7. [Challenge/Bonus]

Suppose A is a nonzero 3×3 matrix with $A^t = -A$, where A^t denotes the transpose of A.

Prove that $\operatorname{rank}(A) = 2$.

(Your proof must work for all nonzero 3×3 matrices A with $A^t = -A$: do not choose a particular matrix.)

$A^T =$ $\quad$ $A = \begin{bmatrix} 1 & 4 & c \\ c & r & l \\ c & c & 0 \end{bmatrix}$ $\quad$ $A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$

For the $A^t = -A$, there must be a diagonal of 0's in A. Since the transpose flips over the diagonal and we need $A^t = -A$, the diagonal row must be 0 for our statement to hold. Now for the matrix here $\begin{bmatrix} 0 & a & 0 \\ -a & 0 & b \\ 0 & -b & 0 \end{bmatrix}$ we have our row of 0's, and when A is flipped, the a's and b's will switch spots, being equivalent to the -A. The rank of this matrix is also 2, if you do some row reducing

$$\begin{bmatrix} 0 & a & 0 \\ -a & 0 & b \\ 0 & -b & 0 \end{bmatrix} = \begin{bmatrix} -a & 0 & b \\ 0 & a & 0 \\ 0 & -b & 0 \end{bmatrix}$$

So the rank(A) = 2 for $A^t = -A$

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