CARLETON UNIVERSITY

MATH 1005 DEFERRED EXAMINATION
June 2012

AUTHORIZED MEMORANDA
NON-PROGRAMMABLE, NON-GRAPHIC CALCULATORS

1. If y is the solution of the initial-value problem $\dfrac{dy}{dx} = \dfrac{4x^3}{3y^2}$, $y(0) = 2$, then $y(1) =$

   (a) $3^{1/3}$ (b) $8^{1/3}$ (c) $2^{1/3}$ (d) $9^{1/3}$

2. If y is the solution of the initial-value problem $x\dfrac{dy}{dx} - y = x^2\cos(x)$, $y(\pi) = \pi$, then $y\left(\dfrac{\pi}{2}\right) =$

   (a) 0 (b) 1 (c) π (d) $\dfrac{\pi}{2}$

3. Solve the differential equation $x^3\dfrac{dy}{dx} - x^2y + \dfrac{1}{2}y^3 = 0$ by a suitable substitution.

   (a) $\dfrac{x^2}{y^2} = \ln|x| + C$ (b) $\dfrac{x}{y} = \ln|x| + C$ (c) $\dfrac{y}{x} = \ln|x| + C$ (d) $\dfrac{y^2}{x^2} = \ln|x| + C$

4. The general solution of the exact equation $(2xy + e^y) + (x^2 + xe^y + 4y)\dfrac{dy}{dx} = 0$ is

   (a) $x^2y - xe^y - 2y^2 = C$ (b) $x^2y - xe^y + 2y^2 = C$ (c) $x^2y + xe^y - 2y^2 = C$
   (d) $x^2y + xe^y + 2y^2 = C$

5. The solution of the initial-value problem $xy' + y = e^x$, $y(1) = 0$, satisfies $y(2) =$

   (a) $\dfrac{e^2 - e}{2}$ (b) $\dfrac{e - e^2}{2}$ (c) 1 (d) 0

6. The general solution of the differential equation $y'' + 3y = 0$ is

   (a) $c_1\cos(3x) + c_2\sin(3x)$ (b) $c_1e^{3x} + c_2e^{-3x}$ (c) $c_1e^{\sqrt{3}\,x} + c_2e^{-\sqrt{3}\,x}$
   (d) $c_1\cos(\sqrt{3}\,x) + c_2\sin(\sqrt{3}\,x)$

7. The general solution of the differential equation $y'' - y' - 6y = 0$ is

   (a) $c_1e^{3x} + c_2e^{-2x}$ (b) $c_1e^{-3x} + c_2e^{2x}$ (c) $c_1e^{3x} + c_2e^{2x}$ (d) $c_1e^{-3x} + c_2e^{-2x}$

8. The solution of the initial-value problem $y'' - 2y' + y = 0$, $y(0) = 0$, $y'(0) = 1$, is

   (a) $y = xe^x$ (b) $y = e^x$ (c) $y = e^x + xe^x$ (d) $y = e^x - xe^x$

9. The general solution of the differential equation $y'' + 4y' + 5y = e^{-2x}$ is

   (a) $y = e^{-2x}[1 + c_1 \cos(x) + c_2 \sin(x)]$ (b) $y = e^{-2x}[c_1 \cos(x) + c_2 \sin(x)]$
   (c) $y = e^{2x}[c_1 \cos(x) + c_2 \sin(x)]$ (d) $y = e^{2x}[1 + c_1 \cos(x) + c_2 \sin(x)]$

10. The general solution of the differential equation $x^2y'' + 2xy' - 6y = 0$ is

    (a) $y = c_1x^3 + c_2x^2$ (b) $y = c_1x^{-3} + c_2x^{-2}$ (c) $y = c_1x^3 + c_2x^{-2}$
    (d) $y = c_1x^{-3} + c_2x^2$

11. Two linearly independent solutions of the system $\left\{ \begin{array}{ll} x' & = 8x - 18y \\ y' & = 3x - 7y \end{array} \right\}$ are

    (a) $e^{2t}\begin{pmatrix} 3 \\ 1 \end{pmatrix}$, $e^{-t}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ (b) $e^{2t}\begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $e^{-t}\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ (c) $e^{2t}\begin{pmatrix} 1 \\ 3 \end{pmatrix}$, $e^{-t}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$
    (d) $e^{2t}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $e^{-t}\begin{pmatrix} 1 \\ 3 \end{pmatrix}$

12. The sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{3^{n+1}}{4^n}$ is

    (a) 9 (b) 12 (c) 4 (d) 1

13. Which of the following three series converge(s)?

    $$\text{(i)} \sum_{n=1}^{\infty} \frac{1}{n^2+1} \qquad \text{(ii)} \sum_{n=1}^{\infty} \frac{2}{\sqrt{n}} \qquad \text{(iii)} \sum_{n=1}^{\infty} \frac{n+1}{3n+5}$$

    (a) (i), (ii) and (iii) (b) (iii) only (c) (i) and (iii) (d) (i) only

14. Which of the following three series converge(s) conditionally?

    $$\text{(i)} \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1} \qquad \text{(ii)} \sum_{n=0}^{\infty} \frac{(-1)^n}{n^3+2} \qquad \text{(iii)} \sum_{n=0}^{\infty} \frac{(-1)^n n}{\sqrt{n^2+1}}$$

    (a) (i)and (iii) (b) (i) only (c) (ii) and (iii) (d) (i), (ii) and (iii)

15. The radius of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{n(x-3)^n}{2^{n+1}}$ is

    (a) $R = 1$ (b) $R = 2$ (c) $R = \frac{1}{2}$ (d) $R = \infty$

16. The interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(x-1)^n}{3^n}$ is

(a) $[-2, 4]$ (b) $(-2, 4]$ (c) $[-2, 4)$ (d) $(-2, 4)$

17. The coefficient of x^3 in the MacLaurin series (Taylor series centred at 0) of $f(x) = \sqrt{1+x}$ is

(a) $-\frac{1}{16}$ (b) $\frac{1}{16}$ (c) $-\frac{3}{8}$ (d) $\frac{3}{8}$

18. The coefficient of $(x-3)^2$ in the Taylor series of $f(x) = \ln(x)$ about $a = 3$ is

(a) $-\frac{1}{9}$ (b) $\frac{1}{9}$ (c) $-\frac{1}{18}$ (d) $\frac{1}{18}$

19. Let $f(x) = x$ for $0 \leq x \leq 2$. The half-range sine series of f is $\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{2}\right)$, where $b_n =$

(a) $\frac{-2(-1)^n}{n\pi}$ (b) $\frac{2(-1)^n}{n\pi}$ (c) $\frac{-4(-1)^n}{n\pi}$ (d) $\frac{4(-1)^n}{n\pi}$

20. Let $f(x) = x$ for $-1 \leq x \leq 1$. The Fourier series of f is

$$\frac{a_0}{2} + \sum_{n=1}^{\infty}[a_n \cos(n\pi x) + b_n \sin(n\pi x)],$$

where

(a) $a_n = 0$, $b_n = \frac{-2(-1)^n}{n\pi}$ (b) $a_0 = 2$, $a_n = \frac{2(-1)^n}{n\pi}$, $b_n = \frac{-2(-1)^n}{n\pi}$

(c) $a_0 = 2$, $a_n = \frac{2(-1)^n}{n\pi}$, $b_n = 0$ (d) $a_0 = 1$, $a_n = \frac{2[1-(-1)^n]}{n^2\pi^2}$, $b_n = 0$