CONCORDIA UNIVERSITY **Department of Mathematics and Statistics**

Course	Number	Section(s)
Mathematics	**MAST 218**	**A, B**

Examination	Date	Time	Pages
Final	**December 2011**	**3 hours**	**2**

Instructors	Course Examiner
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Special Instructions: Calculators permitted. Lined paper booklets.

READ THE QUESTIONS CAREFULLY !!!

SHOW ALL WORK !!! JUSTIFY ALL STEPS !!!

GOOD LUCK !!!

MARKS: marks for each problem are shown in front of the problems.

↓MARKS

10 **Problem 1 :** Consider the plane curve defined by the parametric equations:
$x = t^2 + 2$, $y = 2e^t - 1$.

(a). Find d^2y/dx^2 in terms of t.

(b). Find the values of t at which the plane curve is concave upward.

10 **Problem 2 :** Consider the curves γ_1 and γ_2 defined by polar equations:

$$\gamma_1 : \quad r = 4\cos\theta, \qquad \gamma_2 : \quad r = 2.$$

(a). Identify the curves γ_1 and γ_2 by finding their **Cartesian equations** in $(x, y)-$coordinates, and **sketch the polar curves** γ_1 and γ_2.

(b). Find the area A of the region between the polar curves γ_1 and γ_2.

10 **Problem 3 :** Consider the function

$$f(x) = \int_0^x \frac{1}{4+t^2} dt.$$

(a). Find the Taylor series of $f(x)$ at $x = 0$.

(b). Find the radius R of convergence for the Taylor series obtained in (a).

10 **Problem 4 :** (a). Find an equation of the plane passing through the three points: $A\,(1,2,3)$, $B\,(3,-1,2)$, and $C\,(8,2,4)$.

(b). Find an equation of the line passing through the point $(1,-2,0)$ and perpendicular to the plane obtained in (a).

(c). Find the distance from the point $(1,-2,0)$ to the plane obtained in (a).

10 **Problem 5 :** Consider the space curve $\mathbf{r}(t) = \langle \cos t,\ \sqrt{3}t,\ \sin t\rangle$.

(a). Find an equation of the tangent line to the curve at $\mathbf{r}(1)$.

(b). Find the length of the curve for $t \in [1,3]$.

10 **Problem 6 :** Consider the space curve $\mathbf{r}(t) = \langle t,\ \frac{2}{3}t^3,\ t^2\rangle$.

(a). Find the unit tangent vector $\mathbf{T}(1)$ and the principal normal vector $\mathbf{N}(1)$ of the curve at $t = 1$.

(b). **Use the Chain Rule** to find the partial derivatives $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$, where

$$z = y^2 + e^{2x+y}, \quad x = \frac{s}{t}, \quad y = st.$$

10 **Problem 7 :** Find the limit, if it exists, or show that the limit does not exist:

$$(a).\quad \lim_{(x,y)\to(0,0)} \frac{3xy^3}{2x^2+y^6}, \qquad (b).\quad \lim_{(x,y)\to(0,0)} \frac{(x^4-y^4)\sin(2x^2+2y^2)}{x^2+y^2}.$$

10 **Problem 8 :** Consider the function $f(x,y) = e^y(y^2 - x^2)$

(a). Find all critical points of $f(x,y)$.

(b). Classify those critical points obtained in (a) as points of local minimum, local maximum, or saddle points.

10 **Problem 9 :** For the function $f(x,y) = x^2+y^2+x^2y+4$, find the absolute maximum and minimum values of $f(x,y)$ in the domain

$$D = \big\{(x,y) : |x| \le 1,\ |y| \le 1\big\}.$$

10 **Problem 10 :** Use the **Lagrange Multipliers** to find the maximum and minimum values of $f(x,y) = y^2 - x^2$ subject to the constraint: $x^2 + 4y^2 = 4$.