CONCORDIA UNIVERSITY **Department of Mathematics and Statistics**

Course	Number		Section
Mathematics	**MAST 218**		**AA**
Examination	Date	Time	Pages
Final	**April 2013**	**3 hours**	**2**
Instructor			Course Examiner
Ewa Duma			**Ewa Duma**

Instructions: Only approved calculators permitted. The value for each problem is indicated in square brackets in the margin (out of a possible total of 100). Show all your steps. Write the complete solution on the right hand pages of your examination booklet only.

MARKS: marks for each problem are shown in front of the problems.

↓MARKS

[4] **Problem 1 :** Find and sketch the domain of the function

$$f(x,y) = \sqrt{y} + \sqrt{25 - x^2 - y^2}\ .$$

[14] **Problem 2 :** Consider the curves γ_1 and γ_2 defined by polar equations:

$$\gamma_1 : \quad r = 3 + \sin\theta, \quad 0 \le \theta \le 2\pi; \qquad \gamma_2 : \quad r = 4\sin\theta, \quad 0 \le \theta \le \pi$$

(a) Identify the curve γ_2 by finding its **Cartesian equations** in $(x, y)-$coordinates, and **sketch the polar curves** γ_1 and γ_2.

(b) Find the area A of the region that lies inside the first curve γ_1 and outside the second curve γ_2.

[12] **Problem 3 :** Consider the curve $r(t) = \langle\ \sqrt{3}\ t,\ \sin(t),\ \cos(-t)\ \rangle, \quad 0 \le t \le 1.$

(a) Find the unit tangent vector $\vec{T}(t)$ of the curve.

(b) Find the length of the curve.

[12] **Problem 4 :** Let $P = (1, 1, 1)$, and let L be the line given parametrically

$$L: \quad (0, 1, 0) + t\ \langle 1, 2, 2 \rangle, \qquad -\infty < t < +\infty.$$

(a) Find the distance of point P from line L;

(b) Find the equation of the plane Q passing through point P and the line L;

[12] **Problem 5 :** Let the surface $z = f(x, y)$ be defined implicitly by the equation:

$$x^2 + 2y^2 - 3z^2 = 3\ .$$

(a) Find the partial derivatives $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

(b) Find the equation of the tangent plane at the point $(2, -1, 1)$.

[14] **Problem 6 :** Find the limit, if it exists, or show that the limit does not exist:

$$(a) \qquad \lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2 + y^4}\ , \qquad\qquad (b) \qquad \lim_{(x,y)\to(0,0)} \frac{3x^2 y}{x^2 + y^2}.$$

[18] **Problem 7 :** Consider the function $f(x, y) = x^3 - 6xy + 8y^3$.

(a) Find all critical points of $f(x, y)$.

(b) Classify the critical points obtained in (a) as local minimum, local maximum, or saddle points.

(c) Find the absolute maximum and minimum of $f(x, y)$ on the square bounded by the lines: $x = 0$, $y = 0$, $x = 1$ and $y = 1$.

[14] **Problem 8 :** Use **Lagrange Multipliers** to find the maximum and minimum values of $f(x, y) = y^2 - x^2$ subject to the constraint: $\frac{1}{4}x^2 + y^2 = 1$.

GOOD LUCK !!!