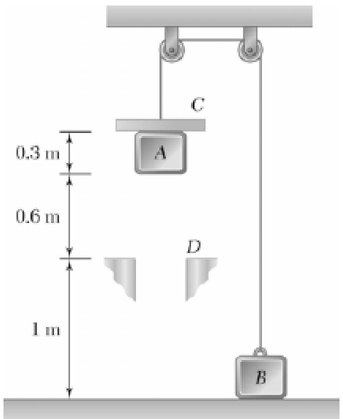
MCG 2108- Fall 2015

DGD questions for Thursday 15th October and Monday 19th October.

Problem 13.33 is the board problem

Problem 13.24, 13.57, 13.143 will be assigned to students



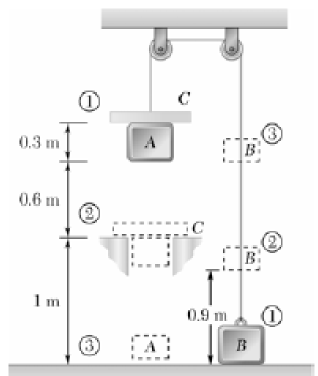
PROBLEM 13.24

Two blocks A and B, of mass 4 kg and 5 kg, respectively, are connected by a cord which passes over pulleys as shown. A 3 kg collar C is placed on block A and the system is released from rest. After the blocks have moved 0.9 m, collar C is removed and blocks A and B continue to move. Determine the speed of block A just before it strikes the ground.

SOLUTION

Position ① to Position ②. $v_1 = 0 \quad T_1 = 0$

At ② before C is removed from the system



$$T_2 = \frac{1}{2}(m_A + m_B + m_C)v_2^2 = \frac{1}{2}(12 \text{ kg})v_2^2 = 6v_2^2$$

$$U_{1-2} = (m_A + m_C - m_B)g(0.9 \text{ m})$$

$$U_{1-2} = (4+3-5)(g)(0.9 \text{ m}) = (2 \text{ kg})(9.81 \text{ m/s}^2)(0.9 \text{ m})$$

$$U_{1-2} = 17.658 \text{ J}$$

$$T_1 + U_{1-2} = T_2:$$

$$0 + 17.658 = 6v_2^2 \qquad v_2^2 = 2.943$$

At Position ②, collar C is removed from the system.

Position ② to Position ③. $T_2' = \frac{1}{2}(m_A + m_B)v_2^2 = \left(\frac{9}{2}\text{ kg}\right)(2.943) = 13.244 \text{ J}$

$$T_3 = \frac{1}{2}(m_A + m_B)(v_3)^2 = \frac{9}{2}v_3^2$$

$$U_{2'-3} = (m_A - m_B)(g)(0.7 \text{ m}) = (-1 \text{ kg})(9.81 \text{ m/s}^2)(0.7 \text{ m}) = -6.867 \text{ J}$$

$$T_2' + U_{2-3} = T_3$$

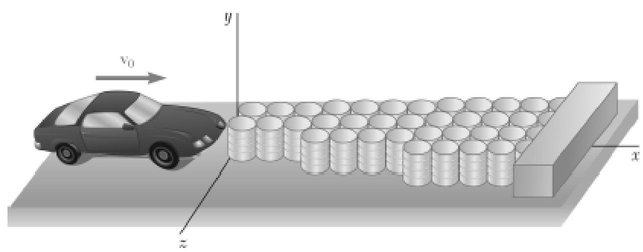
$$13.244 - 6.867 = 4.5v_3^2 \qquad v_3^2 = 1.417$$

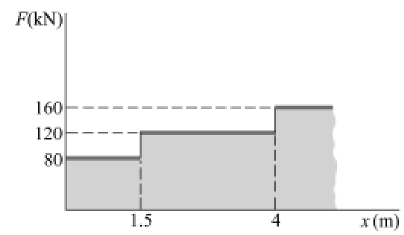
$$v_A = v_3 = 1.190 \text{ m/s}$$

$v_A = 1.190$ m/s ◀

PROBLEM 13.33

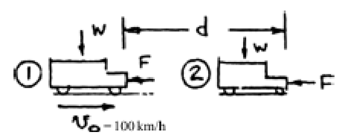
An uncontrolled automobile traveling at 100 km/h strikes squarely a highway crash cushion of the type shown in which the automobile is brought to rest by successively crushing steel barrels. The magnitude **F** of the force required to crush the barrels is shown as a function of the distance x the automobile has moved into the cushion. Knowing that the weight of the automobile is 1000 kg and neglecting the effect of friction, determine (*a*) the distance the automobile will move into the cushion before it comes to rest, (*b*) the maximum deceleration of the automobile.





SOLUTION

(*a*) $100 \text{ km/h} = \dfrac{250}{9} \text{ m/s}$

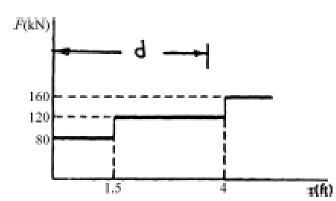


<u>Assume auto stops in $1.5 \text{ m} < d < 4 \text{ m}$.</u>

$$v_1 = \frac{250}{9} \text{ m/s}$$

$$T_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(1000)\left(\frac{250}{9}\right)^2$$

$$T_1 = 385{,}802.5 \text{ J}$$
$$= 385.8025 \text{ kJ}$$



$$v_2 = 0$$
$$T_2 = 0$$

$$U_{1-2} = -\{(80 \text{ kN})(1.5 \text{ m}) + (120 \text{ kN})(d - 1.5)\}$$
$$= -(120 + 120d - 180)$$
$$= -(120d - 60) \text{ kN}\cdot\text{m}$$

$$T_1 + U_{1-2} = T_2$$
$$385.8025 = 120d - 60$$
$$d = 3.715 \text{ m}$$

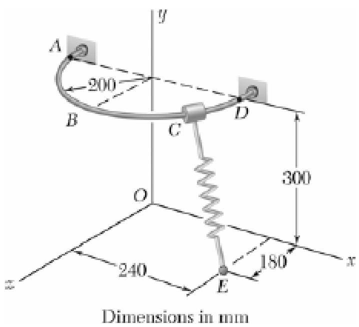
$d = 3.72 \text{ m}$ ◀

Assumption that $d < 4$ m is ok.

(*b*) Maximum deceleration occurs when F is largest. For $d = 3.715$ m, $F = 120$ kN. Thus, $F = ma_D$

$$120{,}000 = 1000\, a_D$$

$a_D = 120 \text{ m/s}^2$ ◀



PROBLEM 13.57

A 600-g collar C may slide along a horizontal, semicircular rod ABD. The spring CE has an undeformed length of 250 mm and a spring constant of 135 N/m. Knowing that the collar is released from rest at A and neglecting friction, determine the speed of the collar (*a*) at B, (*b*) at D.

SOLUTION

First calculate the lengths of the spring when the collar is at positions A, B, and D.

$$l_A = \sqrt{440^2 + 300^2 + 180^2} = 562.14 \text{ mm}$$

$$l_B = \sqrt{240^2 + 300^2 + 20^2} = 384.71 \text{ mm}$$

$$l_D = \sqrt{40^2 + 300^2 + 180^2} = 352.14 \text{ mm}$$

The elongations of springs are given by $e = l - l_0$.

$$e_A = 562.14 - 250 = 312.14 \text{ mm} = 0.31214 \text{ m}$$

$$e_B = 384.71 - 250 = 134.71 \text{ mm} = 0.13471 \text{ m}$$

$$e_D = 352.14 - 250 = 102.14 \text{ mm} = 0.10214 \text{ m}$$

Potential energies: $$V = \frac{1}{2}ke^2$$

$$V_A = \frac{1}{2}(135 \text{ N/m})(0.31214 \text{ m})^2 = 6.5767 \text{ J}$$

$$V_B = \frac{1}{2}(135 \text{ N/m})(0.13471 \text{ m})^2 = 1.2249 \text{ J}$$

$$V_D = \frac{1}{2}(135 \text{ N/m})(0.10214 \text{ m})^2 = 0.7042 \text{ J}$$

Since the semicircular rod ABC is horizontal, there is no change in gravitational potential energy.

Mass of collar: $$m = 600 \text{ g} = 0.600 \text{ kg}$$

Kinetic energies: $$T_A = \frac{1}{2}mv_A^2 = 0.300\,v_A^2 = 0$$

$$T_B = \frac{1}{2}mv_B^2 = 0.300\,v_B^2$$

$$T_D = \frac{1}{2}mv_D^2 = 0.300\,v_D^2$$

PROBLEM 13.57 (Continued)

(*a*) *Speed of collar at B.*

Conservation of energy: $T_A + V_A = T_B + V_B$

$$0 + 6.5767 = 0.300v_B^2 + 1.2249$$

$$v_B^2 = 17.839 \text{ m}^2/\text{s}^2$$

$v_B = 4.22$ m/s ◀

(*b*) *Speed of collar at D.*

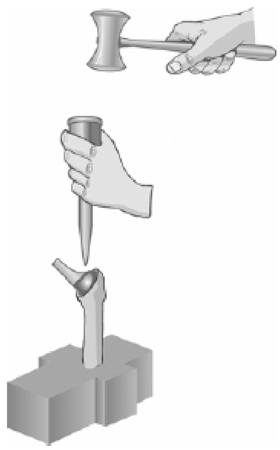
Conservation of energy: $T_A + V_A = T_D + V_D$

$$0 + 6.5767 = 0.300v_0^2 + 0.7042$$

$$v_D^2 = 19.575 \text{ m}^2/\text{s}^2$$

$v_D = 4.42$ m/s ◀

PROBLEM 13.143



The design for a new cementless hip implant is to be studied using an instrumented implant and a fixed simulated femur. Assuming the punch applies an average force of 2 kN over a time of 2 ms to the 200 g implant determine (*a*) the velocity of the implant immediately after impact, (*b*) the average resistance of the implant to penetration if the implant moves 1 mm before coming to rest.

SOLUTION

$$m = 200 \text{ g} = 0.200 \text{ kg}$$
$$F_{ave} = 2 \text{ kN} = 2000 \text{ N}$$
$$\Delta t = 2 \text{ ms} = 0.002 \text{ s}$$

(*a*) Velocity immediately after impact:

Use principle of impulse and momentum:

$$v_1 = 0 \qquad v_2 = ? \qquad \text{Imp}_{1\to 2} = F_{ave}(\Delta t)$$
$$m\mathbf{v}_1 + \mathbf{Imp}_{1\to 2} = m\mathbf{v}_2$$
$$0 + F_{ave}(\Delta t) = mv_2$$
$$v_2 = \frac{F_{ave}(\Delta t)}{m} = \frac{(2000)(0.002)}{0.200}$$

$v_2 = 20.0 \text{ m/s}$ ◀

(*b*) Average resistance to penetration:

$$\Delta x = 1 \text{ mm} = 0.001 \text{ m}$$
$$v_2 = 20.0 \text{ ft/s}$$
$$v_3 = 0$$

Use principle of work and energy.

$$T_2 + U_{2\to 3} = T_3 \qquad \text{or} \qquad \frac{1}{2}mv_2^2 - R_{ave}(\Delta x) = 0$$

$$R_{ave} = \frac{mv_2^2}{2(\Delta x)} = \frac{(0.200)(20.0)^2}{(2)(0.001)} = 40\times 10^3 \text{ N}$$

$R_{ave} = 40.0 \text{ kN}$ ◀