 points.

5. If $P(A) = 0.1$, $P(B) = 0.3$, $P(C) = 0.3$, and events A, B, C are mutually exclusive, determine the following probabilities:

(a) $P(A \cup B \cup C)$

(b) $P(A \cap B \cap C)$

(c) $P(A \cap B)$

(d) $P((A \cup B) \cap C)$

(e) $P(A^c \cap B^c \cap C^c)$

(f) $P[(A \cup B \cup C)^c]$

solution

(a) $P(A \cup B \cup C) = P(A) + P(B) + P(C) = 0.7$, since the events are mutually exclusive.

(b) $P(A \cap B \cap C) = 0$

(c) $P(A \cap B) = 0$

(d) $P((A \cup B) \cap C) = 0$

(e) $P(A^c \cap B^c \cap C^c) = P[(A \cup B \cup C)^c] = 1 - P(A \cup B \cup C) = 0.3$ (draw Venn diagram)

(f) $P[(A \cup B \cup C)^c] = 1 - P(A \cup B \cup C) = 0.3$

Correct answer for each part - 1 point. Total - 6 points.

6. The probability that an electrical switch, which is kept in dry conditions, fails during the guarantee period, is 1%. If the switch is humid, the probability that it will fail during the guarantee period is 8%. Assume that 90% of switches are kept in dry conditions, whereas remaining 10% are kept in humid conditions.

(a) What is the probability that the switch fails during the guarantee period?

(b) If the switch failed during the guarantee period, what is the probability that it was kept in humid conditions?

solution Let F-"failure", H-"humid", D-"dry". Given: $P(F|D) = 0.01$, $P(F|H) = 0.08$, $P(D) = 0.9$, $P(H) = 0.1$

(a)

$$P(F) = P(F|D)P(D) + P(F|H)P(H) = 0.01 * 0.9 + 0.08 * 0.1 = 0.009 + 0.008 = 0.017$$

(b)

$$P(H|F) = \frac{P(H \cap F)}{P(F)} = \frac{P(F|H)P(H)}{P(F)} = 0.4706$$

3 point for each part. Total - 6 points.

7. An inspector working for a manufacturing company has a 95% chance of correctly identifying defective items and 2% chance of incorrectly classifying a good item as defective. The company has evidence that its line produces 1% of defective items.

   (a) What is the probability that an item selected for inspection is classified as defective?

   (b) If an item selected at random is classified as nondefective, what is the probability that it is indeed good?

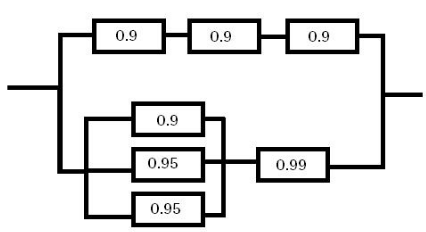
   solution Let A - the event that an item is classified as defective, D - the event that an item is defective; so that D^c is the event that an item is 'good'. What is known is: $P(D) = 0.01$; $P(A|D) = 0.95$, $P(A|D^c) = 0.02$.

   (a)
   $$P(A) = P(A \cap D) + P(A \cap D^c) = P(A|D)P(D) + P(A|D^c)P(D^c) \approx 0.0293.$$

   (b) To compute $P(D^c|A^c)$. From Bayes' formula:
   $$P(D^c|A^c) \quad = \quad \frac{P(A^c|D^c)P(D^c)}{P(A^c)} = \frac{(1 - P(A|D^c))P(D^c)}{1 - P(A)} \approx 0.3243$$

   1 point for each correct value of $P(A|D), P(D), P(A|D^c), P(A^c|D^c)$.
   For part (a): 2 point for formula $P(A|D)P(D) + P(A|D^c)P(D^c)$. For part (b): 2 point for the correct Bayes' formula. Total - 8 points.

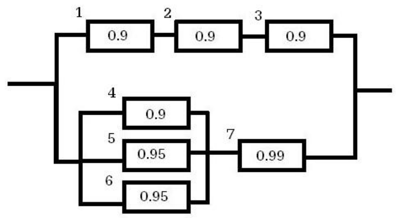
8. The following circuit operates only if there is a path of functional devices from left to right. The probability that each device functions is shown on the graph. Assume that devices fail independently. What is the probability that the circuit operates?



   solution We will number the components.

   Define the events E_i="component i works". We will decompose the circuit into sub-circuits. Consider the components 1, 2 and 3 which are assembled into series. We will denote this component as 8. So

   $$P(E_8) = P(E_1 \cap E_2 \cap E_3) = P(E_1)P(E_2)P(E_3) = (0.9)^3 = 0.729.$$



Consider the components 4, 5 and 6 which are assembled into parallel. We will denote this component as 9. So

$$\begin{aligned} P(E_9) &= P(E_4 \cup E_5 \cup E_6) \\ &= 1 - P(E_4')P(E_5')P(E_6') \\ &= 1 - (0.1)(0.05)(0.05) = 0.99975. \end{aligned}$$

Consider the components 9 and 7 which are assembled in series. We will denote this component as 10. So

$$P(E_{10}) = P(E_9 \cap E_7) = P(E_9)P(E_7) = (0.99975)(0.99) = 0.9897525.$$

The circuit is composed of component 10 and 8 which are assembled in parallel. Therefore, the probability that the circuit operates is

$$P(E_8 \cup E_{10}) = 1 - P(E_8')P(E_{10}') = 1 - (1 - 0.729)(1 - 0.9897525) = 0.9972.$$

2 point each for E8,E9 and E10. 2 points for calculating the probability that the circuit operates-Total 8 points