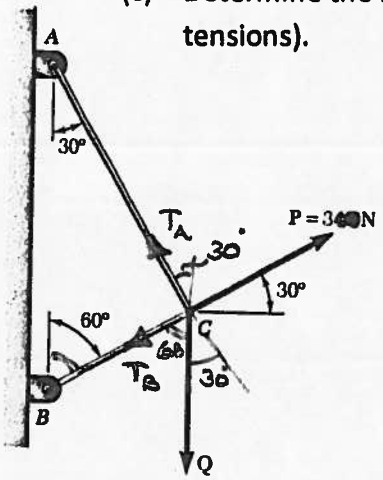
Question 1:

Two cables AC and BC are tied together at C as shown in Figure. Forces P and Q are exerted at C. Here, P = 300 N.

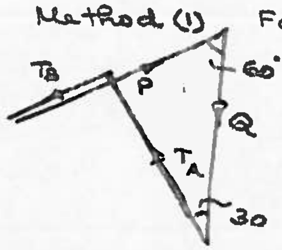
(a) Mark the tensions exerted by the cables AC and BC at C in the given figure itself.

(b) Determine the tensions in the cables AC and BC in terms of Q.

(c) Determine the range of Q for which both cables will remain taut (have positive tensions).



Method (1) Force Polygon Closes



$$\frac{P - T_B}{\sin 30^\circ} = \frac{Q}{\sin 90^\circ} = \frac{T_A}{\sin 60}$$

$$\therefore T_A = \frac{\sqrt{3}}{2} Q \qquad (1)$$

$$P - T_B = \frac{1}{2} Q$$

$$T_B = P - \frac{1}{2} Q \,. \quad (2)$$

P = 300 N

$T_A > 0 \qquad Q > 0$

$T_B > 0 \qquad 300 - \frac{1}{2} Q > 0 \quad \therefore Q < 600.$

$\therefore$ Range : $\quad 0 < Q < 600$

Method (2) Note $AC \perp BC$. Resolving along AC and BC:

$$\nwarrow \quad T_A - Q \cos 30 = 0$$

$$\therefore T_A = \frac{\sqrt{3}}{2} Q \qquad (1)$$

$$\nearrow \quad P - T_B - Q \sin 30^\circ = 0$$

$$\therefore T_B = P - \frac{1}{2} Q \quad (2)$$

Method (3)

$$\rightarrow \quad -T_A \sin 30^\circ - T_B \sin 60 + P \cos 30 = 0.$$

$$-\frac{1}{2} T_A - \frac{\sqrt{3}}{2} T_B + \frac{\sqrt{3}}{2} P = 0$$

$$\therefore T_A + \sqrt{3}\, T_B = \sqrt{3}\, P \qquad (1)$$

$$\uparrow \quad T_A \cos 30^\circ - T_B \cos 60^\circ - Q + P \sin 30^\circ = 0$$

$$\frac{\sqrt{3}}{2} T_A - \frac{1}{2} T_B - Q + \frac{1}{2} P = 0.$$

$$\sqrt{3}\, T_A - T_B = 2Q - P \qquad (2)$$

From (1) $\quad T_A = \sqrt{3} P - \sqrt{3} T_B$

Subs in (2) $\quad \sqrt{3}(\sqrt{3} P - \sqrt{3} T_B) - T_B = 2Q - P$

$$3P - 3T_B - T_B = 2Q - P$$

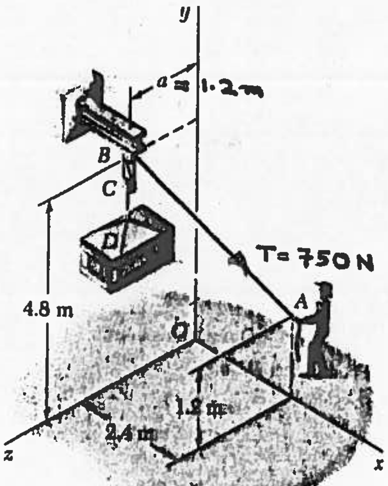
$$T_B = \frac{4P - 2Q}{4} = P - \frac{1}{2} Q$$

Question 2:

The man exerts a 750 N force at the end A of the cable AB shown in Figure. The other end B of the cable is in the yOz plane. Here, the distance a is given as 1.2 m.

(a) Express the force exerted at end A of the cable AB in vector notation

(b) Determine the moments of the force exerted at the end A of the cable AB about the Ox, Oy and Oz axes.



Force: A (2.4 , 1.2 , 0)

B (0 , 4.8 , 1.2)

∴ A|B (2.4 − 0 , 1.2 − 4.8 , 0 − 1.2)

∴ $\vec{r}_{A|B} = 2.4\vec{i} - 3.6\vec{j} - 1.2\vec{k}$ m

$r_{A|B} = \sqrt{2.4^2 + 3.6^2 + 1.2^2} = 4.490$ m

$\vec{T} = 750\left(\frac{2.4\vec{i} - 3.6\vec{j} - 1.2\vec{k}}{4.490}\right)$

$= 400.9\vec{i} - 601.3\vec{j} - 200.4\vec{k}$. N

O is on Ox, Oy and Oz axes.

$\vec{M}_O = \vec{r}_A \times \vec{T}$ $\qquad \vec{r}_A = 2.4\vec{i} + 1.2\vec{j} + 0\vec{k}$.

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2.4 & 1.2 & 0 \\ 4009 & -601.3 & -200.4 \end{vmatrix} = \begin{vmatrix} 1.2 & 0 \\ -601.3 & -200.4 \end{vmatrix}\vec{i} - \begin{vmatrix} 2.4 & 0 \\ 400.9 & -200.4 \end{vmatrix}\vec{j} + \begin{vmatrix} 2.4 & 1.2 \\ 400.9 & -601.3 \end{vmatrix}\vec{k}$$

∴ $\vec{M}_O = -240.5\vec{i} + 481.0\vec{j} - 1942.2\vec{k}$. Nm

$M_{ox} = \vec{M}_O \cdot \vec{i} = -240.5$ Nm

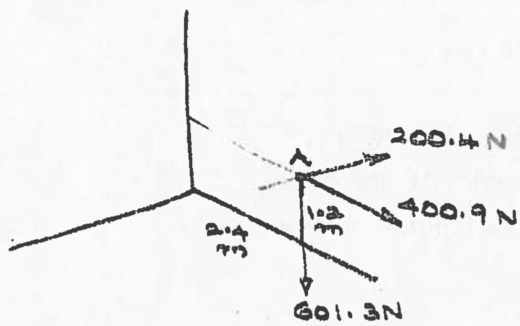
$M_{oy} = \vec{M}_O \cdot \vec{j} = 481.0$ Nm

$M_{oz} = \vec{M}_O \cdot \vec{k} = -1942$ Nm.

Note: $\vec{M}_O = \vec{r}_B \times \vec{T}$ $\qquad \vec{r}_B = 0\vec{i}, 4.8\vec{j} + 1.2\vec{k}$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 4.8 & 1.2 \\ 400.9 & -601.3 & -200.4 \end{vmatrix} = \begin{vmatrix} 4.8 & 1.2 \\ -601.3 & -200.4 \end{vmatrix}\vec{i} - \begin{vmatrix} 0 & 1.2 \\ 400.9 & -200.4 \end{vmatrix}\vec{j} + \begin{vmatrix} 0 & 4.8 \\ 400.9 & -601.3 \end{vmatrix}\vec{k}$$

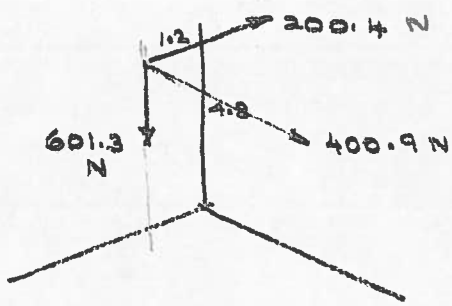
∴ $\vec{M}_O = -240.4\vec{i} + 481.1\vec{j} - 1942.3\vec{k}$.



$M_{ox} = -200.4 \times 1.2$
$= -240.5 \text{ Nm}$

$M_{oy} = +200.4 \times 2.4$
$= +481.0 \text{ Nm}$

$M_{oz} = -400.9 \times 1.2$
-601.3×2.4
$= -1942.2 \text{ Nm}$



$M_{ox} = -200.4 \times 4.8$
$+601.3 \times 1.2$
$= -240.4 \text{ Nm}$

$M_{oy} = +400.9 \times 1.2$
$= +481.1 \text{ Nm}$

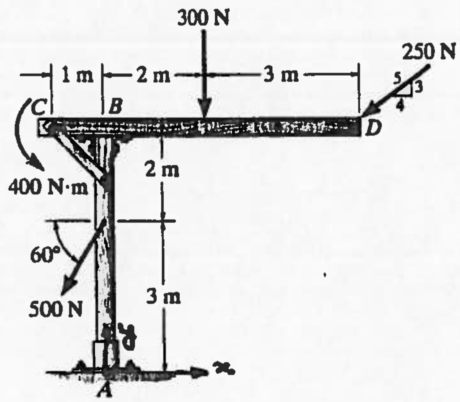
$M_{oz} = -400.9 \times 4.8$
$= -1924.3 \text{ Nm}$

Question 3:

The three forces and the couple acting on the frame is shown in Figure. This force-couple system can be reduced to a force R at A and a couple of moment M_A.

(a) Determine the components R_x and R_y of the force R at A. Show the magnitude and the inclination with x-direction of the force R in a sketch.

(b) The force system can be reduced to a single force. Determine the two distances from B of the points of intersection of the resultant with AB and CD.



Force System:

$R_x = -500\cos 60 - 250 \times \frac{4}{5}$

$= -250 - 200$

$= -450$ N

$R_y = -500\sin 60° - 300 - 250 \times \frac{3}{5}$

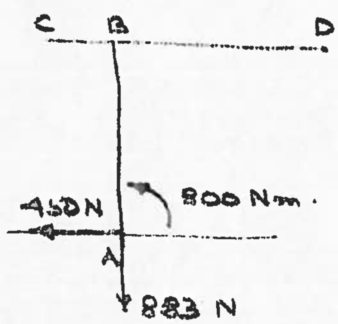
$= -433.0 - 300 - 150$

$= -883.0$ N.

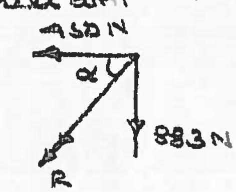
CCW $M_A = 400 + 500\cos 60 \times 3 - 300 \times 2 + 250 \times \frac{4}{5} \times 5 - 250 \times \frac{3}{5} \times 5$

$= 400 + 750 - 600 + 1000 - 750$

$= 800$ Nm.

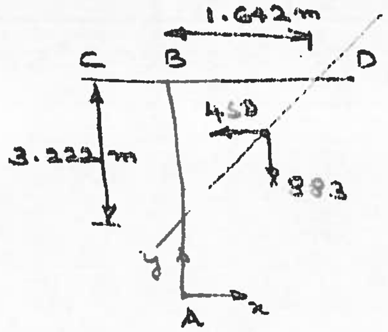


Resultant Force:



$R = \sqrt{450^2 + 883^2} = 991.1$ N

$\alpha = \tan^{-1}\frac{883}{450} = 61.62°$



Equation of the line of action.

A $800 = 450\,y - 883\,x$

AB $x = 0$ $y = \frac{800}{450} = 1.778$ m

$5 - y = 3.222$ m

CD $y = 5$ m $800 = 450 \times 5 - 883\,x$

$x = 1.642$ m.

Note:

$$\circlearrowleft M_B = 400 - 500\cos 60 \times 2 - 300 \times 2 - 250 \times \tfrac{3}{5} \times 5$$

CCW

$$= 400 - 500 - 600 - 750$$

$$= -1450 \text{ Nm.}$$