4b) Using part (a), find all values of e and f so that this system has

i. a unique solution

Rank A = Rank [A|b] = columns = 3

has a unique solution when $e \neq 4$ and $f \neq -6$ not necessary.

ii. infinitely many solutions, or

Rank A = Rank [A|b] < columns

$2 < 3$

has infinitely many solutions when $e = 4$ and $f = -6$ ✓

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iii. no solutions

Rank A < Rank [A|b]

$2 < 3$

has no solutions when $e = 4$ and $f \neq -6$ ✓

4c) In case b(ii) above, give a complete geometric description of the set of solutions.

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & e-4 & f-6 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right] = \quad x_3 = s$$

$$\begin{aligned} x_1 &= -1 - s \\ x_2 &= 2 + s \\ x_3 &= s \end{aligned} \quad = s\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$$

1.5

The set of solutions is a line in $\mathbb{R}^3$ going through the point $(-1, 2, 0)$ with a direction of $(-1, 1, 1)$. ✓