MATH1005H1(BIT2004A) - Test 2: 14:35-15:25, Oct 17, Friday

Total points: 15 **Closed book, non-programmable calculators are allowed!**

Name: **Student Number:**

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[2] 1. Find the general solution of $y'' - 8y' + 16y = 0$.

(a) $c_1e^{3x} + c_2e^{4x}$ (b) $c_1e^{4x} + c_2xe^{4x}$ (c) $c_1e^{4x} + c_2e^{5x}$ (d) $e^{-4x}(c_1 + c_2x)$ (e) $e^{8x}(c_1 + c_2x)$

Solution: (b). The indicial equation is $r^2 - 8r + 16 = 0, \Rightarrow r = 4$. So the general solution is $y(x) = c_1e^{4x} + c_2xe^{4x}$.

[2] 2. Find the general solution of $y'' - 4y' + 13y = 0$.

(a) $c_1e^{3x} + c_2e^{2x}$ (b) $e^{3x}(c_1 \cos 2x + c_2 \sin 2x)$ (c) $e^{2x}(c_1 \cos 3x + c_2 \sin 3x)$
(d) $e^{-3x}(c_1 \cos 2x + c_2 \sin 2x)$ (e) $e^{6x}(c_1 \cos 4x + c_2 \sin 4x)$

Solution: (c). The indicial equation is $r^2 - 4r + 13 = 0, \Rightarrow r = 2 \pm 3i$. So the general solution is $y(x) = e^{2x}(c_1 \cos 3x + c_2 \sin 3x)$.

[2] 3. Find the general solution of $x^2y'' - xy' - 3y = 0$ for $x > 0$.

(a) $c_1x^{-2} + c_2x^{-1}$ (b) $c_1x^2 + c_2x^{-1}$ (c) $c_1x^{-1} + c_2x^3$ (d) $c_1x^2 + c_2x$ (e) $c_1x + c_2x^{-3}$

Solution: (c).

The indicial equation is $r^2 + (-1 - 1)r - 3 = 0, \Rightarrow r = -1, 3$.
The general solution is $y(x) = c_1x^{-1} + c_2x^3$.

[2] 4. Find the general solution of $x^2y'' - 5xy' + 9y = 0$ for $x > 0$.

(a) $c_1x^3 + c_2x^3 \ln(x)$ (b) $c_1x^{-3} + c_2x^3 \ln(x)$ (c) $c_1x^{-3} + c_2x^{-3} \ln(x)$
(d) $c_1x^4 + c_2x^{-1} \ln(x)$ (e) $c_1x^2 + c_2x^2 \ln(x)$

Solution: (a)
The indicial equation is $r^2 - 6r + 9 = 0, \Rightarrow r = 3$.
The general solution is $y(x) = c_1x^3 + c_2x^3 \ln(x)$ or $x^3[c_1 + c_2 \ln(x)]$.

[2] 5. Find the general solution of $x^2y'' + xy' + 9y = 0$ for $x > 0$.

(a) $c_1x^2 + c_2x^{-2}\ln(x)$ (b) $c_1x^{-2} + c_2x^2\ln(x)$ (c) $c_1x^{-2} + c_2x^{-2}\ln(x)$
(d) $c_1\cos(2\ln(3x)) + c_2\sin(2\ln(3x))$ (e) $c_1\cos[3\ln(x)] + c_2\sin[3\ln(x)]$

Solution: (e)
The indicial equation is $r^2 + 9 = 0, \Rightarrow r = \pm 3i$. Thus the general solution is $y(x) = c_1\cos(3\ln(x)) + c_2\sin(3\ln(x))$.

[5] 6. Solve the equation $y'' - 7y' + 10y = 442\cos(3x)$ by undetermined coefficients.

Solution: Step 1: Consider the homogeneous equation $y'' - 7y' + 10y = 0$. The indicial equation is $r^2 - 7r + 10 = 0$, $r = 2, 5$. Thus

$$y_h(x) = c_1e^{2x} + c_2e^{5x}. \qquad 1.5 \textit{ points}$$

Step 2: We try $y_p = A\cos 3x + B\sin(3x)$.

$$\Rightarrow y_p' = -3A\sin 3x + 3B\cos(3x), y_p'' = -9A\cos 3x - 9B\sin(3x).$$

Substitute all of them into the DE:

$$(A - 21B)\cos 3x + (21A + B)\sin(3x) = 442\cos(3x) \Rightarrow$$

$$A - 21B = 442, 21A + B = 0; \Rightarrow A = 1, B = -21.$$

Thus

$$y_p = \cos 3x - 21\sin(3x).$$

Thus the solution is: $y = c_1e^{2x} + c_2e^{5x} + \cos 3x - 21\sin(3x)$.