

**Concordia University  
Department of Economics**

Econ 301  
Intermediate Micro I

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Fall 2015

**Problem set # 1**

Due on Tuesday, Oct. 20 in class

FIRST NAME: \_\_\_\_\_ LAST NAME: \_\_\_\_\_

STUDENT NUMBER: \_\_\_\_\_

**I. True/False/Uncertain - Briefly explain. No credit without an explanation (5 marks each).**

1. The utility function  $U(x_1, x_2) = \ln(x_1 + 2x_2)$  corresponds to preferences for perfect substitutes.  
**True. The utility function is just a monotonic transformation of  $x_1 + 2x_2$  which is preferences for perfect substitutes.**
2. If a person's income increases, then his indifference curves shift up and to the right.  
**False. The indifference curves are a representation of a person's preferences and hence do not depend on income or prices. Changes in income may affect the optimal choice but that happens because the budget line is shifted, not the indifference curves.**
3. If consumers consume only two goods, both of them can be inferior.  
**False (if we assume monotonicity of preferences). Suppose income increases but prices stay constant. If both goods are inferior then quantity demanded for both will have to go down, i.e. since prices didn't change total expenditure must go down. But that means that a person will not spend all her income, which is a contradiction with monotonic preferences (more is better).**
4. If a consumer is making an optimal choice between two goods  $x$  and  $y$ , then, independently of his preferences, the  $MRS = -p_x/p_y$  condition must always hold.  
**False. For example, at a corner solution, the condition fails to hold.**
5. If all prices and income double, the budget constraint shifts to the right and the budget set expands.  
**False. The budget constraint and the budget set are unchanged.**

**II. Problems - You have to show your work. No credit without an explanation (25 marks each).**

1. A consumer consumes only food ( $x_1$ ) and alcohol ( $x_2$ ) and she has a utility function  $U(x_1, x_2) = ax_1 + b \ln x_2$ .
  - (a) Derive the ordinary demand functions for her. (5 marks)  
$$x_1 = \frac{m}{p_1} - \frac{b}{a}, x_2 = \frac{p_1 b}{p_2 a}$$

- (b) Are these goods complements or substitutes? (4 marks)

$$\frac{\partial x_1}{\partial p_2} = 0, x_2 \text{ is neither a complement or substitute for } x_1$$

$$\frac{\partial x_2}{\partial p_1} > 0, x_1 \text{ is a substitute for } x_2$$

Suppose  $p_1 = 4$  and  $p_2 = 2$  and  $m = \$100$ ,  $a = 6$  and  $b = 2$ .

- (c) Write down the budget constraint and derive the consumer's optimal consumption bundle. (4 marks)

$$100 = 4x_1 + 2x_2 \text{ based on part (a) } x_1 = 24.67, x_2 = \frac{2}{3}$$

Suppose the government provides a subsidy of \$1/unit on food, and collects a tax of \$1/unit on alcohol and collects \$10 lump sum income tax.

- (d) What is the new budget constraint? (4 marks)

$$100 - 10 = (4 - 1)x_1 + (2 + 1)x_2$$

- (e) What is the consumer's optimal consumption bundle after this policy? (4 marks)

$$x_1 = 29.67, x_2 = \frac{1}{3}$$

- (f) How much does it cost the government to implement this policy? (4 marks)

$$GR = 10 + \frac{1}{3} - 29.67 * 1 = -19.33$$

2. If Amy spent her entire weekly allowance she could afford 2 candy bars and 5 comic books a week. She could also just afford 8 candy bars and 2 comic books a week. The price of a candy bar is \$1.

- (a) Draw Amy's budget line. (5 marks)

**Put the number of candy bars (b) on the horizontal axis. Given that we know two points on Amy's budget line - namely (2,5) and (8,2) we can connect those and get the budget line. Notice that it is crucial that Amy can just afford these bundles - she spends her entire income on them. If this were not true we can only say they belong to her budget set but we would not be able to draw the budget line.**

- (b) What is Amy's weekly allowance? (5 marks)

**To find Amy's allowance note that we know from (a) that she can just afford the bundles, (2,5) and (8,2). Thus, if her (unknown) allowance is  $m$  and price of a book is  $p_2$  (you are given that  $p_1 = 1$ ) we must have (by the definition of budget line):**

$$2 + 5p_2 = m$$

$$8 + 2p_2 = m$$

**these are two equations in two unknowns. For example, subtract the two to get:  $-6 + 3p_2 = 0$  or,  $p_2 = 2$ . Then plug this into the first equation to get  $m = 2 + 5(2) = 12$ . Thus Amy's budget line can be written as:**

$$b + 2c = 12$$

- (c) If Amy's utility function is  $U(b, c) = \sqrt{bc}$ , where  $b$  is the quantity of candy bars she eats and  $c$  is the quantity of comic books she reads, what is her optimal consumption bundle? (10 marks)

**Amy's consumer (optimal choice/ utility maximization) problem is to maximize utility subject to her budget constraint. Write down her problem first:  $\max_{b,c} \sqrt{bc}$  s.t.  $b + 2c = 12$ .**

**Substitute  $b = 12 - 2c$  from the budget constraint to obtain a simpler equivalent problem of 1 unknown without constraints -  $\max_c \sqrt{(12 - 2c)c}$ .**

**You can use a monotonic transformation on the utility function to simplify the algebra even further (optional step)  $\max_c (12 - 2c)c$ .**

**Maximize the above by taking the derivative and setting to zero:  $12 - 4c = 0$ .**

**to find the optimal amount of comic books Amy reads,  $c^* = 3$ . Now plug this back into the budget line to get:  $b^* = 12 - 2c = 6$ .**

(d) Answer (c) if Amy's utility function were instead  $U(b, c) = b + 2c$ . (5 marks)

**This is a utility function for perfect substitutes so we cannot use derivatives to find the optimal choice - instead just plot the budget line and a few indifference curves. In this particular case the slope of the budget line and the slope of the IC are the same - plot them and see! So Amy is actually indifferent to any bundle on her budget line. Thus, her optimal choice can be any combination of b and c that satisfies  $b + 2c = 12$  (e.g. (2,5), (8,2), (0,6), etc.)**

3. Compute the price (4 marks), cross-price (4 marks) and income (4 marks) elasticities of good 1 when its demand function is  $x_1 = \frac{2m}{p_1 + 2p_2}$ . Is this good ordinary or Giffen (4 marks)? Is this good normal or inferior (4 marks)? What is the relationship between this good and good 2 (5 marks).

**Own price elasticity is given by:**

$$\frac{\partial x_1}{\partial p_1} \frac{p_1}{x_1} = -\frac{p_1}{p_1 + 2p_2} < 0$$

**Thus, the good is ordinary.**

**Cross-price elasticity is given by:**

$$\frac{\partial x_1}{\partial p_2} \frac{p_2}{x_1} = -\frac{2p_2}{p_1 + 2p_2} < 0$$

**Hence it is a complement.**

**The income elasticity is given by:**

$$\frac{d \ln(x_1)}{d \ln(m)} = 1$$

**Hence it is a normal good.**