Lecture Quiz 2, ENGR 233, Fall 2015, Section Q, Solutions

Problem.(a) Given $w = e^{1+xyz}$ and $x = t$, $y = t^2$, $z = \sin(t)$. Find $\frac{dw}{dt}$.
Solution.

$$w^{'}(t) = w_x\, x^{'}(t) + w_y\, y^{'}(t) + w_z\, z^{'}(t) = yze^{1+xyz} + xze^{1+xyz}2t + xye^{1+xyz}\cos(t).$$

(b) Find equations of the tangent plane and the normal line to the surface $z = \ln(x^2 + y^2)$ at the point $(1, 0, 0)$.
Solution. $z - \ln(x^2 + y^2) = 0$, $F(x, y, z) = z - \ln(x^2 + y^2)$,

$$\nabla F(x,y,z) = \left\langle -\frac{2x}{x^2+y^2}, -\frac{2y}{x^2+y^2}, 1 \right\rangle \quad \Rightarrow \quad \nabla F(1,0,0) = \langle -2, 0, 1\rangle.$$

$$\text{the tangent plane: } \langle -2,0,1\rangle \langle x-1, y-0, z-0\rangle = 0 \quad \Rightarrow \quad 2x - z = 2$$

$$\text{the normal line, vector equation: } \langle x,y,z\rangle = \langle 1,0,0\rangle \; + \; t\langle -2,0,1\rangle$$

$$\text{the normal line, parametric equations: } x = 1-2t,\; y = 0,\; z = t.$$

(c) Consider a cubical box $-5 \le x \le 5, -5 \le y \le 5, -5 \le z \le 5$ centered at the origin. The temperature at a point (x, y, z) in the box is $T(x, y, z) = 18+\ln(10+x^2y^2z^4)$. Determine a direction an insect should take, starting at $(1, -1, 1)$ in order to warm up as rapidly as possible.
Solution.

$$\nabla T(x,y,z) = \left\langle \frac{2xy^2z^4}{10+x^2y^2z^4}, \frac{2x^2yz^4}{10+x^2y^2z^4}, \frac{4x^2y^2z^3}{10+x^2y^2z^4} \right\rangle$$

$$\nabla T(1,-1,1) = \left\langle \frac{2}{11}, \frac{-2}{11}, \frac{4}{11} \right\rangle = \frac{2}{11}\langle 1,-1,2\rangle\,.$$

$$\text{The direction of the most rapid warm up is: } \mathbf{u}^* = \left\langle \frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}} \right\rangle.$$