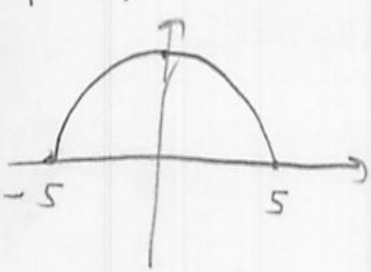
1. a) Use intervals to describe the domain of the function $f(x) = \dfrac{3x^2 - 4x + 2}{4\sqrt{3x - 8}}$.
(2 points)

b) Use intervals to describe the domain of the function $g(x) = \dfrac{\sqrt{25 - x^2}}{4x^2 - 16}$.
(2 points)

a) $3x^2-4x+2$ defined everywhere
$4\sqrt{3x-8}$ cannot be 0 and we must have $3x-8 \geq 0$
$3x-8 \geq 0$ when $x \geq 8/3$ but then $x \neq 8/3$ (otherwise function is undefined)
$\therefore$ dom $f(x)$ is all $x > 8/3 \Rightarrow \boxed{\text{dom } f(x) = (8/3, +\infty)}$

b) need $25-x^2 \geq 0$ and $4x^2-16 \neq 0$
$25-x^2 \geq 0$
$(5-x)(5+x) \geq 0 \Rightarrow -5 \leq x \leq 5$
but $4x^2-16 \neq 0$
$x^2 \neq 4$
$x \neq \pm 2$




$\Rightarrow \boxed{\text{dom } g(x) = [-5,-2) \cup (-2,2) \cup (2,5]}$

2. Consider the function $f(x) = \begin{cases} x^2 + 4x + 8, & \text{if } x \leq -4, \\ -2x^2 - 8x + 9, & \text{if } -4 < x < 1, \\ -7x^4 + 2x^2 + 3x + 1, & \text{if } x \geq 1. \end{cases}$

a) Find $\lim_{x \to -4-} f(x)$, $\lim_{x \to -4+} f(x)$, $\lim_{x \to 1-} f(x)$ and $\lim_{x \to 1+} f(x)$. (2 points)
b) Determine if $\lim_{x \to -4} f(x)$ and $\lim_{x \to 1} f(x)$ exist. (1 point)

a) $\lim_{x \to -4^-} f(x) = \lim_{x \to -4^-} x^2+4x+8 = 16-16+8 = 8$
$\lim_{x \to -4^+} f(x) = \lim_{x \to -4} -2x^2-8x+9 = -32+32+9 = 9$
$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} -2x^2-8x+9 = -2-8+9 = -1$
$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} -7x^4+2x^2+3x+1 = -7+2+3+1 = -1$

b) $\lim_{x \to -4} f(x)$ does not exist
$\lim_{x \to 1} f(x) = -1$

3. Evaluate the following limits :

a) $\lim_{x\to 2}\left(\frac{3}{x} - \frac{\sqrt{3+3x}}{5x-2}\right)$ b) $\lim_{x\to -6}\frac{x^3-x^2-42x}{4x^2+24x}$ c) If $\lim_{x\to 1}\frac{\sqrt{x+3}-2}{x-1}$.

(2 points each)

a) $\lim_{x\to 2} \frac{3}{x} - \frac{\sqrt{3+3x}}{5x-2} = \frac{3}{2} - \frac{\sqrt{9}}{8} = \frac{3}{2} - \frac{3}{8} = \frac{12-3}{8} = \boxed{9/8}$

b) $\lim_{x\to -6} \frac{x^3-x^2-42x}{4x^2+24x} = \frac{0}{0} = ?$

$\lim_{x\to -6} \frac{x(x^2-x-42)}{4x(x+6)} = \lim_{x\to -6} \frac{x(x-7)(x+6)}{4x(x+6)} = \lim_{x\to -6} \frac{x-7}{4} = \boxed{-13/4}$

c) $\lim_{x\to 1} \frac{\sqrt{x+3}-2}{x-1} = \frac{0}{0} = ?$

$\lim_{x\to 1} \frac{\sqrt{x+3}-2}{x-1} = \lim_{x\to 1} \frac{(\sqrt{x+3}-2)(\sqrt{x+3}+2)}{(x-1)(\sqrt{x+3}+2)} = \lim_{x\to 1} \frac{x+3-4}{(x-1)(\sqrt{x+3}+2)} = \lim_{x\to 1} \frac{x-1}{(x-1)(\sqrt{x+3}+2)} = \lim_{x\to 1} \frac{1}{\sqrt{x+3}+2} = \boxed{\frac{1}{4}}$

4. Consider the function $f(x) = \begin{cases} 0.5x^2, & \text{if } x < 1, \\ -2, & \text{if } x = 1, \\ -x+3, & \text{if } x > 1. \end{cases}$

a) Is $f(x)$ discontinuous at $x = 1$? Why or why not? Justify your answer. (1 point)
b) Is $f(x)$ continuous at $x = 2$? Why or why not? Justify your answer. (1 point)

a) $\lim_{x\to 1^-} f(x) = 0.5$ $\lim_{x\to 1^+} f(x) = 2$ $\lim_{x\to 1} f(x)$ does not exist.
So $f(x)$ discontinuous at $x=1$.

b) $\lim_{x\to 2} f(x) = 1$

$f(2) = 1$ and $\lim_{x\to 2} f(x) = f(2)$

$\therefore$ continuous at $x=2$.

5. Find the average rate of change of the function $f(x) = \dfrac{3-2x}{x+4}$ from $x=-2$ to $x=4$. (2 points)

$$\frac{f(4)-f(-2)}{4-(-2)} = \frac{\left(\frac{3-8}{4+4}\right) - \left(\frac{3-2(-2)}{2}\right)}{6}$$

$$= \frac{-\frac{5}{8} - \frac{7}{2}}{6} = \frac{-\frac{5}{8} - \frac{28}{8}}{6} = \boxed{\frac{-33}{48}}$$

$$= \boxed{\frac{-11}{16}} = -0.6875$$

6. 200-mg of a sample of a certain radioactive element is placed into a nuclear reactor. After 30 minutes, the sample has decayed to 140-mg. The equation $N(t) = N_o e^{-\lambda t}$ gives the quantity of the sample at time t (in minutes). $\lambda$ is the disintegration constant.

a) Given the informations above, give a numerical value of what the disintegration constant $\lambda$ must be in this case. (1 point)
b) Determine the half-life of that radioactive element. (1 point)

a) $140 = 200e^{-30\lambda} \Rightarrow \frac{7}{10} = e^{-30\lambda}$

$\Rightarrow \ln(7/10) = \ln(e^{-30\lambda}) = -30\lambda \Rightarrow \boxed{-\ln(7/10)/30}$

$\boxed{\approx -0.012}$

b) $\frac{1}{2}N_0 = N_0 e^{-0.012t}$

$\frac{1}{2} = e^{-0.012t} \Rightarrow \ln(1/2) = -0.012t$

$\Rightarrow t = \frac{\ln(1/2)}{-0.012} \approx \boxed{57.76 \text{ minutes}}$

or any close

[illegible]

7. a) Use the first Principles Definition of the derivative to find the derivative of $f(x) = 4x^2 - 3x - 5$. Show all your work and simplify your answer. **Only a solution using the definition of the derivative will be accepted.** (2 points)

b) Use your answer in 7a to find the equation of the tangent line of $f(x)$ when $x = 1$. (1 point).

a) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(4(x+h)^2 - 3(x+h) - 5) - (4x^2 - 3x - 5)}{h}$

$= \lim_{h \to 0} \frac{(4(x^2+2xh+h^2) - 3x - 3h - 5) - 4x^2 + 3x + 5}{h}$

$= \lim_{h \to 0} \frac{4x^2 + 8xh + 4h^2 - 3x - 3h - 5 - 4x^2 + 3x + 5}{h}$

$= \lim_{h \to 0} \frac{8xh + 4h^2 - 3h}{h} = \lim_{h \to 0} (8x + 4h - 3) = \boxed{8x - 3}$

$\therefore \boxed{f'(x) = 8x - 3}$

b) $m = f'(1) = 8 - 3 = 5$ $\boxed{y = 5x + b}$

$f(1) = 4 - 3 - 5 = -4$

$-4 = 5(1) + b$

$\boxed{-9 = b}$ $\Rightarrow$ $\boxed{y = 5x - 9}$

8. Find the derivatives of the following functions. You do not need to simplify your answers. For this question **you do not need** to use the definition of the derivative. You are expected to use the derivative rules.

a) $f(x) = \dfrac{4}{\sqrt{5x^6 + 3x^4}}$ (2 points) b) $f(x) = (5x^2 + 2x - 4)^7 (3x^4 + 6x^2 + 4x)^6$ (2 points)

c) $f(x) = \ln(4x^3 + 6x + 8)$ (1 point) d) $f(x) = 5^{2x^2 - 5x + 3}$ (1 point)

e) Find $f''(x)$ if $f(x) = \dfrac{3}{x} + 5x^3 + 4.$ (2 points)

a) $f(x) = 4(5x^6 + 3x^4)^{-1/2}$ $\quad f'(x) = -2(5x^6 + 3x^4)^{-3/2}(30x^5 + 12x^3)$

b) $f'(x) = 7(5x^2 + 2x - 4)^6 (10x + 2)(3x^4 + 6x^2 + 4x)^6$
$+ 6(5x^2 + 2x - 4)^7 (3x^4 + 6x^2 + 4x)^5 (12x^3 + 12x + 4)$

c) $f'(x) = \dfrac{12x^2 + 6}{4x^3 + 6x + 8}$

d) $f'(x) = 5^{2x^2 - 5x + 3}(4x - 5)\ln 5$

e) $f(x) = 3x^{-1} + 5x^3 + 4$
$f'(x) = -3x^{-2} + 15x^2$
$f''(x) = 6x^{-3} + 30x$