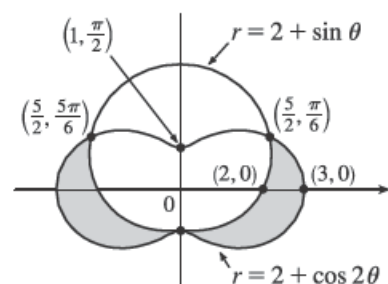


$$\begin{aligned}
36. A &= 2 \int_{-\pi/2}^{\pi/6} \frac{1}{2} [(2 + \cos 2\theta)^2 - (2 + \sin \theta)^2] d\theta \\
&= \int_{-\pi/2}^{\pi/6} [4 \cos 2\theta + \cos^2 2\theta - 4 \sin \theta - \sin^2 \theta] d\theta \\
&= \left[2 \sin 2\theta + \frac{1}{2} \theta + \frac{1}{8} \sin 4\theta + 4 \cos \theta - \frac{1}{2} \theta + \frac{1}{4} \sin 2\theta \right]_{-\pi/2}^{\pi/6} \\
&= \frac{51}{16} \sqrt{3}
\end{aligned}$$



1. $x = \int_1^t \frac{\cos u}{u} du$, $y = \int_1^t \frac{\sin u}{u} du$, so by FTC1, we have $\frac{dx}{dt} = \frac{\cos t}{t}$ and $\frac{dy}{dt} = \frac{\sin t}{t}$. Vertical tangent lines occur when $\frac{dx}{dt} = 0 \Leftrightarrow \cos t = 0$. The parameter value corresponding to $(x, y) = (0, 0)$ is $t = 1$, so the nearest vertical tangent occurs when $t = \frac{\pi}{2}$. Therefore, the arc length between these points is

$$L = \int_1^{\pi/2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \int_1^{\pi/2} \sqrt{\frac{\cos^2 t}{t^2} + \frac{\sin^2 t}{t^2}} dt = \int_1^{\pi/2} \frac{dt}{t} = [\ln t]_1^{\pi/2} = \ln \frac{\pi}{2}$$

25. $\mathbf{n}_1 = \langle 1, 0, -1 \rangle$ and $\mathbf{n}_2 = \langle 0, 1, 2 \rangle$. Setting $z = 0$, it is easy to see that $(1, 3, 0)$ is a point on the line of intersection of $x - z = 1$ and $y + 2z = 3$. The direction of this line is $\mathbf{v}_1 = \mathbf{n}_1 \times \mathbf{n}_2 = \langle 1, -2, 1 \rangle$. A second vector parallel to the desired plane is $\mathbf{v}_2 = \langle 1, 1, -2 \rangle$, since it is perpendicular to $x + y - 2z = 1$. Therefore, the normal of the plane in question is $\mathbf{n} = \mathbf{v}_1 \times \mathbf{v}_2 = \langle 4 - 1, 1 + 2, 1 + 2 \rangle = 3 \langle 1, 1, 1 \rangle$. Taking $(x_0, y_0, z_0) = (1, 3, 0)$, the equation we are looking for is $(x - 1) + (y - 3) + z = 0 \Leftrightarrow x + y + z = 4$.

26.

(a) The vectors $\overrightarrow{AB} = \langle -1 - 2, -1 - 1, 10 - 1 \rangle = \langle -3, -2, 9 \rangle$ and $\overrightarrow{AC} = \langle 1 - 2, 3 - 1, -4 - 1 \rangle = \langle -1, 2, -5 \rangle$ lie in the plane, so $\mathbf{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \langle -3, -2, 9 \rangle \times \langle -1, 2, -5 \rangle = \langle -8, -24, -8 \rangle$ or equivalently $\langle 1, 3, 1 \rangle$ is a normal vector to the plane. The point $A(2, 1, 1)$ lies on the plane so an equation of the plane is $1(x - 2) + 3(y - 1) + 1(z - 1) = 0$ or $x + 3y + z = 6$.

(b) The line is perpendicular to the plane so it is parallel to a normal vector for the plane, namely $\langle 1, 3, 1 \rangle$. If the line passes through $B(-1, -1, 10)$ then symmetric equations are $\frac{x - (-1)}{1} = \frac{y - (-1)}{3} = \frac{z - 10}{1}$ or $x + 1 = \frac{y + 1}{3} = z - 10$.

(c) Normal vectors for the two planes are $\mathbf{n}_1 = \langle 1, 3, 1 \rangle$ and $\mathbf{n}_2 = \langle 2, -4, -3 \rangle$. The angle θ between the planes is given by

$$\cos \theta = \frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{|\mathbf{n}_1| |\mathbf{n}_2|} = \frac{\langle 1, 3, 1 \rangle \cdot \langle 2, -4, -3 \rangle}{\sqrt{1^2 + 3^2 + 1^2} \sqrt{2^2 + (-4)^2 + (-3)^2}} = \frac{2 - 12 - 3}{\sqrt{11} \sqrt{29}} = -\frac{13}{\sqrt{319}}$$

Thus $\theta = \cos^{-1}\left(-\frac{13}{\sqrt{319}}\right) \approx 137^\circ$ or $180^\circ - 137^\circ = 43^\circ$.

(d) From part (c), the point $(2, 0, 4)$ lies on the second plane, but notice that the point also satisfies the equation of the first plane, so the point lies on the line of intersection of the planes. A vector $\mathbf{v}$ in the direction of this intersecting line is perpendicular to the normal vectors of both planes, so take $\mathbf{v} = \mathbf{n}_1 \times \mathbf{n}_2 = \langle 1, 3, 1 \rangle \times \langle 2, -4, -3 \rangle = \langle -5, 5, -10 \rangle$ or equivalently we can take $\mathbf{v} = \langle 1, -1, 2 \rangle$. Parametric equations for the line are $x = 2 + t$, $y = -t$, $z = 4 + 2t$.

10. The parametric value corresponding to the point $(1, 0, 1)$ is $t = 0$.

$$\mathbf{r}'(t) = e^t \mathbf{i} + e^t(\cos t + \sin t) \mathbf{j} + e^t(\cos t - \sin t) \mathbf{k} \Rightarrow |\mathbf{r}'(t)| = e^t \sqrt{1 + (\cos t + \sin t)^2 + (\cos t - \sin t)^2} = \sqrt{3} e^t$$

$$\text{and } s(t) = \int_0^t e^u \sqrt{3} du = \sqrt{3}(e^t - 1) \Rightarrow t = \ln\left(1 + \frac{1}{\sqrt{3}}s\right).$$

$$\text{Therefore, } \mathbf{r}(t(s)) = \left(1 + \frac{1}{\sqrt{3}}s\right) \mathbf{i} + \left(1 + \frac{1}{\sqrt{3}}s\right) \sin \ln\left(1 + \frac{1}{\sqrt{3}}s\right) \mathbf{j} + \left(1 + \frac{1}{\sqrt{3}}s\right) \cos \ln\left(1 + \frac{1}{\sqrt{3}}s\right) \mathbf{k}.$$

$$14. g(u, v) = \frac{u + 2v}{u^2 + v^2} \Rightarrow g_u = \frac{(u^2 + v^2)(1) - (u + 2v)(2u)}{(u^2 + v^2)^2} = \frac{v^2 - u^2 - 4uv}{(u^2 + v^2)^2},$$

$$g_v = \frac{(u^2 + v^2)(2) - (u + 2v)(2v)}{(u^2 + v^2)^2} = \frac{2u^2 - 2v^2 - 2uv}{(u^2 + v^2)^2}$$

45. $f(x, y) = x^2 e^{-y} \Rightarrow \nabla f = \langle 2xe^{-y}, -x^2 e^{-y} \rangle$, $\nabla f(-2, 0) = \langle -4, -4 \rangle$. The direction is given by $\langle 4, -3 \rangle$, so

$$\mathbf{u} = \frac{1}{\sqrt{4^2 + (-3)^2}} \langle 4, -3 \rangle = \frac{1}{5} \langle 4, -3 \rangle \text{ and } D_{\mathbf{u}} f(-2, 0) = \nabla f(-2, 0) \cdot \mathbf{u} = \langle -4, -4 \rangle \cdot \frac{1}{5} \langle 4, -3 \rangle = \frac{1}{5}(-16 + 12) = -\frac{4}{5}.$$

62.

$$f(x, y, z) = x^2 + 2y^2 + 3z^2, \quad g(x, y, z) = x + y + z = 1, \quad h(x, y, z) = x - y + 2z = 2 \Rightarrow$$

$$\nabla f = \langle 2x, 4y, 6z \rangle = \lambda \nabla g + \mu \nabla h = \langle \lambda + \mu, \lambda - \mu, \lambda + 2\mu \rangle \text{ and } 2x = \lambda + \mu \text{ (1), } 4y = \lambda - \mu \text{ (2), } 6z = \lambda + 2\mu \text{ (3),}$$

$x + y + z = 1$ (4), $x - y + 2z = 2$ (5). Then six times (1) plus three times (2) plus two times (3) implies

$12(x + y + z) = 11\lambda + 7\mu$, so (4) gives $11\lambda + 7\mu = 12$. Also six times (1) minus three times (2) plus four times (3) implies

$12(x - y + 2z) = 7\lambda + 17\mu$, so (5) gives $7\lambda + 17\mu = 24$. Solving $11\lambda + 7\mu = 12$, $7\lambda + 17\mu = 24$ simultaneously gives

$\lambda = \frac{6}{23}$, $\mu = \frac{30}{23}$. Substituting into (1), (2), and (3) implies $x = \frac{18}{23}$, $y = -\frac{6}{23}$, $z = \frac{11}{23}$ giving only one point. Then

$f\left(\frac{18}{23}, -\frac{6}{23}, \frac{11}{23}\right) = \frac{33}{23}$. Now since $(0, 0, 1)$ satisfies both constraints and $f(0, 0, 1) = 3 > \frac{33}{23}$, $f\left(\frac{18}{23}, -\frac{6}{23}, \frac{11}{23}\right) = \frac{33}{23}$ is an

absolute minimum, and there is no absolute maximum.