

CONCORDIA UNIVERSITY  
Department of Mathematics & Statistics

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Course	Number	Section(s)	
Mathematics	208/2	All except EC	
Examination	Date	Time	Pages
Final	December 2011	3 Hours	3
Instructors	Course Examiner		
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**FORMULAE:**

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$$A = P(1 + i)^n, \quad A = Pe^{rt}, \quad FV = PMT \frac{(1 + i)^n - 1}{i}, \quad PV = PMT \frac{1 - (1 + i)^{-n}}{i}$$

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**Special Instructions:**

- ▷ Answer all questions.
  - ▷ Only approved calculators are allowed.
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**MARKS**

[10] 1. Given the quadratic function  $f(x) = 1.2 + 0.96x - 0.12x^2$

- (A) Find  $x$  and  $y$  intercepts algebraically.
- (B) Find the vertex form of  $f$ .
- (C) Find the vertex and the maximum or minimum.
- (D) Find the range of  $f$ .

[10] 2. Solve for  $x$  in the following equations:

(A)  $\left(\frac{3}{4}\right)^x = \frac{16}{9}$

(B)  $(0.5)^{-3x^2+15x-72} = (0.5)^{-x^2+35x-22}$

(C)  $\log_3\left(\frac{x}{5}\right) + \log_3 7 + 2\log_3 \sqrt{5} = 3\log_3 \sqrt[3]{175} + 5\log_3 1$

(D)  $\log_a x + \log_a(x + 1) = \log_a 6$

(E)  $\log_2(\sqrt{2x^2}) - 1 = \frac{3}{2}$

[10] 3. For  $f(x) = -12x + 16$  and  $g(x) = 3(0.8)^x$  find the following:

(A)  $\sum_{k=0}^{49} f(k) = f(0) + f(1) + f(2) + \cdots + f(49).$

(B)  $\sum_{h=0}^{24} g(h) = g(0) + g(1) + g(2) + \cdots + g(24).$

[10] 4. Kelly sells some land in Quebec. She will be paid a lump sum of \$60,000 in 7 years. Until then, the buyer pays 8% simple interest quarterly.

(A) Find the amount of each quarterly interest payment.

(B) The buyer sets up a sinking fund so that enough money will be present to pay off the \$60,000. The buyer wants to make semiannual payments into the sinking fund; the account pays 6% compounded semiannually. Find the amount of each payment into the fund.

(C) What is the amount in the sinking fund after the first two deposits.

[10] 5. A person purchased a house 10 years ago for \$160,000. The house was financed by paying 20% down and signing a 30-year mortgage at 7.75% on the unpaid balance with payments made monthly.

(A) What is the unpaid balance after 120th payment?

(B) After the 120th payment, the owner wishes to refinance the house due to a need for additional cash. If the loan company agrees to a new 30-year mortgage of 80% of the new appraised value of the house, which is \$225,000, how much cash will the owner receive after repaying the balance of the original mortgage?

[10] 6. Solve by using Gauss-Jordan Elimination:

$$2x_1 + 6x_2 + 15x_3 = -12$$

$$4x_1 + 7x_2 + 13x_3 = -10$$

$$3x_1 + 6x_2 + 12x_3 = -9$$

No other method of solving these systems of equations will be accepted!

- [10] 7. An economy is based on three sectors, agriculture, energy, and manufacturing. Production of a dollar's worth of agriculture requires an input of \$0.20 from the agriculture sector and \$0.40 from the energy sector. Production of a dollar's worth of energy requires an input of \$0.20 from the energy sector and \$0.40 from the manufacturing sector. Production of a dollar's worth of manufacturing requires an input of \$0.10 from the agriculture sector, \$0.10 from the energy sector, and \$0.30 from the manufacturing sector.
- (A) Write the technological matrix  $M$  for this economy.
- (B) If a final demand of \$20 billion for agriculture, \$10 billion for energy, and \$30 billion for manufacturing is to be met, then set up the equation to be satisfied by the inputs from the respective sectors.
- (C) Solve the respective inputs satisfying these demands.
- [10] 8. Extremize  $P(x, y) = 30x + 10y$  subject to
- $$2x + 2y \geq 4, \quad 6x + 4y \leq 36, \quad 2x + y \leq 10, \quad x \geq 0, \quad y \geq 0.$$
- [10] 9. A small town has two radio stations: an AM station and an FM station. A survey of 100 town residents produced the following results: In the last 30 days, 65 people have listened to the AM station, 45 have listened to the FM station, and 30 have listened to both stations. During this 30-day period,
- (A) How many people in the survey have listened to the AM station but not to the FM station?
- (B) How many people in the survey have listened to the FM station but not to the AM station?
- (C) How many people in the survey have not listened to either station?
- [10] 10. A study on body types gave the following results: 45% were short, 25% were short and overweight, and 24% were tall and not overweight. Find the probabilities that a person is the following:
- (A) Overweight
- (B) Short, but not overweight
- (C) Tall and overweight

Concordia University - MAT208 - Solution to the December 2011 Final exam

**Question 1** (A)  $f(x) = -0.12x^2 + 0.96x + 1.2$  is a quadratic function. Using the quadratic formula, the  $x$  intercepts are

$$x = \frac{-0.96 \pm \sqrt{.96^2 - 4 \times (-0.12) \times 1.2}}{2 \times (-0.12)}$$

$$x \simeq -1.1 \text{ or } x \simeq 9.1$$

The  $y$ -intercept is  $f(0) = 1.2$ .

(B) Vertex form:  $f(x) = -0.12(x - 4)^2 + 3.12$ .

(C) The vertex is  $(4, 3.12)$ . The maximum of  $f$  is 3.12 and there is no minimum.

(D) The range of  $f$  is  $(-\infty, 3.12)$ .

**Question 2**

(A)  $x = -2$

(B) The equation is equivalent to

$$-3x^2 + 15x - 72 = -x^2 + 35x - 22$$

$$\Leftrightarrow$$

$$2x^2 + 20x + 50 = 0$$

$$\Leftrightarrow$$

$$x^2 + 10x + 25 = 0$$

$$(x - 5)^2 = 0$$

So the solution is  $x = 5$ .

(C) Using the properties of logarithms, we see that the equation is equivalent to

$$\frac{x}{5} \times 7 \times \sqrt{5^2} = 175$$

So the solution is  $x = 5$ .

(D)

$$\log_a(x) + \log_a(x + 1) = \log_a(6)$$

$$x(x + 1) = 6$$

$$x^2 + x - 6 = 0$$

$$(x + 3)(x - 2) = 0$$

Solution to quadratic equation:  $x = -3$  or  $x = 2$ . We do not allow take  $x = -3$  because  $\log_a(-3)$  D.N.E. So the solution to the original equation is :  $x = 2$ .

(E)

$$\log_2(\sqrt{2x^2}) - 1 = \frac{3}{2}$$

$$\Leftrightarrow$$

$$\begin{aligned}
\log_2(\sqrt{2x^2}) &= \frac{5}{2} \\
&\Leftrightarrow \\
\sqrt{2x^2} &= 2^{\frac{5}{2}} \\
&\Leftrightarrow \\
2x^2 &= 2^5 \\
&\Leftrightarrow \\
x^2 &= 2^4 = 16
\end{aligned}$$

So the solution is:  $x = -4$  or  $x = 4$ .

**Question 3**

(A) Since  $f(k)$  is an arithmetic progression,

$$\sum_{k=0}^{49} f(k) = \sum_{k=0}^{49} [-12k + 16] = \frac{f(0) + f(49)}{2} \times 50 = \frac{16 - 572}{2} \times 50 = -13900.$$

(B) Since  $g(h)$  is a geometric progression,

$$\sum_{h=0}^{24} g(h) = \sum_{h=0}^{24} 3(0.8)^h = 3 \times \frac{1 - (0.8)^{25}}{1 - (0.8)} \simeq 14.93$$

**Question 4**

(A) The quarterly interest payment is

$$I = \$60,000 \times 0.08 = \$4800.$$

(B) We use the formula

$$FV = PMT \times \frac{(1+i)^n - 1}{i},$$

with  $FV = \$60,000$ ,  $i = 0.06/2 = 0.03$  and  $n = 7 \times 2 = 14$ .

$$\frac{(1+i)^n - 1}{i} \simeq 17.0863$$

so

$$PMT \simeq \$60,000/17.0863 \simeq \$3511.58$$

(C) We use the formula

$$FV_2 = PMT \times \frac{(1+i)^n - 1}{i},$$

with  $PMT = \$3511.58$ ,  $i = 0.03$  and  $n = 2$  and get

$$FV_2 \simeq 7128.51.$$

**Question 5** The solution below is extracted from the solution manual of the textbook.

Amortized amount =  $\$160,000 - (160,000)(0.20) = \$128,000$ .

Thus,  $PV = \$128,000$ ,  $i = 0.00646$ ,  $n = 360$ .

$$PMT = 128,000 \frac{0.00646}{1 - (1.00646)^{-360}} = \$917.18$$

Balance after 10 years:

$PMT = \$917.18$ ,  $i = 0.00646$ ,  $n = 20(12) = 240$ ,

$$\text{So the balance after 10 years} = 917.18 \frac{1 - (1.00646)^{-240}}{0.00646} = \$111,705.19$$

New loan amount is  $(225,000)(0.80) = \$180,000$ , so the cash the owner will receive will be  $180,000 - 111,705.19 = \$68,294.81$ .

**Question 6** The augmented matrix corresponding to the system is

$$\left( \begin{array}{ccc|c} 2 & 6 & 15 & -12 \\ 4 & 7 & 13 & -10 \\ 3 & 6 & 12 & -9 \end{array} \right),$$

and Gaussian elimination gives the Row Reduced Echelon Form

$$\left( \begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -2 \end{array} \right).$$

The solution to the system is therefore  $x_1 = -3$ ,  $x_2 = 4$ ,  $x_3 = -2$ .

**Question 7**

(A)

$$M = \begin{pmatrix} 0.2 & 0 & 0.1 \\ 0.4 & 0.2 & 0.1 \\ 0 & 0.4 & 0.3 \end{pmatrix}$$

(B) The equation is

$$(I - M)X = D,$$

where  $D = \begin{pmatrix} 20 \\ 10 \\ 30 \end{pmatrix}$  is the demand vector (measured in millions of \$),  $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$  is the

unknown vector with the inputs from the respective sectors (measured in millions of \$), and  $I$  is the  $3 \times 3$  identity matrix.

(C) The augmented matrix corresponding to the system in  $B$  is

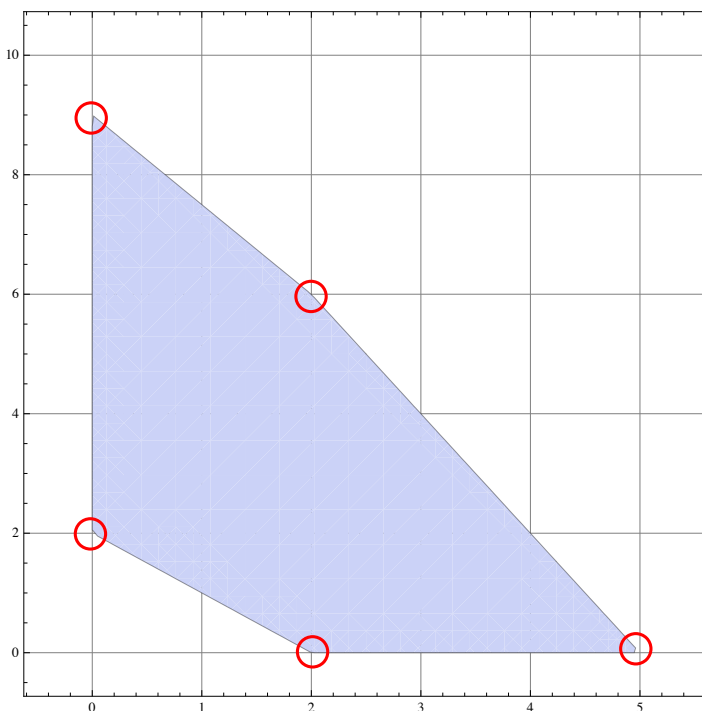
$$\left( \begin{array}{ccc|c} 0.8 & 0 & -0.1 & 20 \\ -0.4 & 0.8 & -0.1 & 10 \\ 0 & -0.4 & 0.7 & 30 \end{array} \right)$$

and Gaussian elimination gives the Row Reduced Echelon Form

$$\begin{pmatrix} 1 & 0 & 0 & 33. \\ 0 & 1 & 0 & 37. \\ 0 & 0 & 1 & 64. \end{pmatrix}.$$

Therefore the outputs are  $x_1 = 33M\$$  from the agriculture sector  $x_2 = 37M\$$  from the energy sector and  $x_3 = 64M\$$  from the manufacturing sector.

**Question 8** The feasible region in the  $(x, y)$ -plane defined by the inequality is given below.



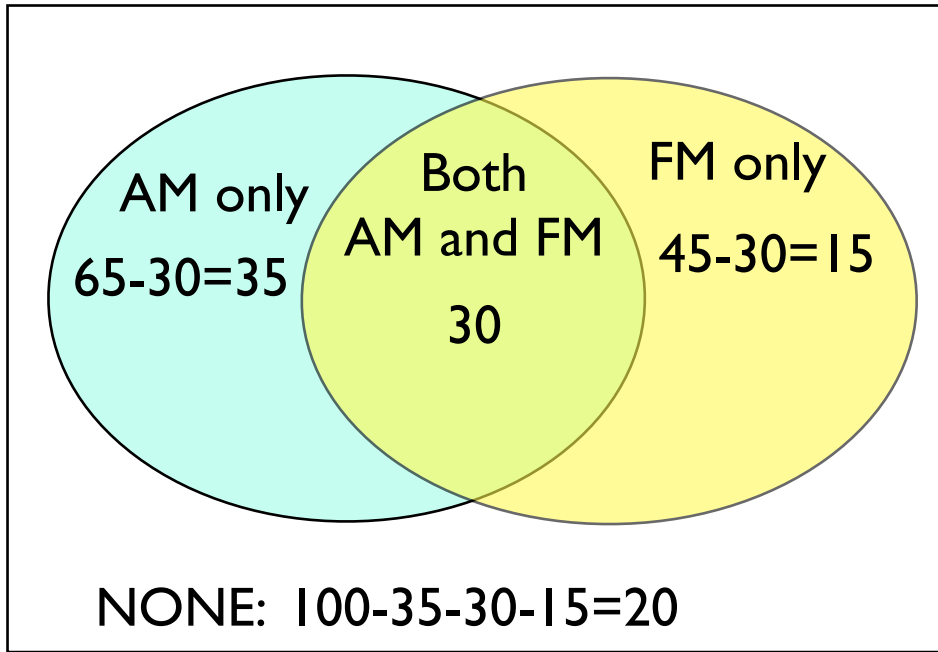
We compute the value of the objective function  $P(x, y) = 30x + 10y$  at the corner points.

Corner	$(x,y)$	$P(x,y)$
	$(0,2)$	20
	$(0,9)$	90
	$(2,6)$	120
	$(5,0)$	150
	$(2,0)$	60

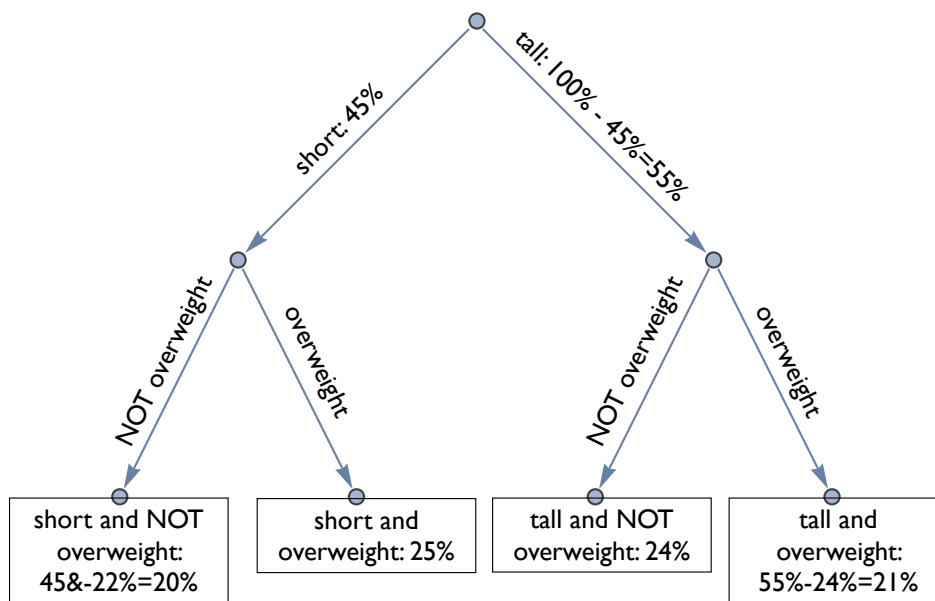
The minimum of  $P(x, y)$  is equal to 20 and is attained at the point  $(0, 2)$ . The maximum of  $P(x, y)$  is equal to 150 and is attained at the point  $(5, 0)$ .

**Question 9**

- (A) 35 people
- (B) 15 people
- (C) 20 people



Question 10



- (A) overweight: 25% + 21% = 46%
- (B) short but not overweight: 20%
- (C) tall and overweight: 21%