Ryerson University
Department of Mathematics
MTH 140 Fall 2009 – Final Exam

LAST NAME: _____ FIRST NAME: _____
(Please print) (Please print)

I.D. NUMBER:_____ SIGNATURE:_____

Date: December 7, 2009 Duration: 2 hours **Version: White HBF-348⊕**

Please indicate your section (check the appropriate box):

Prof. Pascal: ☐ Sec. 1 ☐ Sec. 2 ☐ Sec. 3 ☐ Sec. 4 ☐ Sec. 5 ☐ Sec. 6

Prof. Wang: ☐ Sec. 7 ☐ Sec. 8 ☐ Sec. 9 ☐ Sec. 10 ☐ Sec. 11

Prof. Bonato: ☐ Sec. 12 ☐ Sec. 13 ☐ Sec. 14 ☐ Sec. 15 ☐ Sec. 16 ☐ Sec. 17

Prof. Ferrando: ☐ Sec. 18 ☐ Sec. 19 ☐ Sec. 20 ☐ Sec. 21 ☐ Sec. 22 ☐ Sec. 23

INSTRUCTIONS:

- Verify that your exam has 9 pages including this page.
- The use of notes, formula sheets, books or calculators is not allowed.
- For full-answer questions:

  Give full justification for your answers; correct answers alone may be worth nothing. Cross out or erase all rough work not relevant to your solution. Write your solutions in the space provided. If you need more space, use the back of the page. Indicate this fact on the original page, making sure that your solution cannot be confused with any rough work which may be there.
- For multiple choice questions:

  Make sure to write your answers in the box at the end of each question carefully. There are no part marks in the multiple-choice section and only the answer in the box will be marked. The correct response gets full marks, an incorrect response or no response gets no marks.

For markers' use only:

Page	Value	Mark
2	13	
3	8	
4	13	
5	13	
6	10	
7	8	
8	10	
9	10	
Total	85	

1. (3 pts.) (multiple-choice question) Consider the asymptotic behaviour of the function

$$f(x) = \frac{2x^5 + 3x - 1}{x^4 + 1}$$

Find the asymptote of the graph of f.

Select the correct answer.

A) $y = 2$ **B)** $y = x - 1$

C) $y = 3x - 1$ **D)** $y = x$ **E)** $y = 2x$

Write the (capital) letter of the answer in this box ⟶ 1. [ )]

2. (10 pts.) Use the **disk/washer method** to find the volume of the solid obtained by rotating the region bounded by the curve $y = x^2 + 1$ and the line $y = 2x + 1$ about the x-axis.

answer: [ ]

3. (3 pts.) (multiple-choice question) Use implicit differentiation to find $\frac{dy}{dx}$ if $x = \int_3^{y^2} \sqrt{t^3+1}\, dt$.

Select the correct answer.

A) $\frac{1}{\sqrt{y^3+1}}$

B) $\frac{1}{\sqrt{y^6+1}}$

C) $\sqrt{y^3+1} - 9$

D) $\frac{1}{2y\sqrt{y^6+1}}$

E) $\frac{x}{\sqrt{y^6+1}}$

Write the (capital) letter of the answer in this box ⟶ 3. [ ]

4. (5 pts.) Find the absolute maximum and minimum values of the function $f(x) = x + \sin(2x)$ on the interval $[0, \pi]$.
Hint: *Some* of the following approximate values may be useful: $\pi \approx 3$, $\sqrt{3} \approx \frac{7}{4}$, $\sqrt{2} \approx \frac{3}{2}$.

answer: [ ]

5. (3 pts.) (multiple-choice question) Suppose $f(2) = 6$, $f(4) = 3$, and $f'(x) < 0$ for $x \in [2, 4]$. Let $I = \int_2^4 f(x)dx$. Which of the following are true?

A) $6 \leq I \leq 12$ **B)** $I = 0$

C) $I = 3$ **D)** $3 \leq I \leq 6$

E) $I = -3$

Write the (capital) letter of the answer in this box ⟶ 5. ( )

6. (10 pts.) Find the area of the region bounded by the curve $y = x^3 - 1$ and the line $y = 8x - 8$ for $0 \leq x \leq 2$.

answer:

7. (3 pts.)(multiple-choice question) Evaluate the integral $\displaystyle\int_{-1}^{1} \frac{3x^2+1}{x^3+x+3}\,dx$.

Select the correct answer.

A) 0 **B)** $\displaystyle 2\int_{0}^{1} \frac{3x^2+1}{x^3+x+3}\,dx$

C) $\displaystyle -\frac{16}{5}$ **D)** $\ln 5$

E) $\ln 4$

Write the (capital) letter of the answer in this box ⟶ 7. ( )

8. (10 pts.) Use the **method of cylindrical shells** to find the volume of the solid obtained by rotating the region bounded by the curve $y = 2x - x^2$ and the line $y = x$ about the line $x = 1$.

answer:

9. (10 pts.) Evaluate the following integrals:

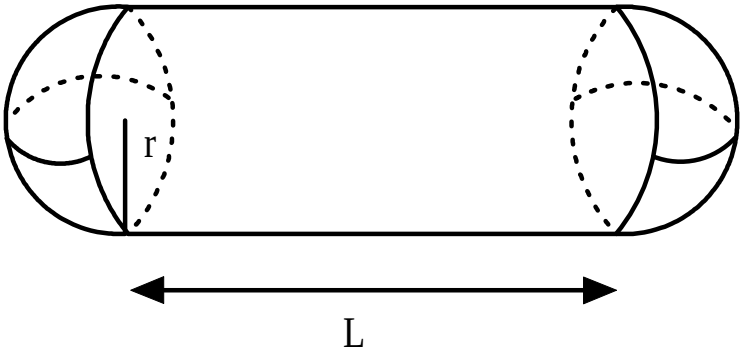
(a) $\displaystyle\int_1^{e^{\frac{\pi}{4}}} \frac{\sec^2(\ln x)}{x}\, dx$

answer:

(b) $\displaystyle\int \frac{dx}{\sqrt{x}(1+x)}$

answer:

10. (8 pts.) A balloon has the shape of a right circular cylinder of radius r and length L with a hemisphere at each end of radius r. The balloon is being filled at a rate of 10 cm^3/s in such a way that L increases twice as fast as r. Find the rate of change of r when $r = 8$ cm and $L = 26$ cm.
(For a sphere of radius r the volume is $V_s = \frac{4}{3}\pi r^3$, and for a right circular cylinder of radius r and length L the volume is $V_c = \pi r^2 L$.)



answer:

11. (10 pts.) Consider the function:

$$f(x) = x^2e^{-x} \quad \text{with} \quad f'(x) = x(2-x)e^{-x} \quad \text{and} \quad f''(x) = (x^2-4x+2)e^{-x}$$

(a) Find all the asymptotes of the graph of f.

(b) Find the local (or relative) maximum and minimum values of f.

(c) Find the intervals where the graph of f is concave up and concave down.

(d) Use the information from parts (a) - (c) to sketch the graph of f.

12. (10 pts.) Find the largest volume of a **closed** right circular cylinder whose total surface area is 6π ft^2. (For a right circular cylinder of radius r and height h the volume is $V = \pi r^2 h$ and the surface area of **the sides** is $S_1 = 2\pi rh$.)

answer: