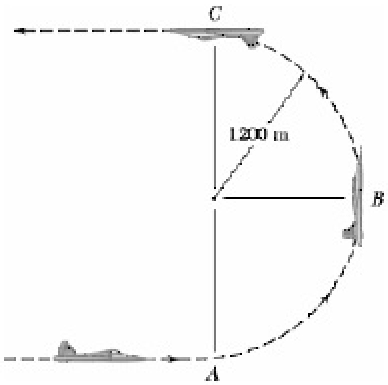
MCG 2108 - Fall 2015

DGD Week 4 (Monday 5th October)

Chapter 12 (Part 2)

Problem **12.50** is the board problem

Problem **12.46, 12.67** will be assigned to students



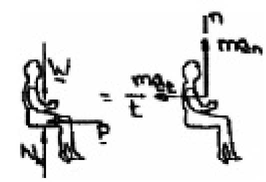
PROBLEM 12.50

A 54-kg pilot flies a jet trainer in a half vertical loop of 1200-m radius so that the speed of the trainer decreases at a constant rate. Knowing that the pilot's apparent weights at Points A and C are 1680 N and 350 N, respectively, determine the force exerted on her by the seat of the trainer when the trainer is at Point B.

SOLUTION

First we note that the pilot's apparent weight is equal to the vertical force that she exerts on the seat of the jet trainer.

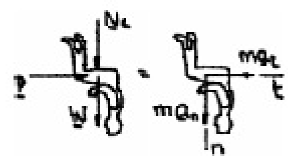
At A: $+\uparrow \Sigma F_n = ma_n$: $N_A - W = m\dfrac{v_A^2}{\rho}$



or $$v_A^2 = (1200\text{ m})\left(\frac{1680\text{ N}}{54\text{ kg}} - 9.81\text{ m/s}^2\right)$$

$$= 25{,}561.3\text{ m}^2/\text{s}^2$$

At C: $+\downarrow \Sigma F_n = ma_n$: $N_C + W = m\dfrac{v_C^2}{\rho}$



or $$v_C^2 = (1200\text{ m})\left(\frac{350\text{ N}}{54\text{ kg}} + 9.81\text{ m/s}^2\right)$$

$$= 19{,}549.8\text{ m}^2/\text{s}^2$$

Since $a_t =$ constant, we have from A to C

$$v_C^2 = v_A^2 + 2a_t\,\Delta s_{AC}$$

or $$19{,}549.8\text{ m}^2/\text{s}^2 = 25{,}561.3\text{ m}^2/\text{s}^2 + 2a_t(\pi \times 1200\text{ m})$$

or $$a_t = -0.79730\text{ m/s}^2$$

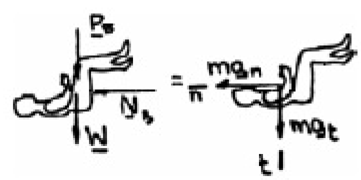
Then from A to B

$$v_B^2 = v_A^2 + 2a_t\,\Delta s_{AB}$$

$$= 25{,}561.3\text{ m}^2/\text{s}^2 + 2(-0.79730\text{ m/s}^2)\left(\frac{\pi}{2} \times 1200\text{ m}\right)$$

$$= 22{,}555\text{ m}^2/\text{s}^2$$

PROBLEM 12.50 (Continued)



At B: $\overset{+}{\leftarrow} \Sigma F_n = ma_n$: $N_B = m\dfrac{v_B^2}{\rho}$

or $$N_B = 54 \text{ kg}\frac{22{,}555 \text{ m}^2/\text{s}^2}{1200 \text{ m}}$$

or $$\mathbf{N}_B = 1014.98 \text{ N} \longleftarrow$$

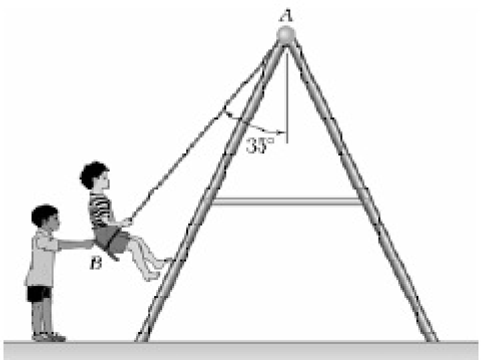
$+\downarrow \Sigma F_t = ma_t$: $W + P_B = m|a_t|$

or $$P_B = (54 \text{ kg})(0.79730 - 9.81) \text{ m/s}^2$$

or $$\mathbf{P}_B = 486.69 \text{ N} \uparrow$$

Finally, $$(F_{\text{pilot}})_B = \sqrt{N_B^2 + P_B^2} = \sqrt{(1014.98)^2 + (486.69)^2} = 1126 \text{ N}$$

or $$(\mathbf{F}_{\text{pilot}})_B = 1126 \text{ N} \searrow 25.6° \blacktriangleleft$$



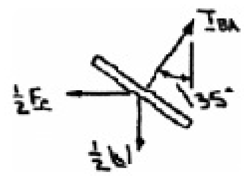
PROBLEM 12.46

A child having a mass of 22 kg sits on a swing and is held in the position shown by a second child. Neglecting the mass of the swing, determine the tension in rope AB (*a*) while the second child holds the swing with his arms outstretched horizontally, (*b*) immediately after the swing is released.

SOLUTION

Note: The factors of "$\frac{1}{2}$" are included in the following free-body diagrams because there are two ropes and only one is considered.

(*a*) For the swing at rest



$$\Sigma F_y = 0: \quad T_{BA}\cos 35° - \frac{1}{2}W = 0$$

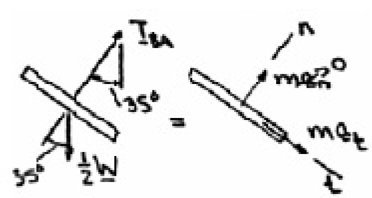
or

$$T_{BA} = \frac{22\text{ kg} \times 9.81\text{ m/s}^2}{2\cos 35°}$$

or

$T_{BA} = 131.7\text{ N}$ ◀

(*b*) At $t = 0$, $v = 0$, so that $a_n = \frac{v^2}{\rho} = 0$



$$+\nearrow \Sigma F_n = 0: \quad T_{BA} - \frac{1}{2}W\cos 35° = 0$$

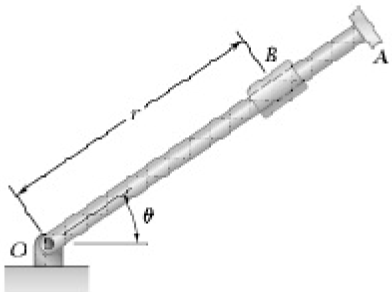
or

$$T_{BA} = \frac{1}{2}(22\text{ kg})(9.81\text{ m/s}^2)\cos 35°$$

or

$T_{BA} = 88.4\text{ N}$ ◀

PROBLEM 12.67



Rod OA oscillates about O in a horizontal plane. The motion of the 2.5 kg collar B is defined by the relations $r = 250/(t+4)$ and $\theta = (2/\pi)\sin \pi t$, where r is expressed in mm, t in seconds, and θ in radians. Determine the radial and transverse components of the force exerted on the collar when (*a*) $t = 1$ s, (*b*) $t = 6$ s.

SOLUTION

We have $\quad r = \dfrac{250}{t+4}\text{ mm} = \dfrac{0.25}{t+4}\text{ m} \qquad \theta = \left(\dfrac{2}{\pi}\sin \pi t\right)\text{ rad}$

Then $\quad \dot{r} = -\dfrac{0.25}{(t+4)^2}\text{ m/s} \qquad \dot{\theta} = (2\cos \pi t)\text{ rad/s}$

and $\quad \ddot{r} = \dfrac{0.5}{(t+4)^3}\text{ m/s}^2 \qquad \ddot{\theta} = -(2\pi \sin \pi t)\text{ rad/s}^2$

(*a*) At $t = 1$ s: $\quad r = 0.05$ m

$\dot{r} = -0.01$ m/s $\qquad \theta = -2$ rad/s

$\ddot{r} = 0.004$ m/s^2 $\qquad \ddot{\theta} = 0$

Now

$$a_r = \ddot{r} - r\dot{\theta}^2$$
$$= (0.004\text{ m/s}^2) - (0.05\text{ m})(-2\text{ rad/s})^2$$
$$= -0.196\text{ m/s}^2$$

and

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$
$$= 0 + 2(-0.01)(-2)$$
$$= 0.04\text{ m/s}^2$$

Finally

$$F_r = m_B a_r$$
$$= 2.5\text{ kg}(-0.196\text{ m/s}^2)$$

or $\qquad F_r = -0.49\text{ N}$ ◀

$$F_\theta = m_B a_\theta$$
$$= (2.5\text{ kg})(0.04\text{ m/s}^2)$$

or $\qquad F_\theta = 0.1\text{ N}$ ◀

PROBLEM 12.67 (Continued)

(*b*) At $t = 6$ s: $r = 0.025$ m

$\dot{r} = -0.0025$ m/s $\quad \dot{\theta} = 2$ rad/s

$\ddot{r} = 5\times10^{-4}$ m/s^2 $\quad \ddot{\theta} = 0$

Now $a_r = \ddot{r} - r\dot{\theta}^2 = 5\times10^{-4}(0.025^{\text{m}})(2 \text{ rad/s}^2) = -0.0995 \text{ m/s}^2$

and $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-0.0025 \text{ m/s})(2 \text{ rad/s}) = -0.01 \text{ m/s}^2$

Finally $F_r = m_B a_r$

$= (2.5 \text{ kg})(-0.0995 \text{ m/s}^2)$

or $F_r = -0.249$ N ◀

$F_\theta = m_B a_\theta$

$= (2.5 \text{ kg})(-0.01 \text{ m/s}^2)$

or $F_\theta = -0.0250$ N ◀