

Assignment 1

Assigned : Jan 11 Due Jan 25: 6:00 PM

Total Points: 50 (4 points for each correctly solved problem+10 points for presentation)

For full marks the solution should be clear and neat with clear diagram if needed

In the case of longer problems the main arguments should be presented in the space provided, while detailed calculations should be presented on the back of the page.

1. The driver of a car slams on the brakes when he sees a tree blocking the road. The car slows uniformly with an acceleration of -5.60 m/s^2 for 4.20 s , making straight skid marks 62.4 m long ending at the tree. With what speed does the car then strike the tree?

SOLUTION: This is a motion with constant a so we may use kinematic equations:

$$\left\{ \begin{array}{l} v_{xf} = v_{xi} + a_x t \\ x_f - x_i = \frac{1}{2}(v_{xi} + v_{xf})t \end{array} \right\} \text{ we have } \left\{ \begin{array}{l} v_{xf} = v_{xi} - (5.60 \text{ m/s}^2)(4.20 \text{ s}) \\ 62.4 \text{ m} = \frac{1}{2}(v_{xi} + v_{xf})(4.20 \text{ s}) \end{array} \right\}.$$

So substituting for v_{xi} gives $62.4 \text{ m} = \frac{1}{2}[v_{xf} + (56.0 \text{ m/s}^2)(4.20 \text{ s}) + v_{xf}](4.20 \text{ s})$

$$14.9 \text{ m/s} = v_{xf} + \frac{1}{2}(5.60 \text{ m/s}^2)(4.20 \text{ s}).$$

Thus

$$v_{xf} = \boxed{3.10 \text{ m/s}}.$$

2. A test rocket is fired vertically upward from a well. A catapult gives it initial velocity 80.0 m/s at ground level. Its engines then fire and it accelerates upward at 4.00 m/s^2 until it reaches an altitude of $1\,000 \text{ m}$. At that point its engines fail and the rocket goes into free fall, with an acceleration of -9.80 m/s^2 . (a) How long is the rocket in motion above the ground? (b) What is its maximum altitude? (c) What is its velocity just before it collides with the Earth? (You will need to consider the motion while the engine is operating separate from the free-fall motion)

SOLUTION:

Let point 0 be at ground level and point 1 be at the end of the engine burn. Let point 2 be the highest point the rocket reaches and point 3 be just before impact. Below are the data found for each phase of the rocket's motion.

(0 to 1) $v_f^2 - (80.0)^2 = 2(4.00)(1\,000)$ so $v_f = 120 \text{ m/s}$
 $120 = 80.0 + (4.00)t$ giving $t = 10.0 \text{ s}$

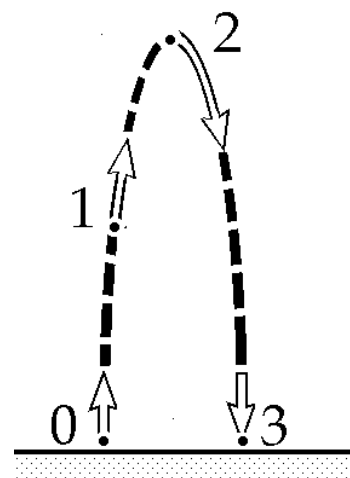
(1 to 2) $0 - (120)^2 = 2(-9.80)(x_f - x_i)$ giving $x_f - x_i = 735 \text{ m}$
 $0 - 120 = -9.80t$ giving $t = 12.2 \text{ s}$
 This is the time of maximum height of the rocket.

(2 to 3) $v_f^2 - 0 = 2(-9.80)(-1\,735)$
 $v_f = -184 = (-9.80)t$ giving $t = 18.8 \text{ s}$

(a) $t_{\text{total}} = 10 + 12.2 + 18.8 = \boxed{41.0 \text{ s}}$

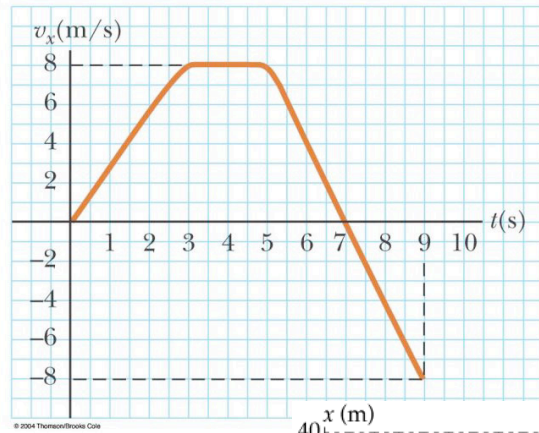
(b) $(x_f - x_i)_{\text{total}} = \boxed{1.73 \text{ km}}$

(c) $v_{\text{final}} = \boxed{-184 \text{ m/s}}$



		t	x	v	a
0	Launch	0.0	0	80	+4.00
#1	End Thrust	10.0	1 000	120	+4.00
#2	Rise Upwards	22.2	1 735	0	-9.80
#3	Fall to Earth	41.0	0	-184	-9.80

- 3 A student drives a moped along a straight road as described by the velocity-versus-time graph in Figure P2.54. Sketch this graph in the middle of a sheet of graph paper. (a) Directly above your graph, sketch a graph of the position versus time, aligning the time coordinates of the two graphs. (b) Sketch a graph of the acceleration versus time directly below the v_x - t graph, again aligning the time coordinates. On each graph, show the numerical values of x and a_x for all points of inflection. (c) What is the acceleration at $t = 6$ s? (d) Find the position (relative to the starting point) at $t = 6$ s. (e) What is the moped's final position at $t = 9$ s?



SOLUTIONS:

- (a) See the graphs at the right.

Choose $x = 0$ at $t = 0$.

$$\text{At } t = 3 \text{ s, } x = \frac{1}{2}(8 \text{ m/s})(3 \text{ s}) = 12 \text{ m.}$$

$$\text{At } t = 5 \text{ s, } x = 12 \text{ m} + (8 \text{ m/s})(2 \text{ s}) = 28 \text{ m.}$$

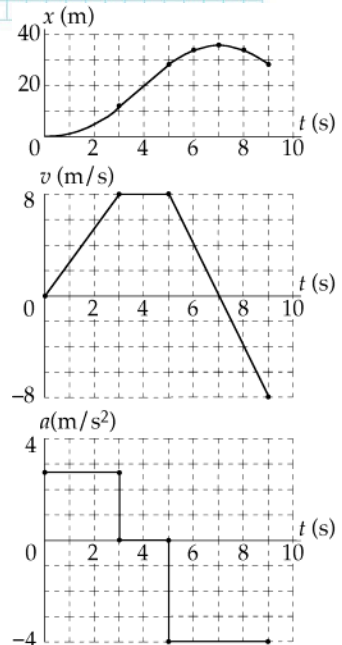
$$\text{At } t = 7 \text{ s, } x = 28 \text{ m} + \frac{1}{2}(8 \text{ m/s})(2 \text{ s}) = 36 \text{ m.}$$

- (b) For $0 < t < 3$ s, $a = \frac{8 \text{ m/s}}{3 \text{ s}} = 2.67 \text{ m/s}^2$.
For $3 < t < 5$ s, $a = 0$.

(c) For $5 < t < 9$ s, $a = -\frac{16 \text{ m/s}}{4 \text{ s}} = \boxed{-4 \text{ m/s}^2}$.

(d) At $t = 6$ s, $x = 28 \text{ m} + (6 \text{ m/s})(1 \text{ s}) = \boxed{34 \text{ m}}$.

(e) At $t = 9$ s, $x = 36 \text{ m} + \frac{1}{2}(-8 \text{ m/s})(2 \text{ s}) = \boxed{28 \text{ m}}$.



- 4 The height of a helicopter above the ground is given by $h = 3.00t^3$, where h is in meters and t is in seconds. After 2.00 s, the helicopter releases a small mailbag. How long after its release does the mailbag reach the ground?

SOLUTION:

$$y = 3.00t^3: \text{ At } t = 2.00 \text{ s, } y = 3.00(2.00)^3 = 24.0 \text{ m and}$$

$$v_y = \frac{dy}{dt} = 9.00t^2 = 36.0 \text{ m/s } \uparrow.$$

If the helicopter releases a small mailbag at this time, the equation of motion of the mailbag is

$$y_b = y_{bi} + v_i t - \frac{1}{2}gt^2 = 24.0 + 36.0t - \frac{1}{2}(9.80)t^2.$$

Setting $y_b = 0$,

$$0 = 24.0 + 36.0t - 4.90t^2.$$

Solving for t , (only positive values of t count), $\boxed{t = 7.96 \text{ s}}$.

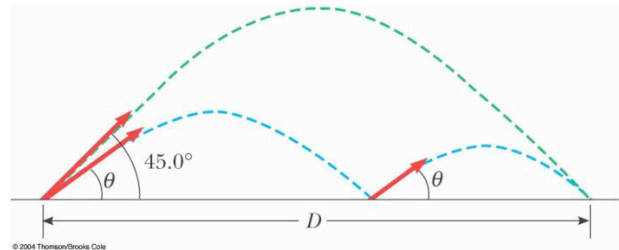
- 5 A rock is dropped from rest into a well. (a) The sound of the splash is heard 2.40 s after the rock is released from rest. How far below the top of the well is the surface of the water? The speed of sound in air (at the ambient temperature) is 336 m/s. (b) **What If?** If the travel time for the sound is neglected, what percentage error is introduced when the depth of the well is calculated? /Provide the full solution on the back page/

SOLUTION:

$$\begin{aligned}
 \text{(a)} \quad d &= \frac{1}{2}(9.80)t_1^2 & d &= 336t_2 \\
 t_1 + t_2 &= 2.40 & 336t_2 &= 4.90(2.40 - t_2)^2 \\
 4.90t_2^2 - 359.5t_2 + 28.22 &= 0 & t_2 &= \frac{359.5 \pm \sqrt{359.5^2 - 4(4.90)(28.22)}}{9.80} \\
 t_2 &= \frac{359.5 \pm 358.75}{9.80} = 0.0765 \text{ s} & \text{so} \quad d &= 336t_2 = \boxed{26.4 \text{ m}}
 \end{aligned}$$

(b) Ignoring the sound travel time, $d = \frac{1}{2}(9.80)(2.40)^2 = 28.2 \text{ m}$, an error of $\boxed{6.82\%}$

6 When baseball players throw the ball in from the outfield, they usually allow it to take one bounce before it reaches the infield, on the theory that the ball arrives sooner that way. Suppose that the angle at which a bounced ball leaves the ground is the same as the angle at which the outfielder threw it, as in Figure P4.55, but that the ball's speed after the bounce is one half of what it was before the bounce. (a) Assuming the ball is always thrown with the same initial speed, at what angle θ should the fielder throw the ball to make it go the same distance D with one bounce (blue path) as a ball thrown upward at 45.0° with no bounce (green path)? (b) Determine the ratio of the times for the one-bounce and no-bounce throws.



SOLUTION:

The special conditions allowing use of the horizontal range equation applies.

For the ball thrown at 45° ,
$$D = R_{45} = \frac{v_i^2 \sin 90}{g}$$

For the bouncing ball,
$$D = R_1 + R_2 = \frac{v_i^2 \sin 2\theta}{g} + \frac{\left(\frac{v_i}{2}\right)^2 \sin 2\theta}{g}$$

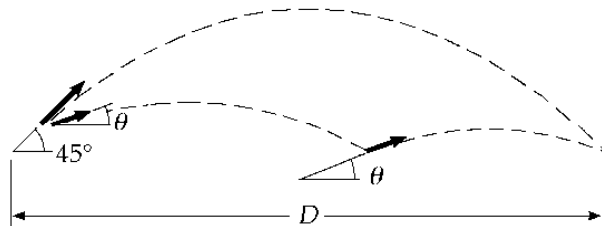
where θ is the angle it makes with the ground when thrown and when bouncing.

(a) We require:

$$\frac{v_i^2}{g} = \frac{v_i^2 \sin 2\theta}{g} + \frac{v_i^2 \sin 2\theta}{4g}$$

$$\sin 2\theta = \frac{4}{5}$$

$$\theta = \boxed{26.6^\circ}$$



(b) The time for any symmetric

parabolic flight is given by

$$y_f = v_{yi}t - \frac{1}{2}gt^2$$

$$0 = v_i \sin \theta_i t - \frac{1}{2}gt^2$$

If $t = 0$ is the time the ball is thrown, then $t = \frac{2v_i \sin \theta_i}{g}$ is the time at landing.

$$\text{So for the ball thrown at } 45.0^\circ \quad t_{45} = \frac{2v_i \sin 45.0^\circ}{g}.$$

$$\text{For the bouncing ball,} \quad t = t_1 + t_2 = \frac{2v_i \sin 26.6^\circ}{g} + \frac{2\left(\frac{v_i}{2}\right) \sin 26.6^\circ}{g} = \frac{3v_i \sin 26.6^\circ}{g}.$$

$$\text{The ratio of this time to that for no bounce is} \quad \frac{\frac{3v_i \sin 26.6^\circ}{g}}{\frac{2v_i \sin 45.0^\circ}{g}} = \frac{1.34}{1.41} = \boxed{0.949}.$$

7. A tire 0.500 m in radius rotates at a constant rate of 200 rev/min. Find the speed and acceleration of a small stone lodged in the tread of the tire (on its outer edge).

$$r = 0.500 \text{ m};$$

$$v_t = \frac{2\pi r}{T} = \frac{2\pi(0.500 \text{ m})}{\frac{60.0 \text{ s}}{200 \text{ rev}}} = 10.47 \text{ m/s} = \boxed{10.5 \text{ m/s}}$$

$$a = \frac{v^2}{R} = \frac{(10.47)^2}{0.5} = \boxed{219 \text{ m/s}^2 \text{ inward}}$$

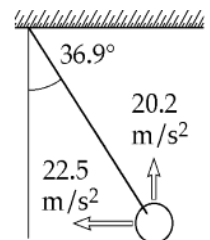
8. A ball swings in a vertical circle at the end of a rope 1.50 m long. When the ball is 36.9° past the lowest point on its way up, its total acceleration is $(-22.5\hat{i} + 20.2\hat{j}) \text{ m/s}^2$. At that instant, (a) sketch a vector diagram showing the components of its acceleration, (b) determine the magnitude of its radial acceleration, and (c) determine the speed and velocity of the ball.

(a) See figure to the right.

(b) The components of the 20.2 and the 22.5 m/s^2 along the rope together constitute the centripetal acceleration:

$$a_c = (22.5 \text{ m/s}^2) \cos(90.0^\circ - 36.9^\circ) + (20.2 \text{ m/s}^2) \cos 36.9^\circ = \boxed{29.7 \text{ m/s}^2}$$

(c) $a_c = \frac{v^2}{r}$ so $\mathbf{v} = \boxed{6.67 \text{ m/s at } 36.9^\circ \text{ above the horizontal}}$



9. A river has a steady speed of 0.500 m/s. A student swims upstream a distance of 1.00 km and swims back to the starting point. If the student can swim at a speed of 1.20 m/s in still water, how long does the trip take? Compare this with the time the trip would take if the water were still.

$$\text{Total time in still water } t = \frac{d}{v} = \frac{2000}{1.20} = \boxed{1.67 \times 10^3 \text{ s}}.$$

Total time = time upstream plus time downstream:

$$t_{\text{up}} = \frac{1000}{(1.20 - 0.500)} = 1.43 \times 10^3 \text{ s}$$

$$t_{\text{down}} = \frac{1000}{1.20 + 0.500} = 588 \text{ s.}$$

Therefore, $t_{\text{total}} = 1.43 \times 10^3 + 588 = \boxed{2.02 \times 10^3 \text{ s}}$.

10 A dive bomber has a velocity of

280 m/s at an angle θ below the horizontal. When the altitude of the aircraft is 2.15 km, it releases a bomb, which subsequently hits a target on the ground. The magnitude of the displacement from the point of release of the bomb to the target is 3.25 km. Find the angle θ .

When the bomb has fallen a vertical distance 2.15 km, it has traveled a horizontal distance x_f given by

$$x_f = \sqrt{(3.25 \text{ km})^2 - (2.15 \text{ km})^2} = 2.437 \text{ km}$$

$$y_f = x_f \tan \theta - \frac{gx_f^2}{2v_i^2 \cos^2 \theta_i}$$

$$-2150 \text{ m} = (2437 \text{ m}) \tan \theta_i - \frac{(9.8 \text{ m/s}^2)(2437 \text{ m})^2}{2(280 \text{ m/s})^2 \cos^2 \theta_i}$$

$$\therefore -2150 \text{ m} = (2437 \text{ m}) \tan \theta_i - (371.19 \text{ m})(1 + \tan^2 \theta_i)$$

$$\therefore \tan^2 \theta - 6.565 \tan \theta_i - 4.792 = 0$$

$$\therefore \tan \theta_i = \frac{1}{2} \left(6.565 \pm \sqrt{(6.565)^2 - 4(1)(-4.792)} \right) = 3.283 \pm 3.945.$$

Select the negative solution, since θ_i is below the horizontal.

$$\therefore \tan \theta_i = -0.662, \quad \boxed{\theta_i = -33.5^\circ}$$