MCG 2108: DGD –Monday 9th Nov- Week 7

Problem : **14.45** : Board problem

Problem(s) **14.35,14.106** Assigned problems

PROBLEM 14.35

Two automobiles A and B, of mass m_A and m_B, respectively, are traveling in opposite directions when they collide head on. The impact is assumed perfectly plastic, and it is further assumed that the energy absorbed by each automobile is equal to its loss of kinetic energy with respect to a moving frame of reference attached to the mass center of the two-vehicle system. Denoting by E_A and E_B, respectively, the energy absorbed by automobile A and by automobile B, (*a*) show that $E_A/E_B = m_B/m_A$, that is, the amount of energy absorbed by each vehicle is inversely proportional to its mass, (*b*) compute E_A and E_B, knowing that $m_A = 1600$ kg and $m_B = 900$ kg and that the speeds of A and B are, respectively, 90 km/h and 60 km/h.

SOLUTION

Velocity of mass center:

$$(m_A + m_B)\overline{\mathbf{v}} = m_A \mathbf{v}_A + m_B \mathbf{v}_B$$

$$\overline{\mathbf{v}} = \frac{m_A \mathbf{v}_A + m_B \mathbf{v}_B}{m_A + m_B}$$

Velocities relative to the mass center:

$$\mathbf{v}'_A = \mathbf{v}_A - \overline{\mathbf{v}} = \mathbf{v}_A - \frac{m_A \mathbf{v}_A + m_B \mathbf{v}_B}{m_A + m_B} = \frac{m_B(\mathbf{v}_A + \mathbf{v}_B)}{m_A + m_B}$$

$$\mathbf{v}'_B = \mathbf{v}_B - \overline{\mathbf{v}} = \mathbf{v}_B - \frac{m_A \mathbf{v}_A + m_B \mathbf{v}_B}{m_A + m_B} = \frac{m_A(\mathbf{v}_A + \mathbf{v}_B)}{m_A + m_B}$$

Energies:

$$E_A = \frac{1}{2} m_A \mathbf{v}'_A \cdot \mathbf{v}'_A = \frac{m_A m_B^2 (\mathbf{v}_A + \mathbf{v}_B)\cdot(\mathbf{v}_A + \mathbf{v}_B)}{2(m_A + m_B)^2}$$

$$E_B = \frac{1}{2} m_B \mathbf{v}'_B \cdot \mathbf{v}'_B = \frac{m_A^2 m_B (\mathbf{v}_A + \mathbf{v}_B)\cdot(\mathbf{v}_A + \mathbf{v}_B)}{2(m_A + m_B)^2}$$

(*a*) Ratio: $\dfrac{E_A}{E_B} = \dfrac{m_B}{m_A}$ ◀

(*b*)

$$\mathbf{v}_A = 90 \text{ km/h} = 25 \text{ m/s} \longrightarrow$$

$$\mathbf{v}_B = 60 \text{ km/h} = 16.667 \text{ m/s} \longleftarrow$$

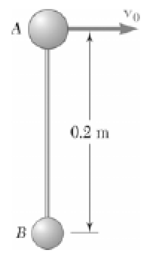
$$\mathbf{v}_A + \mathbf{v}_B = 41.667 \text{ m/s} \longrightarrow$$

$$E_A = \frac{(1600)(900)^2(41.667)^2}{(2)(2500)^2} = 180.0 \times 10^3 \text{ J}$$

$E_A = 180.0$ kJ ◀

$$E_B = \frac{(1600)^2(900)(41.667)^2}{(2)(2500)^2} = 320 \times 10^3 \text{ J}$$

$E_B = 320$ kJ ◀



PROBLEM 14.45

Two small spheres A and B, of mass 2.5 kg and 1 kg, respectively, are connected by a rigid rod of negligible weight. The two spheres are resting on a horizontal, frictionless surface when A is suddenly given the velocity $\mathbf{v}_0 = (3.5\text{ m/s})\mathbf{i}$. Determine (*a*) the linear momentum of the system and its angular momentum about its mass center G, (*b*) the velocities of A and B after the rod AB has rotated through 180°.

SOLUTION

Position of mass center:

$$\bar{y} = \sum \frac{m_i y_i}{m_i} = \frac{2.5(0) + 1(0.2)}{2.5 + 1} = 0.057143\text{ m}$$

(*a*) *Linear and angular momentum*:

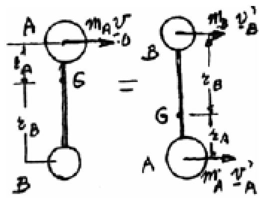
$$L = m_A v_0 = 2.5\text{ kg}(3.5\text{ m/s})\mathbf{i} = (8.75\text{ kg}\cdot\text{m/s})\mathbf{i}$$

$$\mathbf{L} = (8.75\text{ kg}\cdot\text{m/s})\mathbf{i} \blacktriangleleft$$

$$\mathbf{H}_G = \overline{GA} \times m_A \mathbf{v}_0 = (0.05714285\text{ m})\mathbf{j} \times (8.75\text{ kg}\cdot\text{m/s})\mathbf{i}$$
$$= -(0.50000\text{ kg}\cdot\text{m}^2/\text{s})\mathbf{k}$$

$$\mathbf{H}_G = -(0.500\text{ kg}\cdot\text{m}^2/\text{s})\mathbf{k} \blacktriangleleft$$

(*b*) *Velocities of A and B after 180° rotation*



Conservation of linear momentum:

$$m_A v_0 = m_A v'_A + m_B v'_B$$
$$(2.5)(3.5) = (2.5)v'_A + (1.0)v'_B$$

$$2.55 v'_A + v'_B = 8.75 \qquad (1)$$

Conservation of angular momentum about G':

$$+\circlearrowleft: \quad r_A m_A v_0 = -r_A m_A v'_A + r_B m_B v'_B$$

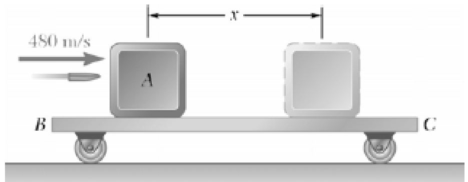
$$r_B = 0.20 - r_A = 0.14286\text{ m}$$
$$(0.057143)(2.5)(3.5) = -(0.057143)(2.5)v'_A + (0.14286)(1.0)v'_B$$

Dividing by 0.057143: $\quad -2.5v'_A + \dfrac{0.14286}{0.057143} v'_B = 8.75 \qquad (2)$

Add Eqs. (1) and (2): $\quad 3.5 v'_B = 17.5 \qquad v'_B = +5.00\text{ m/s}$

From Eq. (1): $\quad 2.5 v'_A + (5) = 8.75 \qquad v'_A = +1.50\text{ m/s}$

$$\mathbf{v}'_A = (1.50\text{ m/s})\mathbf{i}; \quad \mathbf{v}'_B = (5.00\text{ m/s})\mathbf{i} \blacktriangleleft$$

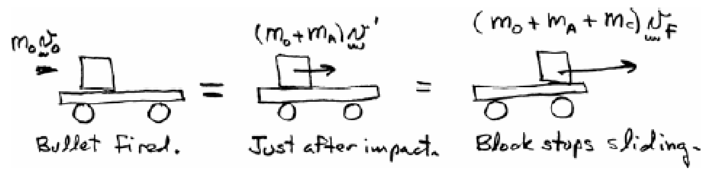


PROBLEM 14.106

A 30-g bullet is fired with a velocity of 480 m/s into block A, which has a mass of 5 kg. The coefficient of kinetic friction between block A and cart BC is 0.50. Knowing that the cart has a mass of 4 kg and can roll freely, determine (*a*) the final velocity of the cart and block, (*b*) the final position of the block on the cart.

SOLUTION

(*a*) Conservation of linear momentum:



$$m_0 v_0 = (m_0 + m_A)v' = (m_0 + m_A + m_C)v_f$$

$$(0.030 \text{ kg})(480 \text{ m/s}) = (5.030 \text{ kg})v' = (9.030 \text{ kg})v_f$$

$$v' = \frac{0.030}{5.030}(480 \text{ m/s}) = 2.863 \text{ m/s}$$

$$v_f = \frac{0.030}{9.030}(480 \text{ m/s}) = 1.5947 \text{ m/s} \qquad v_f = 1.595 \text{ m/s} \blacktriangleleft$$

(*b*) Work-energy principle:

Just after impact: $T' = \frac{1}{2}(m_0 + m_A)v'^2$

$= \frac{1}{2}(5.030 \text{ kg})(2.863 \text{ m/s})^2$

$= 20.615 \text{ J}$

Final kinetic energy: $T_f = \frac{1}{2}(m_0 + m_A + m_C)\frac{1}{2}v_f^2$

$= \frac{1}{2}(9.030 \text{ kg})(1.5947 \text{ m/s})^2$

$= 11.482 \text{ J}$

Work of friction force: $F = \mu_k N$

$= \mu_k (m_0 + m_A)g$

$= 0.50(5.030)(9.81)$

$= 24.672 \text{ N}$

Work $= U = -Fx = -24.672x$

$$T' + U = T_f: \quad 20.615 - 24.672x = 11.482 \qquad x = 0.370 \text{ m} \blacktriangleleft$$