Solutions

School of Mathematics and Statistics
Carleton University
Math. 2004A, Fall 2013

TEST 3

Any non-programmable calculator permitted, 1 blank sheet permitted for roughs

Print Name : ______

Student Number: ______

Tutorial Section (A1, A4, ...): ______

PART I: Multiple Choice Questions

(Choose and CIRCLE only ONE answer - No part marks here.)

1. **[2 marks]** Find the directional derivative of the function $x^3 - x^2y^2 + xy + y^2$ at the point $P(-1,1)$ in the direction of the vector $u = \mathbf{i} - 2\mathbf{j}$.
   (a) $\sqrt{5}/5$, (b) 5, ~~(c)~~ $-2\sqrt{5}/5$, (d) 0. e) $8/\sqrt{5}$! everyone gets 2 extra marks
2. **[2 marks]** Calculate the gradient vector of the function xe^{yz} at $P(1,0,2)$.
   (a) $\mathbf{i}+\mathbf{j}$, (b) $2\mathbf{i}+3\mathbf{k}$, (c) $\mathbf{i}-\mathbf{j}+3\mathbf{k}$, (d) $\mathbf{i}+2\mathbf{j}$
3. **[2 marks]** What is the maximum value of the directional derivative of $f(x,y) = ye^x$?
   (a) 0, (b) 2, (c) 1, (d) $\sqrt{2}$, (e) none of these.
4. **[2 marks]** Find a normal vector to the surface defined by $4x^2 + y^2 + 4z^2 - 16 = 0$ at the point $P(1,2,\sqrt{2})$.
   (a) $(8,4,8\sqrt{2})$, (b) $(2,1,\sqrt{2})$, (c) $(8,1,0)$, (d) $(2,1,2)$ (e) none of these
5. **[2 marks]** Which of the following vectors is perpendicular to the normal line of the surface $x^2 + 4y^2 + 9z^2 - 17 = 0$ at the point $(2,1,1)$?
   (a) $(0,2,0)$, (b) $(4,-2,0)$, (c) $(1,0-1)$, (d) $(2,-1,3)$ (e) none of these

PART II: Show all work here and give details.
No additional pages will be accepted

6. **[10 marks]** a) Use the method of Lagrange multipliers to find the maximum and minimum values of the function $f(x,y) = xy$ subject to the constraint $x^2 + 4y^2 = 1$.

$g(x,y) = 0 \iff g(x,y) = x^2+4y^2-1$. $\leftarrow$ 1

Let $\ell(x,y,\lambda) = xy - \lambda(x^2+4y^2-1) \Rightarrow$ 1) $\ell_1 = y - 2\lambda x = 0$ $\leftarrow$ 1
2) $\ell_2 = x - 8\lambda y = 0$ $\leftarrow$ 1
3) $\ell_3 = x^2+4y^2-1 = 0$. $\leftarrow$ 1

$\therefore$ 1) $\Rightarrow y = 2\lambda x$ + 2) $\Rightarrow x = 8\lambda y$. $\therefore y = 16\lambda^2 y$.

Case 1: $y \neq 0 \Rightarrow 16\lambda^2 = 1 \Rightarrow \lambda = \pm 1/4$.

a) $\lambda = 1/4$ $\therefore y = 2\frac{1}{4}x = x/2$ + 3) $\Rightarrow x^2 + x^2 - 1 = 0 \Rightarrow 2x^2 = 1$; $x = \pm\frac{1}{\sqrt{2}}$

$\therefore \left(\frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}\right)$ and $\left(-\frac{1}{\sqrt{2}}, \frac{-1}{2\sqrt{2}}\right)$ are critical points

b) $\lambda = -1/4$ $\therefore y = -x/2 \Rightarrow x = \pm 1/\sqrt{2}$ $\therefore y = \mp 1/2\sqrt{2}$ $\therefore \left(\frac{1}{\sqrt{2}}, -\frac{1}{2\sqrt{2}}\right), \left(\frac{-1}{\sqrt{2}}, \frac{+1}{2\sqrt{2}}\right)$

Case 2: $y = 0 \Rightarrow x = \pm 1$ by 3). Also 1) $\Rightarrow \lambda = 0$ $\therefore x = 0$ by 2)
a contradiction $\therefore y \neq 0$ + so Case 1 applies.

$\therefore$ 4 CP's.

	Value of f :	
$\left(\frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}\right) \leftarrow$ 1	$1/4$	max
$\left(-\frac{1}{\sqrt{2}}, -\frac{1}{2\sqrt{2}}\right) \leftarrow$ 1	$1/4$	max
$\left(\frac{1}{\sqrt{2}}, -\frac{1}{2\sqrt{2}}\right) \leftarrow$ 1	$-1/4$	min
$\left(-\frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}\right) \leftarrow$ 1	$-1/4$	min.

$\therefore$ max $= 1/4$ $\leftarrow$ 1   1 $\rightarrow$ min $= -1/4$