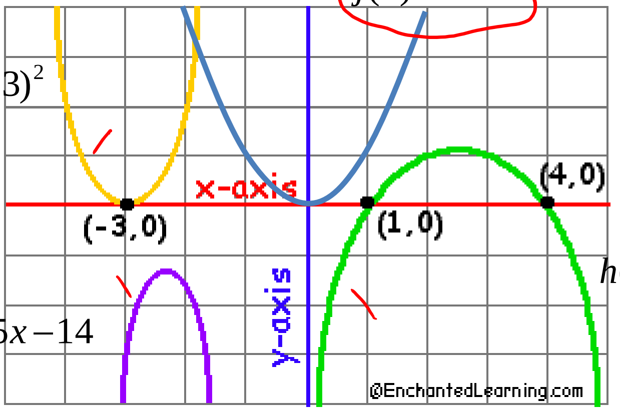
Idea: Parent Functions

Family of Functions: Functions that have similar characteristics/shape

Ex - Parabolas form a family of functions as they all have a common shape.

$j(x) = x^2$

$f(x) = 2(x+3)^2$



$h(x) = -0.33(x-1)(x-4)$

$g(x) = -2x^2 - 5x - 14$

Parent Function: A function within a family that has the simplest equation.

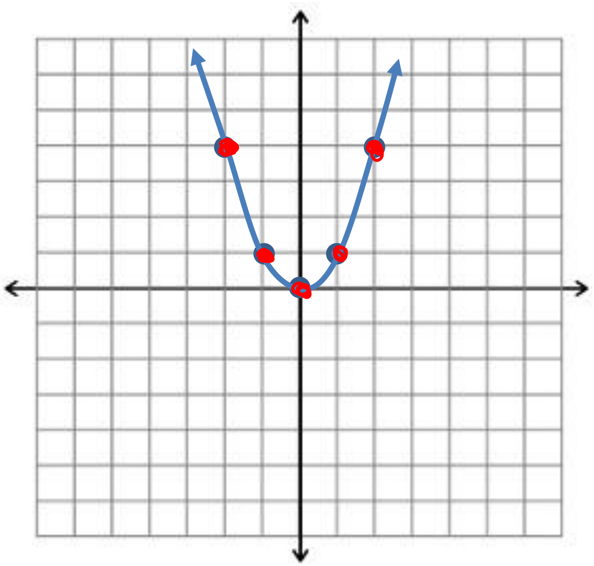
From the above diagram, you can see that j(x) is a parabola with the simplest equation. Thus the parent functions for parabolas is $j(x) = x^2$

Graphing from Table of Values

To graph any equation, we can simply make a table of values, plot these points, and connect them with a smooth curve.

For example, if we want to graph $f(x) = x^2$, we can make the following table:

x	f(x)
-2	4
-1	1
0	0
1	1
2	4



Pros:
- Easy to do ✓
- Works on any graph ✓

Cons:
- Can be time consuming
- You may have trouble finding key characteristics (zeros, vertex, etc…)

Idea: Transformations

Transformations are shifts, reflections, or stretches that augment a parent function to produce another function within the family.

→ old function we are transforming

Transformation Formula: $y = a[f(b[x-c])] + d$ (f is the parent function)

$f(x)$

a: multiply the y-coordinates by a

(*) b: divide the x-coordinates by b

c: add c to the x-coordinates

$bx - q$

d: add d to the y-coordinates

Note: You may think that b and c behave a bit counter-intuitively. You may think we would multiply the x-coordinates by b and subtract by c, as those are the operations that appear in the formula. We need to work to isolate the x to see the true transformations:

$bx - q \rightarrow$ add q $\rightarrow$ divide b $\rightarrow x$

$$b[x-c] \xrightarrow{\text{Divide by b}} x - c \xrightarrow{\text{Add c}} x$$

Here you can see that by dividing by b and adding c we get x isolated, and the true transformations appear.

Graphing Using Transformations

First we would need to plot the parent function, you can use our table of values strategy if you forget what the parent function looks like.

Next, you would determine the transformations using the transformation formula: $y = a[f(b[x-c])] + d$ <u>(f is the parent function)</u>

(note that the b may not be factored out, so you may have to do this yourself)

a: multiply the y-coordinates by a

b: divide the x-coordinates by b

c: add c to the x-coordinates

d: add d to the y-coordinates

$h(x) = 3x^3 - 5x^2 + x - 1$ $\quad$ $g(x) = x^3$

Finally, you apply the appropriate transformations by multiplying, dividing, adding, or subtracting the x-coordinates and y-coordinates based on the transformations you uncovered.

Pros: - Gets more characteristics
- Can be a lot faster

Cons: - Doesn't give accurate graphs for every function (ex higher degree polynomials).

Example 1

Graph $g(x) = 3x^2 - 2$

Here we see that the parent function is $f(x) = x^2$

If we write the transformation formula: $y = a[f(b[x - c])] + d$

$f(b(x-c))$

$(b(x-c))$

$1(x - 0)$

We see that $\underbrace{a = 3}_{xy}$, $\underbrace{b = 1}_{x \div ?}$, $\underbrace{c = 0}_{+x}$, and d = -2. $+y$

This means we need to multiply the y-coordinates by 3 then subtract 2 to the y-coordinates. (dividing x-coordinates by 1 and adding 0 does not change anything, so we do not really need to do these if you don't want to)

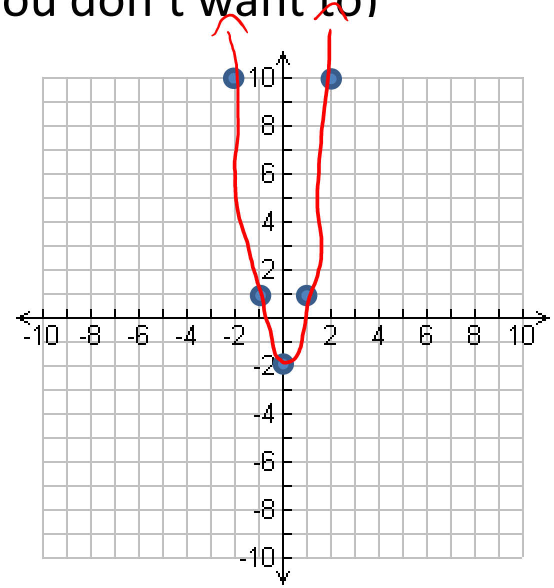
x	f(x)
-2	4
-1	1
0	0
1	1
2	4

3y-2 →

$\frac{x}{1} + 0$

x	g(x)
-2	10
-1	1
0	-2
1	1
2	10

→



Example 2

Graph $g(x) = -(2^{2(x-1)}) - 2$

Here we see that the parent function is $f(x) = 2^x$

If we write the transformation formula: $y = a[f(b[x-c])] + d$

We see that a = -1, b = 2, c = 1, and d = -2.

x y x ÷ 2 +x +y

$2^{-2} = \frac{1}{2^2}$

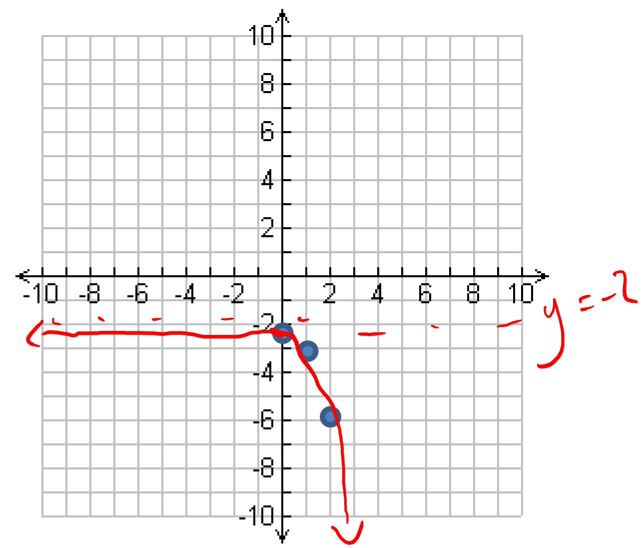
This means we need to multiply the y-coordinates by -1 then subtract 2 to the y-coordinates, we divide the x-coordinates by 2 and then add 1 to the x-coordinates.

x	f(x)
-2	0.25
-1	0.5
0	1
1	2
2	4

-y-2

½ x +1

x	g(x)
0	-2.25
0.5	-2.5
1	-3
1.5	-4
2	-6



Example 3

$b(x-c)$

$-6-3x$

$-3(2+x)$

$-3(x+2)$

Graph $g(x)=\sqrt{-6-3x}+2$

Here we see that the parent function is $f(x)=\sqrt{x}$

If we write the transformation formula: $y=a[f(b[x-c])]+d$

We need to make sure that it is written correctly: $g(x)=\sqrt{-3(x+2)}+2$

We see that a = 1, b = -3, c = -2, and d = 2.

This means we need to divide the x-coordinates by -3 then subtract 2 from the x-coordinates, and add 2 to the y-coordinates .

x	f(x)
-1	und
0	0
1	1
4	2
9	3

y+2

$-\frac{1}{3}x-2$

x	f(x)
-1.67	und
-2	2
-2.33	3
-3.33	4
-5	5