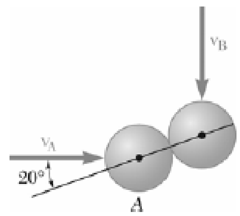
MCG 2108: DGD –Monday 2nd Nov- Week 6

Problem : **13-167** : Board problem

Problem(s) **14.6, 14.8** Assigned problems

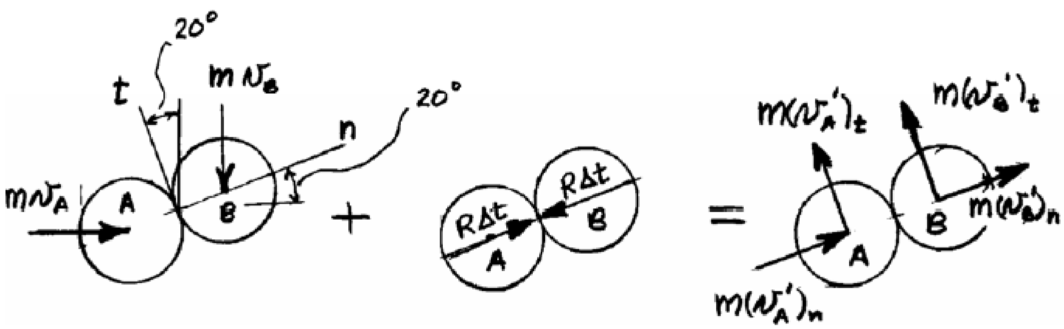
PROBLEM 13.167



Two identical hockey pucks are moving on a hockey rink at the same speed of 3 m/s and in perpendicular directions when they strike each other as shown. Assuming a coefficient of restitution e = 0.9, determine the magnitude and direction of the velocity of each puck after impact.

SOLUTION

Use principle of impulse-momentum: $\Sigma m\mathbf{v}_1 + \Sigma \mathbf{Imp}_{1\to 2} = \Sigma m\mathbf{v}_2$



t-direction for puck A:

$$-mv_A \sin 20° + 0 = m(v'_A)_t$$
$$(v'_A)_t = v_A \sin 20° = 3 \sin 20° = 1.0261 \text{ m/s}$$

t-direction for puck B:

$$-mv_B \cos 20° + 0 = m(v'_B)_t$$
$$(v'_B)_t = v_B \cos 20° = -3 \cos 20° = -2.8191 \text{ m/s}$$

n-direction for both pucks:

$$mv_A \cos 20° - mv_B \sin 20° = m(v'_A)_n + m(v'_B)_n$$
$$(v'_A)_n + (v'_B)_n = v_A \cos 20° - v_B \sin 20°$$
$$= 3 \cos 20° - 3 \sin 20° \quad (1)$$

Coefficient of restitution: $e = 0.9$

$$(v'_B)_n - (v'_A)_n = e[(v_A)_n - (v_B)_n]$$
$$= 0.9[3 \cos 20° - (-3) \sin 20°] \quad (2)$$

Solving Eqs. (1) and (2) simultaneously,

$$(v'_A)_n = -0.8338 \text{ m/s} \qquad (v'_B)_n = 2.6268 \text{ m/s}$$

PROBLEM 13.167 (Continued)

Summary:

$(v'_A)_n = 0.8338$ m/s ∠ 20°

$(v'_A)_t = 1.0261$ m/s ∠ 70°

$(v'_B)_n = 2.6268$ m/s ∠ 20°

$(v'_B)_t = 2.8191$ m/s ∠ 70°

$$v_A = \sqrt{(0.8338)^2 + (1.0261)^2} = 1.322 \text{ m/s}$$

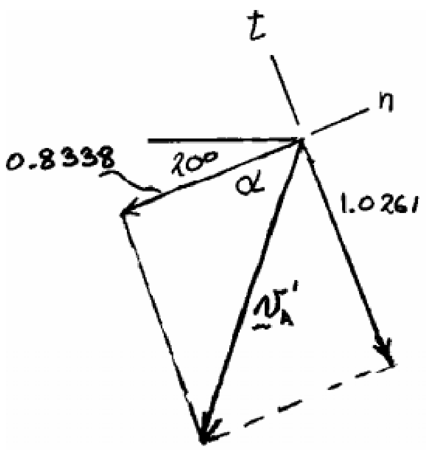
$$\tan\alpha = \frac{1.0261}{0.8338} \qquad \alpha = 50.9° \qquad \alpha + 20° = 70.9°$$

$v'_A = 1.322$ m/s ∠ 70.9° ◀

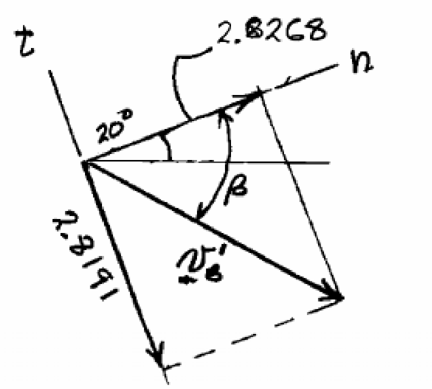
$$v'_B = \sqrt{(2.6268)^2 + (2.8191)^2} = 3.85 \text{ m/s}$$

$$\tan\beta = \frac{2.8191}{2.6268} \qquad \beta = 47.0° \qquad \beta - 20° = 27.0°$$

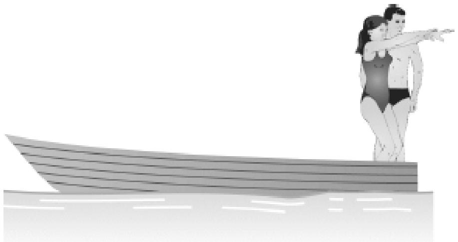
$v'_B = 3.85$ m/s ∠ 27.0° ◀



Velocity of Puck A



Velocity of Puck B



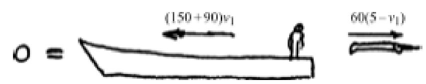
PROBLEM 14.6

A 90-kg man and a 60 kg woman stand side by side at the same end of a 150-kg boat ready to dive, each with a 5-m/s velocity relative to the boat. Determine the velocity of the boat after they have both dived, if (*a*) the woman dives first, (*b*) the man dives first.

SOLUTION

(*a*) <u>Woman dives first.</u>

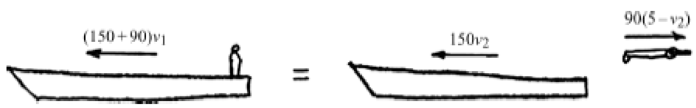
Conservation of momentum:



$$60(5 - v_1) - (150 + 90)v_1 = 0$$

$$v_1 = \frac{300}{300} = 1 \text{ m/s} \longleftarrow$$

Man dives next. Conservation of momentum:



$$-240v_1 = -150v_2 + 90(5 - v_2)$$

$$v_2 = \frac{240v_1 + 450}{240} = 2.875 \text{ m/s} \qquad \mathbf{v}_2 = 2.88 \text{ m/s} \longleftarrow \blacktriangleleft$$

(*b*) <u>Man dives first.</u>

Conservation of momentum:

$$90(5 - v_1') - 210v_1' = 0$$

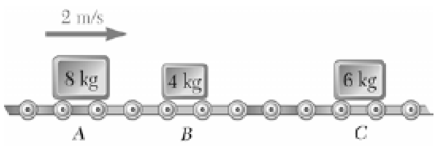
$$v_1' = \frac{450}{300} = 1.5 \text{ m/s}$$

Woman dives next. Conservation of momentum:

$$-210v_1' = -150v_2' + 60(5 - v_2')$$

$$v_2' = \frac{210v_1' + 300}{210} = 2.9286 \text{ m/s} \qquad \mathbf{v}_2' = 2.93 \text{ m/s} \longleftarrow \blacktriangleleft$$

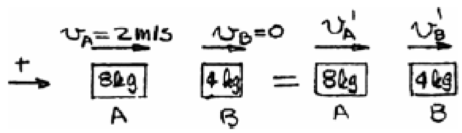
PROBLEM 14.8



Packages in an automobile parts supply house are transported to the loading dock by pushing them along on a roller track with very little friction. At the instant shown, packages B and C are at rest and package A has a velocity of 2 m/s. Knowing that the coefficient of restitution between the packages is 0.3, determine (*a*) the velocity of package C after A hits B and B hits C, (*b*) the velocity of A after it hits B for the second time.

SOLUTION

(*a*) Packages A and B:



Total momentum conserved:

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$$

$$(8\text{ kg})(2\text{ m/s}) + 0 = (8\text{ kg})v'_A + (4\text{ kg})v'_B$$

$$4 = 2v'_A + v'_B \qquad (1)$$

Relative velocities.

$$(v_A - v_B)e = (v'_B - v'_A)$$

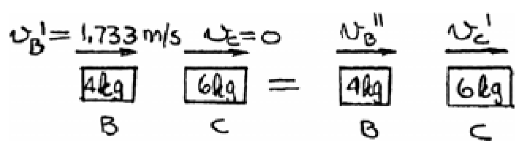
$$(2)(0.3) = v'_B - v'_A \qquad (2)$$

Solving Equations (1) and (2) simultaneously,

$$v'_A = 1.133\text{ m/s} \longrightarrow$$

$$v'_B = 1.733\text{ m/s} \longrightarrow$$

Packages B and C:



$$\xrightarrow{+} m_B v'_B + m_C v_C = m_B v''_B + m_C v'_C$$

$$(4\text{ kg})(1.733\text{ m/s}) + 0 = 4v''_B + 6v'_c$$

$$6.932 = 4v''_B + 6v'_C \qquad (3)$$

PROBLEM 14.8 (Continued)

Relative velocities:

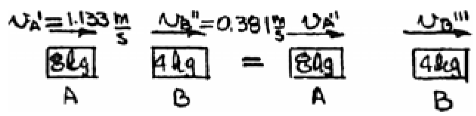
$$(v'_B - v_C)e = v'_C - v''_B$$

$$(1.733)(0.3) = 0.5199 = v'_C - v''_B \qquad (4)$$

Solving equations (3) and (4) simultaneously,

$$\mathbf{v}'_C = 0.901 \text{ m/s} \longrightarrow \blacktriangleleft$$

(*b*) Packages *A* and *B* (second time),



Total momentum conserved:

$$(8)(1.133) + (4)(0.381) = 8v''_A + 4v'''_B$$

$$10.588 = 8v''_A + 4v''_B \qquad (5)$$

Relative velocities:

$$(v'_A - v''_B)e = v'''_B - v''_A$$

$$(1.133 - 0.381)(0.3) = 0.2256 = v'''_B - v''_A \qquad (6)$$

Solving (5) and (6) simultaneously,

$$v''_A = 0.807 \text{ m/s} \qquad \mathbf{v}''_A = 0.807 \text{ m/s} \longrightarrow \blacktriangleleft$$