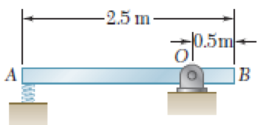
MCG 2108: DGD –Thursday 3rd Dec- Week 10

Problem : **17.61** : Board problem

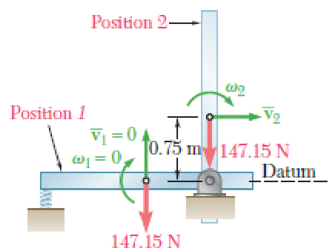
Problem(s) **17.17. 17.77** Assigned problems (for grading)

PROBLEM 17.17



A 15-kg slender rod AB is 2.5 m long and is pivoted about a point O which is 0.5 m from end B. The other end is pressed against a spring of constant k = 300 kN/m until the spring is compressed 40 mm. The rod is then in a horizontal position. If the rod is released from this position, determine its angular velocity and the reaction at the pivot O as the rod passes through a vertical position.

SOLUTION



Position 1. ***Potential Energy.*** Since the spring is compressed 40 mm, we have x_1 = 40 mm.

$$V_e = \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}(300{,}000\ \text{N/m})(0.040\text{m}^2) = 240\ \text{J}$$

Choosing the datum as shown, we have $V_g = 0$; therefore,

$$V_1 = V_e + V_g = 240\ \text{J}$$

Kinetic Energy. Since the velocity in position *1* is zero, we have $T_1 = 0$.

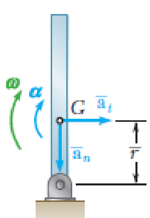
Position 2. ***Potential Energy.*** The elongation of the spring is zero, and we have $V_e = 0$. Since the center of gravity of the rod is now 0.75 m above the datum,

$$V_g = (147.15\ \text{N})(0.75\ \text{m}) = 110.4\ \text{J}$$
$$V_2 = V_e + V_g = 110.4\ \text{J}$$

Kinetic Energy. Denoting by $\boldsymbol{\omega}_2$ the angular velocity of the rod in position 2, we note that the rod rotates about O and write $\overline{v}_2 = \overline{r}\omega_2 = 0.75\omega_2$.

$$\overline{I} = \tfrac{1}{12}ml^2 = \frac{1}{12}(15\ \text{kg})(2.5\text{m})^2 = 7.81\,\text{kg-m}^2$$

$$T_2 = \tfrac{1}{2}m\overline{v}_2^2 + \tfrac{1}{2}\overline{I}\omega_2^2 = \frac{1}{2}\times(15)(0.75\,\omega_2)^2 + \frac{1}{2}\times(7.81)\omega_2^2 = 8.12\,\omega_2^2$$



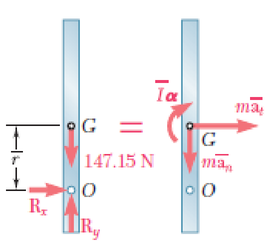
Conservation of Energy

$$T_1 + V_1 = T_2 + V_2$$
$$0 + 240\ \text{J} = 8.12\,\omega_2^2 + 110.4\ \text{J}$$

$$\omega_2 = 3.995\ \text{rad/s}\circlearrowright$$ ◀

Reaction in Position 2. Since ω_2 = 3.995 rad/s, the components of the acceleration of G as the rod passes through position 2 are

$$\overline{a}_n = \overline{r}\omega_2^2 = (0.75\,\text{m})(3.995\,\text{rad/s})^2 = 11.97\,\text{m/s}^2 \qquad \overline{\mathbf{a}}_n = 11.97\,\text{m/s}^2\downarrow$$
$$\overline{a}_t = \overline{r}\alpha \qquad \overline{\mathbf{a}}_t = \overline{r}\alpha\ \rightarrow$$

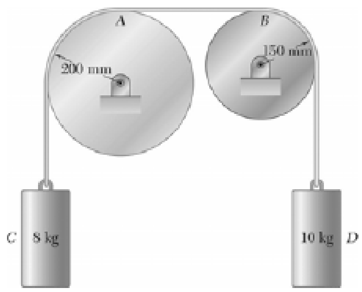


We express that the system of external forces is equivalent to the system of effective forces represented by the vector of components $m\overline{\mathbf{a}}_t$ and $m\overline{\mathbf{a}}_n$ attached at G and the couple $\overline{I}\boldsymbol{\alpha}$.

$$+\circlearrowright\Sigma M_O = \Sigma(M_O)_{\text{eff}}: \qquad 0 = \overline{I}\alpha + m(\overline{r}\alpha)\overline{r} \qquad \alpha = 0$$
$$\xrightarrow{+}\Sigma F_x = \Sigma(F_x)_{\text{eff}}: \qquad R_x = m(\overline{r}\alpha) \qquad R_x = 0$$
$$+\uparrow\Sigma F_y = \Sigma(F_y)_{\text{eff}}: \qquad R_y - 147.15\ \text{N} = -m\overline{a}_n$$

$$R_y - 147.15\ \text{N} = -(15\ \text{kg})\,(11.97\,\text{m/s}^2)$$

$$R_y = -32.4\ \text{N} \qquad \mathbf{R} = 32.4\ \text{N}\downarrow$$ ◀



PROBLEM 17.61

Two uniform disks and two cylinders are assembled as indicated. Disk A has a mass of 10 kg and disk B has a mass of 6 kg. Knowing that the system is released from rest, determine the time required for cylinder C to have a speed of 0.5 m/s.

The cylinders are attached to a single cord that passes over the disks. Assume that no slipping occurs between the cord and the disks.

SOLUTION

Moments of inertia.

Disk A: $I_A = \frac{1}{2} m_A r_A^2 = \frac{1}{2}(10\text{ kg})(0.200\text{ m})^2 = 0.2\text{ kg}\cdot\text{m}^2$

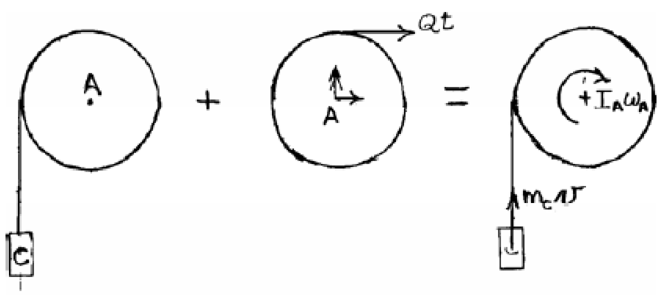
Disk B: $I_B = \frac{1}{2} m_B r_B^2 = \frac{1}{2}(6\text{ kg})(0.150\text{ m})^2 = 0.0675\text{ kg}\cdot\text{m}^2$

Kinematics: $\mathbf{v}_C = v\uparrow \qquad \mathbf{v}_D = v\downarrow$

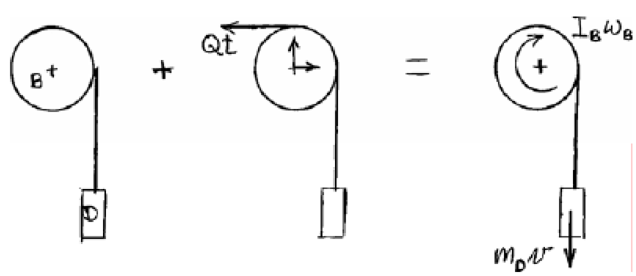
$\omega_A = \frac{v}{r_A} \circlearrowright \qquad \omega_B = \frac{v}{r_B} \circlearrowright$

Principle of impulse and momentum.

Disk A and cylinder C



Disk B and cylinder D



Syst. Momenta$_1$ + **Syst. Ext. Imp.**$_{1\to 2}$ = **Syst. Momenta**$_2$

Disk A and cylinder C. $\circlearrowleft +$ Moments about A:

$$Qtr_A - m_C g t r_A = m_A v r_A + I_A \omega_A \tag{1}$$

Disk B and cylinder D. $\circlearrowleft +$ Moments about B:

$$-Qtr_B - m_D g t r_B = m_D r_B v + I_B \omega_B \tag{2}$$

To eliminate Qt divide Equation (1) by r_A and Equation (2) by r_B, and then add the resulting equations.

$$(m_D - m_C) g t = \left(m_A + \frac{I_A}{r_A^2} + m_B + \frac{I_B}{r_B^2} \right) v \tag{3}$$

Data: $v = 0.5 \text{ m/s} \qquad t = ?$

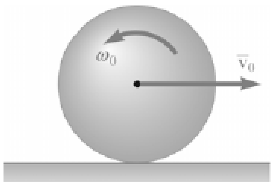
$$(m_D - m_C) g = (2 \text{ kg})(9.81 \text{ m/s}^2) = 19.62 \text{ N}$$

$$m_C + \frac{I_A}{r_A^2} + m_D + \frac{I_B}{r_B^2} = 8 \text{ kg} + \frac{0.2 \text{ kg} \cdot \text{m}^2}{(0.200 \text{ m})^2} + 10 \text{ kg} + \frac{0.0675 \text{ kg} \cdot \text{m}^2}{(0.150 \text{ m})^2} = 26 \text{ kg}$$

Equation (3) becomes $(19.62 \text{ N})t = (26 \text{ kg})(0.5 \text{ m})$

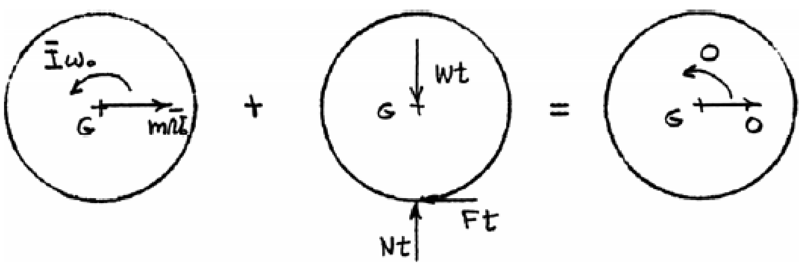
$$t = 0.66259 \text{ s}$$

$t = 0.663$ s ◀

	**PROBLEM 17.77** A sphere of radius r and mass m is projected along a rough horizontal surface with the initial velocities shown. If the final velocity of the sphere is to be zero, express (*a*) the required magnitude of $\boldsymbol{\omega}_0$ in terms of v_0 and r, (*b*) the time required for the sphere to come to rest in terms of v_0 and coefficient of kinetic friction μ_k.

SOLUTION

Moment of inertia. Solid sphere. $\overline{I} = \frac{2}{5}mr^2$



Syst. Momenta$_1$ + **Syst. Ext. Imp.$_{1\to 2}$** = **Syst. Momenta$_2$**

$+\uparrow$ y components: $Nt - Wt = 0 \qquad N = W = mg$ (1)

$\xrightarrow{+}$ x components: $m\overline{v}_0 - Ft = 0 \qquad Ft = m\overline{v}_0$ (2)

$+\circlearrowright$ Moments about G: $\overline{I}\omega_0 - Ftr = 0$ (3)

$$\frac{2}{5}mr^2\omega_0 - m\overline{v}_0 r = 0$$

(*a*) *Solving for* ω_0, $\omega_0 = \frac{5}{2}\frac{\overline{v}_0}{r}$ ◀

(*b*) *Time to come to rest.*

From Equation (2), $t = \frac{mv_0}{F} = \frac{m\overline{v}_0}{\mu_k mg}$ $t = \frac{\overline{v}_0}{\mu_k g}$ ◀