Question 1:

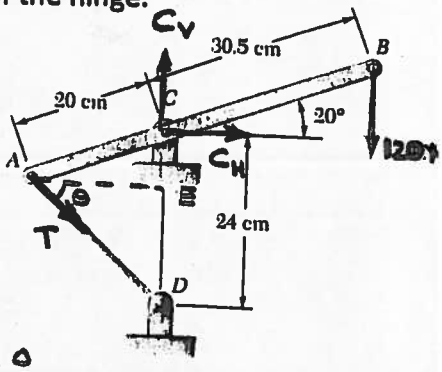
Lever ACB is smoothly hinged at C as shown in Figure. It carries a 120 N load at end B. Cable AD keeps the lever in the position shown.

(a) Mark the forces acting on the lever in the given Figure.

(b) Determine the tension in the cable AD.

(c) Determine the horizontal and vertical components of the reaction of the hinge.

(d) Is this lever a two-force body or a three-force body? Explain.



Method (1) $AE = 0.20 \cos 20^\circ = 0.1879$ m

$ED = 0.24 - 0.20 \sin 20 = 0.1716$ m

$\tan\theta = \frac{ED}{AE} = 0.9133$

$\theta = 42.40^\circ$

(b) (C $\quad T \times 0.20 \sin(\theta + 20^\circ) - 120 \times 0.305 \cos 20^\circ = 0$

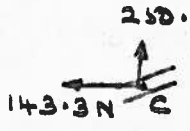
$\therefore T = 194.0$ N.

(c) $\rightarrow \quad C_H + T\cos\theta = 0$

$\therefore C_H = -143.3$ N.

$\uparrow \quad C_V - 120 - T\sin\theta = 0$

$\therefore C_V = 250.8$ N.

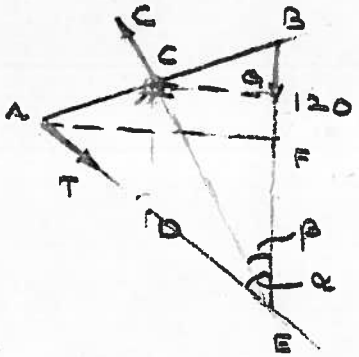


Reaction of Hinge C.

(d) Three forces are acting at A, B and C.

$\therefore$ Three-force body.

Method (2)



Similar Δs.

$\frac{BE}{CD} = \frac{AB}{AC} \quad \therefore BE = CD \times \frac{AB}{AC}$

$= \frac{0.24 \times 0.305}{0.20} = 0.606$ m

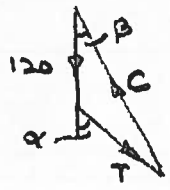
$\tan\alpha = \frac{AF}{FE} = \frac{0.305\cos 20^\circ}{0.606 - 0.305\sin 20^\circ} = 1.095$

$\therefore \alpha = 47.60^\circ$

$\tan\beta = \frac{CG}{EG} = \frac{0.305\cos 20^\circ}{0.606 - 0.305\sin 20^\circ} = 0.5713$

$\beta = 29.74^\circ$

Force Δ:

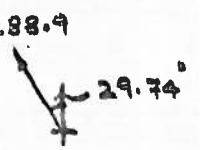


Sine Rule:

$\frac{120}{\sin(\alpha - \beta)} = \frac{C}{\sin(180 - \alpha)} = \frac{T}{\sin\beta}$

$\therefore T = 194.1$ N

$C = 288.9$ N

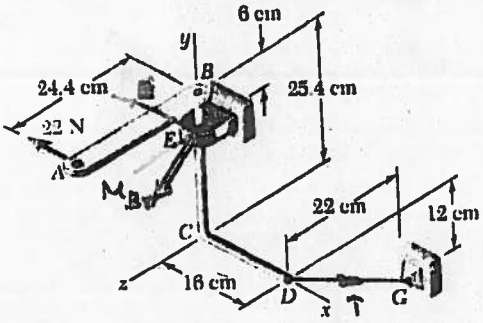


Question 2:

Bent-rod BCD is supported in bearing E as shown in Figure. This bearing allows the rod to turn about the axis BC. Consequently, the couple exerted by the bearing has components in the x and z directions only. The bearing prevents the movement of the rod in the downward direction. Cable DG keeps the rod in the position shown.

(a) Determine the tension in the cable.

(b) Determine the three components of the force exerted by the bearing,



Reaction of Bearing at E.

Force = $\vec{E}$

Moment = $\vec{M}_{Br} = (M_{Br})_x \vec{i} + (M_{Br})_z \vec{k}$

Note: The reaction moment does not have component in axial direction of $\vec{T}$

$$\vec{T} = \frac{-0.12\vec{j} - 0.22\vec{k}}{\sqrt{0.12^2 + 0.22^2}} \cdot T$$

$$= 0\vec{i} - 0.4789T\vec{j} - 0.8779T\vec{k}$$

(a) Taking moments about E

$$\vec{M}_E = \vec{0} = \vec{M}_{Br} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0.06 & 0.244 \\ -22 & 0 & 0 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.16 & -0.194 & 0 \\ 0 & -0.4789T & -0.8779T \end{vmatrix}$$

$$\therefore \vec{M}_{Br} + (0\vec{i} - 5.368\vec{j} + 1.32\vec{k}) + (0.1703T\vec{i} + 0.1405T\vec{j} - 0.07662T\vec{k}) = \vec{0}$$

$\vec{M}_{Br}$ does not have component along $\vec{j}$ direction $= \vec{0}$.

$$\therefore -5.368 + 0.1405T = 0$$

$$T = 38.21 \text{ N}$$

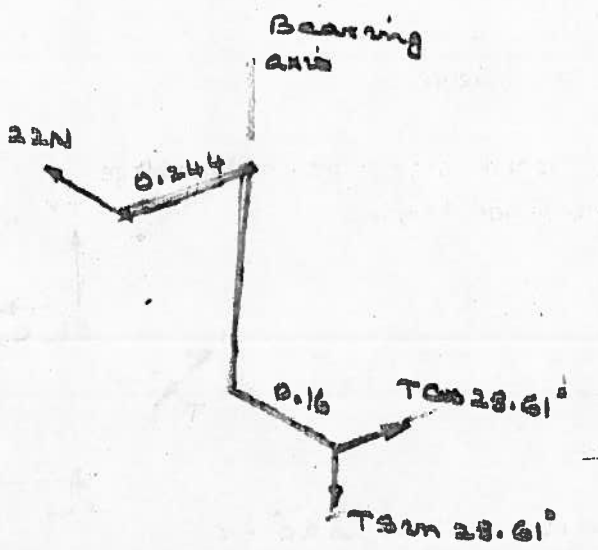
(b) $\vec{T} = 38.21(0\vec{i} - 0.4987\vec{j} - 0.8779\vec{k})$

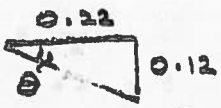
$= 0\vec{i} - 19.06\vec{j} - 33.54\vec{k}$ N

$\Sigma \vec{F} = \vec{0}$

$$\therefore -22\vec{i} + (0\vec{i} - 19.06\vec{j} - 33.54\vec{k}) + \vec{E} = \vec{0}$$

$$\therefore \vec{E} = 22\vec{i} + 19.06\vec{j} + 33.54\vec{k}$$





$\tan\theta = \frac{0.12}{0.22}$

$\theta = 28.61°$

Taking moments about the axis

$$-22 \times 0.244 + (T\cos 28.61) \times 0.16 = 0$$

$$\therefore T = 38.22\ N$$

For equilibrium $\sum \vec{F} = 0$

x: $E_x - 22 = 0$

$\therefore E_x = 22\ N$

y: $E_y - T\sin 28.61° = 0$

$\therefore E_y = 18.30\ N$

z: $E_z - T\cos 28.61 = 0$

$\therefore E_z = 33.55\ N$

Question 3:

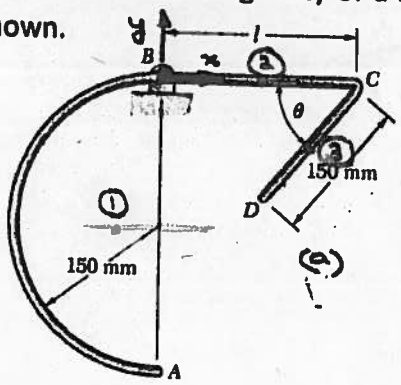
A uniform rod ABCD is bent in the shape shown in Figure. Here, the length l of BC is 150 mm and the angle θ is 45°. Use the coordinate axes located at B as shown for your answers.

(a) Express the coordinates of the center of gravity of the parts AB, BC and CD of the rod. Present your answers in a table

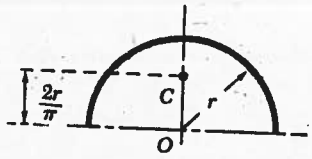
(b) Determine the coordinates of the center of gravity of the whole rod.

(c) Determine the first moments of the line ABCD about the x and y axes.

Note: The center of gravity of a semi-circle of radius r is at distance $2r/\pi$ from the center as shown.



(a)



The wire has three components. AB, BC and CD

	L	$\bar{x}$	$\bar{y}$
AB	$L_1 = 150\pi$	$\bar{x}_1 = -\frac{2\times150}{\pi}$	$\bar{y}_1 = -150$
BC	$L_2 = 150$	$\bar{x}_2 = 75$	$\bar{y}_2 = 0$
CD	$L_3 = 150$	$\bar{x}_3 = 150 - 75\cos\theta$ $= 96.97$	$\bar{y}_3 = -75\sin\theta$ $= -53.03$

(b)

$$\bar{x} = \frac{L_1\bar{x}_1 + L_2\bar{x}_2 + L_3\bar{x}_3}{L_1 + L_2 + L_3} = -24.90 \text{ mm}$$

$$\bar{y} = \frac{L_1\bar{y}_1 + L_2\bar{y}_2 + L_3\bar{y}_3}{L_1 + L_2 + L_3} = -102.0 \text{ mm}$$

(c)

$$L = L_1 + L_2 + L_3 = 771.2 \text{ mm}$$

$$Q_x = L\bar{y} = -78.66\times10^3 \text{ mm}^2$$

$$Q_y = L\bar{x} = -19.20\times10^3 \text{ mm}^2$$