MCG 2108 - Fall 2015

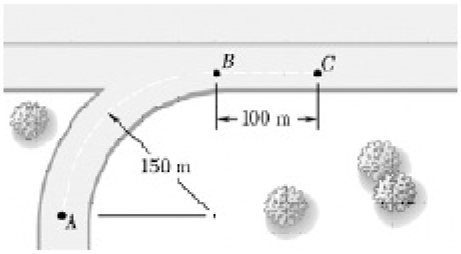
DGD Week 2(21st and 24th Sept 2015)

Chapter 11 (Part 2)

Question **11.141** will be solved by the TA

Question **11. 140, 11.147** will be assigned to the students

PROBLEM 11.140



A motorist starts from rest at Point A on a circular entrance ramp when $t = 0$, increases the speed of her automobile at a constant rate and enters the highway at Point B. Knowing that her speed continues to increase at the same rate until it reaches 100 km/h at Point C, determine (*a*) the speed at Point B, (*b*) the magnitude of the total acceleration when $t = 20$ s.

SOLUTION

Speeds: $v_0 = 0 \qquad v_1 = 100 \text{ km/h} = 27.78 \text{ m/s}$

Distance: $s = \frac{\pi}{2}(150) + 100 = 335.6 \text{ m}$

Tangential component of acceleration: $v_1^2 = v_0^2 + 2a_t s$

$$a_t = \frac{v_1^2 - v_0^2}{2s} = \frac{(27.78)^2 - 0}{(2)(335.6)} = 1.1495 \text{ m/s}^2$$

At Point B, $v_B^2 = v_0^2 + 2a_t s_B$ where $s_B = \frac{\pi}{2}(150) = 235.6 \text{ m}$

$$v_B^2 = 0 + (2)(1.1495)(235.6) = 541.69 \text{ m}^2/\text{s}^2$$

$$v_B = 23.27 \text{ m/s} \qquad v_B = 83.8 \text{ km/h} \blacktriangleleft$$

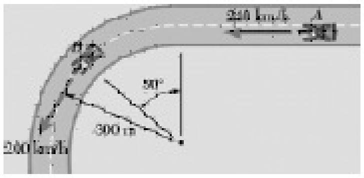
(*a*) At $t = 20$ s, $v = v_0 + a_t t = 0 + (1.1495)(20) = 22.99 \text{ m/s}$

Since $v < v_B$, the car is still on the curve. $\rho = 150 \text{ m}$

Normal component of acceleration: $a_n = \frac{v^2}{\rho} = \frac{(22.99)^2}{150} = 3.524 \text{ m/s}^2$

(*b*) Magnitude of total acceleration: $|a| = \sqrt{a_t^2 + a_n^2} = \sqrt{(1.1495)^2 + (3.524)^2} \qquad |a| = 3.71 \text{ m/s}^2 \blacktriangleleft$

PROBLEM 11.141



Racecar A is traveling on a straight portion of the track while racecar B is traveling on a circular portion of the track. At the instant shown, the speed of A is increasing at the rate of 10 m/s^2, and the speed of B is decreasing at the rate of 6 m/s^2. For the position shown, determine (*a*) the velocity of B relative to A, (*b*) the acceleration of B relative to A.

SOLUTION

Speeds:

$$v_A = 240 \text{ km/h} = 66.67 \text{ m/s}$$
$$v_B = 200 \text{ km/h} = 55.56 \text{ m/s}$$

Velocities:

$$\mathbf{v}_A = 66.67 \text{ m/s} \longleftarrow$$
$$\mathbf{v}_B = 55.56 \text{ m/s} \measuredangle 50^\circ$$

(*a*) Relative velocity:

$$\mathbf{v}_{B/A} = \mathbf{v}_B - \mathbf{v}_A$$
$$\mathbf{v}_{B/A} = (55.56\cos 50^\circ) \leftarrow + 55.56\sin 50^\circ \downarrow + 66.67 \longrightarrow$$
$$= 30.96 \rightarrow + 42.56 \downarrow$$
$$= 52.63 \text{ m/s} \searrow 53.96^\circ$$

$\mathbf{v}_{B/A} = 189.5 \text{ km/h} \searrow 54.0^\circ$ ◀

Tangential accelerations:

$$(\mathbf{a}_A)_t = 10 \text{ m/s}^2 \longleftarrow$$
$$(\mathbf{a}_B)_t = 6 \text{ m/s}^2 \measuredangle 50^\circ$$

Normal accelerations:

$$a_n = \frac{v^2}{\rho}$$

Car A: $(\rho = \infty)$ $\qquad (\mathbf{a}_A)_n = 0$

Car B: $(\rho = 300 \text{ m})$

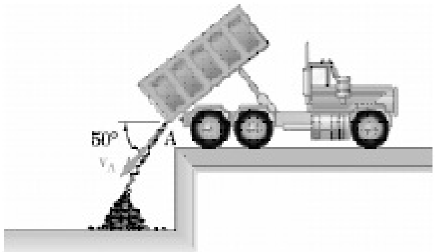
$$(\mathbf{a}_B)_n = \frac{(55.56)^2}{300} = 10.288 \qquad (\mathbf{a}_B)_n = 10.288 \text{ m/s}^2 \searrow 40^\circ$$

(*b*) Acceleration of B relative to A:

$$\mathbf{a}_{B/A} = \mathbf{a}_B - \mathbf{a}_A$$
$$\mathbf{a}_{B/A} = (\mathbf{a}_B)_t + (\mathbf{a}_B)_n - (\mathbf{a}_A)_t - (\mathbf{a}_A)_n$$
$$= 6 \measuredangle 50^\circ + 10.288 \searrow 40^\circ + 10 \rightarrow + 0$$
$$= (6\cos 50^\circ + 10.288\cos 40^\circ + 10) \longrightarrow$$
$$+ (6\sin 50^\circ - 10.288\sin 40^\circ) \uparrow$$
$$= 21.738 \rightarrow + 2.017 \downarrow$$

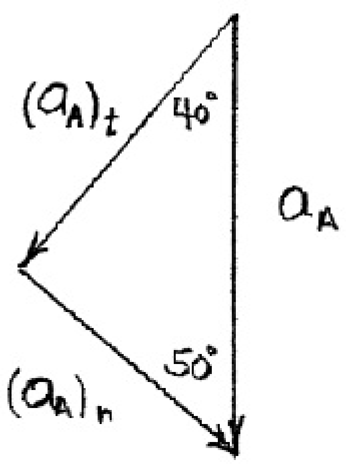
$\mathbf{a}_{B/A} = 21.8 \text{ m/s}^2 \searrow 5.3^\circ$ ◀

PROBLEM 11.147



Coal is discharged from the tailgate A of a dump truck with an initial velocity $\mathbf{v}_A = 2$ m/s $\measuredangle 50°$. Determine the radius of curvature of the trajectory described by the coal (*a*) at Point A, (*b*) at the point of the trajectory 1 m below Point A.

SOLUTION



(*a*) At Point A. $a_A = g \downarrow = 9.81 \text{ m/s}^2 \downarrow$

Sketch tangential and normal components of acceleration at A.

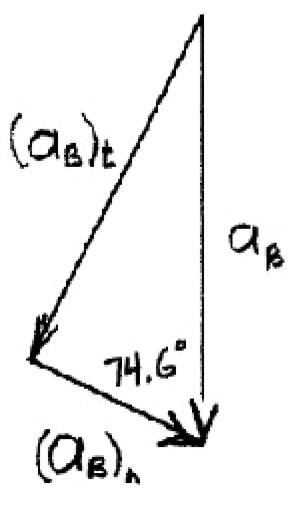
$$(a_A)_n = g\cos 50°$$

$$\rho_A = \frac{v_A^2}{(a_A)_n} = \frac{(2)^2}{9.81\cos 50°} \qquad \rho_A = 0.634 \text{ m} \blacktriangleleft$$

(*b*) At Point B, 1 meter below Point A.

Horizontal motion: $(v_B)_x = (v_A)_x = 2\cos 50° = 1.286 \text{ m/s} \longleftarrow$

Vertical motion:
$$(v_B)_y^2 = (v_A)_y^2 + 2a_y(y_B - y_A)$$
$$= (2\cos 40°)^2 + (2)(-9.81)(-1)$$
$$= 21.97 \text{ m}^2/\text{s}^2$$
$$(v_B)_y = 4.687 \text{ m/s} \downarrow$$



$$\tan\theta = \frac{(v_B)_y}{(v_B)_x} = \frac{4.687}{1.286}, \qquad \text{or} \qquad \theta = 74.6°$$

$$a_B = g\cos 74.6°$$

$$\rho_B = \frac{v_B^2}{(a_B)_n} = \frac{(v_B)_x^2 + (v_B)_y^2}{g\cos 74.6°}$$

$$= \frac{(1.286)^2 + 21.97}{9.81\cos 74.6°} \qquad \rho_B = 9.07 \text{ m} \blacktriangleleft$$