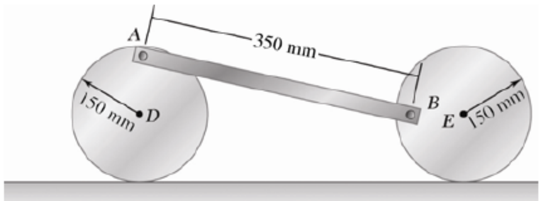
MCG 2108: DGD –Monday 16 th Nov- Week 8

Problem : **15.112** : Board problem

Problem(s) **15.70, 15.123** Assigned problems (for grading)

Problem : 15.121 (**only if time left in DGD**, otherwise give some hints and assigned it to students to attempt on their own after the DGD)

No QUIZ this week



PROBLEM 15.70

Both 150 mm-radius wheels roll without slipping on the horizontal surface. Knowing that the distance *AD* is 125 mm, the distance *BE* is 100 mm and *D* has a velocity of 150 mm/s to the right, determine the velocity of Point *E*.

SOLUTION

Disk *D*: Velocity at the contact Point *P* with the ground is zero.

$$\mathbf{v}_0 = 150 \text{ mm/s} \longrightarrow$$

$$\omega_D = \frac{v_D}{r_{D/P}} = \frac{150 \text{ mm/s}}{150 \text{ mm}} = 1 \text{ rad/s} \qquad \boldsymbol{\omega}_D = 1 \text{ rad/s} \curvearrowright$$

At Point *A*,

$$v_A = r_{A/P}\omega_D = (150 \text{ mm} + 125 \text{ mm})(1 \text{ rad/s}) = 275 \text{ mm/s}$$

$$\mathbf{v}_A = 275 \text{ mm/s} \longrightarrow$$

Disk *E*: Velocity at the contact Point *Q* with the ground is zero. $\boldsymbol{\omega}_E = \omega_E \circlearrowleft = \omega_E \mathbf{k}$.

$$\mathbf{r}_{B/Q} = -(100 \text{ mm})\mathbf{i} + (150 \text{ mm})\mathbf{j}$$

$$\mathbf{v}_B = \mathbf{v}_{B/Q} = \boldsymbol{\omega}_E \times \mathbf{r}_{B/Q} = \omega_E \mathbf{k} \times (-100\mathbf{i} + 150\mathbf{j})$$

$$\mathbf{v}_B = -150\omega_E \mathbf{i} - 100\omega_E \mathbf{j} \qquad (1)$$

Connecting rod *AB*:

$$\mathbf{r}_{B/A} = (\sqrt{(350)^2 - (125)^2})\mathbf{i} - 125\mathbf{j} \text{ in mm.}$$

$$\mathbf{v}_{B/A} = \sqrt{106875}\mathbf{i} - 125\mathbf{j} \qquad \boldsymbol{\omega}_{AB} = \omega_{AB}\mathbf{k}$$

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} = \mathbf{v}_A + \omega_{AB}\mathbf{k} \times (\sqrt{106875}\mathbf{i} - 125\mathbf{j})$$

$$= 275\mathbf{i} + 125\omega_{AB}\mathbf{i} + \sqrt{106875}\omega_{AB}\mathbf{j} \qquad (2)$$

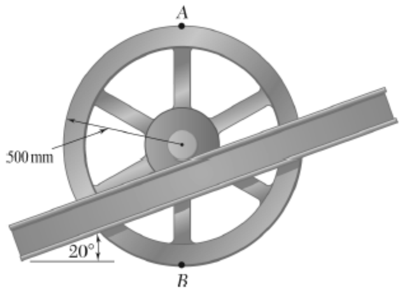
Equating expressions (1) and (2) for $\mathbf{v}_B$ gives

$$-150\omega_E \mathbf{i} - 100\omega_E \mathbf{j} = 275\mathbf{i} + 125\omega_{AB}\mathbf{i} + \sqrt{106875}\omega_{AB}\mathbf{j}$$

Equating like components and transposing terms,

i: $$125\omega_{AB} + 150\omega_E = -275 \qquad (3)$$

j: $$\sqrt{106875}\omega_{AB} + 100\omega_E = 0 \qquad (4)$$



PROBLEM 15.112

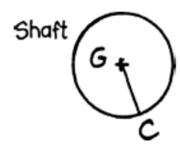
The 500 mm-radius flywheel is rigidly attached to a 40 mm-radius shaft that can roll along parallel rails. Knowing that at the instant shown the center of the shaft has a velocity of 30 mm/s and an acceleration of $10\ \text{mm/s}^2$, both directed down to the left, determine the acceleration (*a*) of Point *A*, (*b*) of Point *B*.

SOLUTION

Velocity analysis.

Let Point *G* be the center of the shaft and Point *C* be the point of contact with the rails. Point *C* is the instantaneous center of the wheel and shaft since that point does not slip on the rails.

$$v_G = r\omega, \quad \omega = \frac{v_G}{r} = \frac{30}{40} = 0.75 \text{ rad/s} \circlearrowright$$



Acceleration analysis.

Since the shaft does not slip on the rails,

$$\mathbf{a}_C = a_C \nwarrow 20°$$

Also,

$$\mathbf{a}_G = [10 \text{ mm/s}^2 \swarrow 20°]$$

$$\mathbf{a}_C = \mathbf{a}_G + (\mathbf{a}_{C/G})_t + (\mathbf{a}_{C/G})_n$$

$$[a_C \nwarrow 20°] = [10 \text{ mm/s}^2 \swarrow 20°] + [40\alpha \nearrow 20°] + [40\omega^2 \nwarrow 20°]$$

Components $\swarrow$ 20°: $\quad 10 = -40\alpha \quad \alpha = 0.25 \text{ rad/s}^2 \circlearrowright$

(*a*) *Acceleration of Point A.*

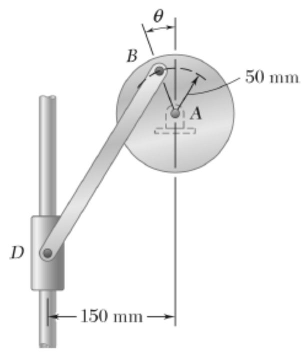
$$\mathbf{a}_A = \mathbf{a}_G + (\mathbf{a}_{A/G})_t + (\mathbf{a}_{A/G})_n$$
$$= [10 \swarrow 20°] + [500\alpha \leftarrow] + [500\omega^2 \downarrow]$$
$$= [9.3969 \leftarrow] + [3.4202 \downarrow] + [125 \leftarrow] + [281.25 \downarrow]$$
$$= [134.3969 \leftarrow] + [284.6702 \downarrow]$$

$\mathbf{a}_A = 315 \text{ mm/s}^2 \swarrow 64.7°$ ◀

(*b*) *Acceleration of Point B.*

$$\mathbf{a}_B = \mathbf{a}_G + (\mathbf{a}_{B/G})_t + (\mathbf{a}_{B/G})_n$$
$$= [10 \swarrow 20°] + [500\alpha \rightarrow] + [500\omega^2 \uparrow]$$
$$= [9.3969 \leftarrow] + [3.4202 \downarrow] + [125 \rightarrow] + [281.25 \uparrow]$$
$$= [115.603 \rightarrow] + [277.8298 \uparrow]$$

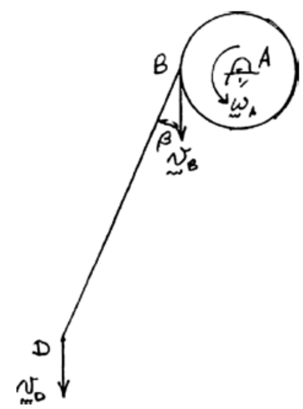
$\mathbf{a}_B = 301 \text{ mm/s}^2 \nearrow 67.4°$ ◀



PROBLEM 15.123

The disk shown has a constant angular velocity of 500 rpm counterclockwise. Knowing that rod BD is 250 mm long, determine the acceleration of collar D when (*a*) $\theta = 90°$, (*b*) $\theta = 180°$.

SOLUTION



<u>Disk *A*.</u>

$$\boldsymbol{\omega}_A = 500 \text{ rpm} \circlearrowleft = 52.36 \text{ rad/s} \circlearrowleft$$

$$\boldsymbol{\alpha}_A = 0, \quad (AB) = 50 \text{ mm} = 0.05 \text{ m}$$

$$v_B = (AB)\omega_A = (0.05)(52.36) = 2.618 \text{ m/s}$$

$$a_B = (AB)\omega_A^2 = (0.05)(52.36)^2 = 137.08 \text{ m/s}^2$$

(*a*) $\theta = 90°$.

$$\mathbf{v}_B = 2.618 \text{ m/s}\downarrow, \quad \mathbf{v}_D = v_D\downarrow$$

$$\sin\beta = \frac{100 \text{ mm}}{250 \text{ mm}} = 0.4 \quad \beta = 23.58°$$

$\mathbf{v}_D$ and $\mathbf{v}_B$ are parallel.

$$\omega_{BD} = 0$$

$$\mathbf{a}_B = 137.08 \text{ m/s}^2 \rightarrow, \quad \mathbf{a}_D = a_D\uparrow, \quad \boldsymbol{\alpha}_{BD} = \alpha_{BD} \circlearrowleft$$

$$\mathbf{a}_{D/B} = [(BD)\alpha_{BD} \searrow \beta] + [(BD)\omega_{BD}^2 \nearrow \beta]$$

$$= [0.25\ \alpha_{BD} \searrow \beta] + 0$$

$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{D/B}$ Resolve into components.

$$\xrightarrow{+} 0 = 137.08 + (0.25 \cos\beta)\alpha_{BD} \qquad \alpha_{BD} = -589.3 \text{ rad/s}^2$$

$$+\uparrow a_D = 0 - (0.25 \sin\beta)(-589.3) + 0 = 59.83 \text{ m/s}^2$$

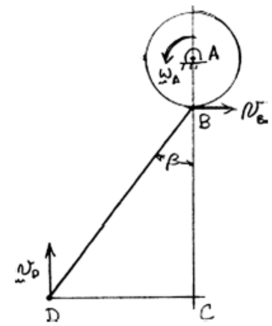
$$\mathbf{a}_D = 59.8 \text{ m/s}^2\uparrow \blacktriangleleft$$

PROBLEM 15.123 (Continued)

(*b*) $\theta = 180°$. $\mathbf{v}_B = 2.618 \text{ m/s} \rightarrow, \quad \mathbf{v}_D = v_D \uparrow$

$$\sin\beta = \frac{150 \text{ mm}}{250 \text{ mm}} = 0.6 \quad \beta = 36.87°$$

$$\mathbf{v}_B = 2.618 \text{ m/s} \rightarrow, \quad \mathbf{v}_D = v_D \uparrow$$



Instantaneous center of bar *BD* lies at Point *C*.

$$\omega_{BD} = \frac{v_B}{(BD)} = \frac{2.618}{0.25 \cos\beta} = 13.09 \text{ rad/s}$$

$$\mathbf{a}_B = 137.08 \text{ m/s}^2 \uparrow, \quad \mathbf{a}_D = a_D \uparrow, \quad \boldsymbol{\alpha}_{BD} = \alpha_{BD} \circlearrowleft$$

$$\mathbf{a}_{D/B} = [(BD)\alpha_{BD} \searrow \beta] + [(BD)\omega_{BD}^2 \nearrow \beta]$$

$$= [0.25\alpha_{BD} \searrow \beta] + [42.837 \nearrow \beta]$$

$$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{D/B} \quad \text{Resolve in components.}$$

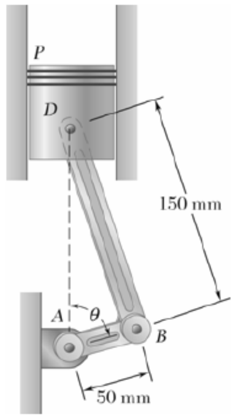
$$\xrightarrow{+} 0 = 0 + (0.25 \cos\beta)\alpha_{BD} + 42.837 \sin\beta$$

$$\alpha_{BD} = -128.51 \text{ rad/s}^2$$

$$+\!\uparrow a_D = 137.08 - (0.25 \sin\beta)(-128.51) + 42.837 \cos\beta$$

$$= 190.63 \text{ m/s}^2$$

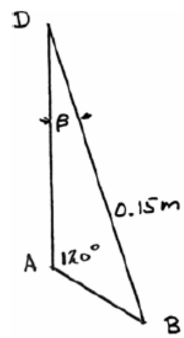
$\mathbf{a}_D = 190.6 \text{ m/s}^2 \uparrow$ ◀



PROBLEM 15.121

Knowing that crank AB rotates about Point A with a constant angular velocity of 900 rpm clockwise, determine the acceleration of the piston P when $\theta = 120°$.

SOLUTION



Law of sines. $\frac{\sin\beta}{0.05} = \frac{\sin 120°}{0.15}, \qquad \beta = 16.779°$

Velocity analysis. $\omega_{AB} = 900 \text{ rpm} = 30\pi \text{ rad/s} \circlearrowright$

$$\mathbf{v}_B = 0.05\omega_{AB} = 1.5\pi \text{ m/s} \measuredangle 60°$$

$$\mathbf{v}_D = v_D \downarrow \qquad \omega_{BD} = \omega_{BD} \circlearrowleft$$

$$\mathbf{v}_{D/B} = 0.15\omega_{BD} \measuredangle \beta$$

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B}$$

$$[v_D \downarrow] = [1.5\pi \measuredangle 60°] + [0.15\omega_{BD} \measuredangle \beta]$$

Components $\xrightarrow{+}$: $0 = -1.5\pi \cos 60° - 0.15\omega_{BD} \cos\beta$

$$\omega_{BD} = -\frac{1.5\pi \cos 60°}{0.15 \cos\beta} = 16.4065 \text{ rad/s} \circlearrowright$$

Acceleration analysis. $\alpha_{AB} = 0$

$$\mathbf{a}_B = 0.05\omega_{AB}^2 = (0.05)(30\pi)^2 = 444.13 \text{ m/s}^2 \measuredangle 30°$$

$$\mathbf{a}_D = a_D \downarrow \qquad \alpha_{BD} = \alpha_{BD} \circlearrowright$$

$$\mathbf{a}_{D/B} = [0.15\alpha_{AB} \measuredangle \beta] + [0.15\omega_{BD}^2 \measuredangle \beta]$$

$$= [6\alpha_{BD} \measuredangle \beta] + [40.376 \measuredangle \beta]$$

$\mathbf{a}_D = \mathbf{a}_B + \mathbf{a}_{D/B}$ Resolve into components.

PROBLEM 15.121 (Continued)

$\xrightarrow{+}$: $0 = -444.13\cos 30° + 0.15\alpha_{BD}\cos\beta + 40.376\sin\beta$

$\alpha_{BD} = 2597.0 \text{ rad/s}^2$

$+\downarrow$: $a_D = -444.13\sin 30° - (0.15)(2597.0)\sin\beta + 40.376\cos\beta$

$= -296 \text{ m/s}^2 \qquad \mathbf{a}_P = \mathbf{a}_D$ $\qquad \mathbf{a}_P = 296 \text{ m/s}^2 \uparrow$ ◀