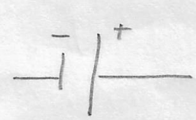
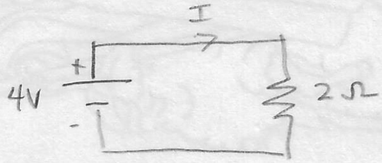
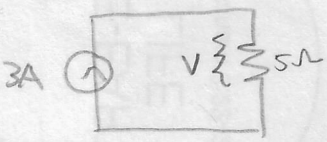
Resistor Networks

Power Sources: Voltage Source  constant V across

Current Source  constant I across



$I = V/R = 4/2 = 2A$



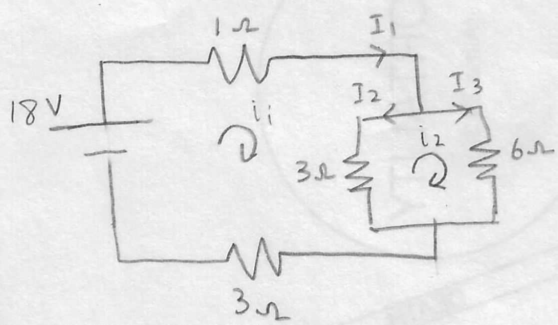
Voltage drop across resistor $= (3)(5) = 15V$

Kirchoff's Laws

Sum of voltage drops around a closed loop $= 0$

Sum of current entering a node $= 0$

E.g.



Find I_1, I_2, I_3:

$I_1 = I_2 + I_3$

$(1+3)I_1 + 3I_2 - 18 = 0$

$6I_3 - 3I_2 = 0$

$I_1 = 3A$

$I_2 = 2A$

$I_3 = 1A$

$(1+3)i_1 + 3(i_1 - i_2) - 18 = 0$

$\Rightarrow 7i_1 - 3i_2 = 18$

$(6)i_2 + (3)(i_2 - i_1) = 0$

$\Rightarrow 9i_2 = 3i_1$

$\Rightarrow 3i_2 = i_1$

$i_1 = 3A$

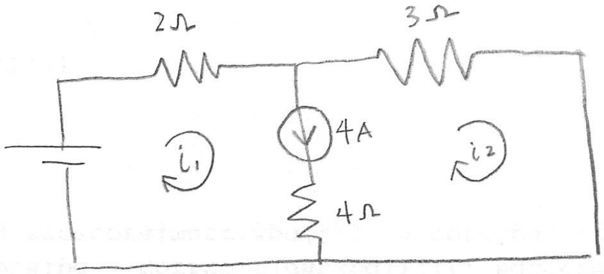
$i_2 = 1A$

$I_1 = i$

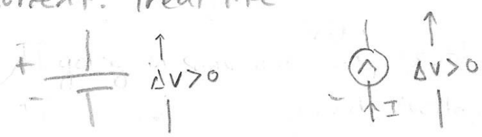
$I_2 = i_1 - i_2$

$I_3 = i_2$

loop current method



The current src (↑) has an associated voltage increase in same direction of current. Treat like



variables: - "loop current" (i) for each elementary loop $\quad i_1, i_2$

- voltage increase (v) for each current source $\quad v$

equations: - voltage drop around each elementary loop

- match current source and loop current for each current source

loop 1: $(2\Omega)i_1 - v + (4\Omega)(i_1 - i_2) - 18 = 0$

loop 2: $(3\Omega)i_2 + (4\Omega)(i_2 - i_1) + v = 0$

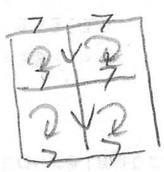
current src: $4 = i_1 - i_2$

$i_1 = 6, \quad i_2 = 2 \quad v = 10$

e.g. $V_{3\Omega} = (i_2)(3\Omega) = 6\,V$

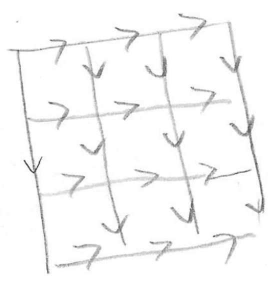
Advantages

- more systematic
- less variables in large network



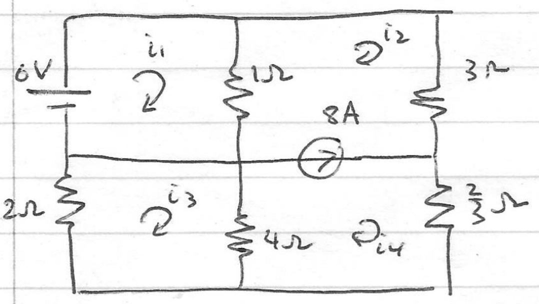
4 vars in loop current method

8 vars in trad method



9 vars in loop method

22 vars in trad method



Find loop currents
(sign in CW direction)

Find direction of current through the 4Ω w.

Vdrop is +ve

Let v be the voltage increase across the current source.

Calculates based on voltage drops

$l1$	$(1\Omega)(i_1 - i_2) = 6$	$i_1 - i_2 = 6$
$l2$	$(1\Omega)(i_2 - i_1) + (3\Omega)\, i_2 + v = 0$	$-i_1 + 4i_2 + v = 0$
$l3$	$(2\Omega)\, i_3 + (4\Omega)(i_3 - i_4) = 0$	$6i_3 = 4i_4$
$l4$	$(\frac{2}{3}\Omega)\, i_4 + (4\Omega)(i_4 - i_3) - v = 0$	$\frac{14}{3} i_4 - 4i_3 - v = 0$
7	$8 = i_4 - i_2$	$i_4 - i_2 = 8$

GE:

$i_1 = 4$

$i_2 = -2$

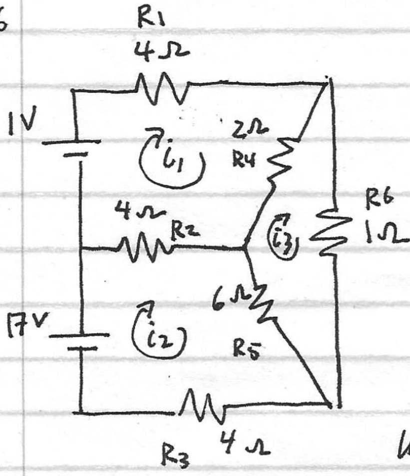
$i_3 = 4$

$i_4 = 6$

$v = 12$

current through 4Ω wire:
$i_4 - i_3 = 6 - 4 = 2A$ (upwards)

WW 5.6



Use Kirchoff's voltage law to find an eq'n for voltage drop 'round

loop 1: $1 = (4)i_1 + (2)(i_1 - i_3) + (4)(i_1 - i_2)$

$1 = 10i_1 - 4i_2 - 2i_3$

loop 2: $17 = (4)(i_2 - i_1) + (6)(i_2 - i_3) + 4i_2$

$17 = -4i_1 + 14i_2 - 6i_3$

loop 3: $0 = (2)(i_3 - i_1) + 1(i_3) + (6)(i_3 - i_2)$

$0 = -2i_1 - 6i_2 + 9i_3$

Find i_1, i_2, i_3 (+ for cw direction)

$$\left[\begin{array}{ccc|c} 10 & -4 & -2 & 1 \\ -4 & 14 & -6 & 17 \\ -2 & -6 & 9 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 2 & 6 & -9 & 0 \\ 0 & 26 & -24 & 17 \\ 0 & -34 & 43 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 2 & 6 & -9 & 0 \\ 0 & 26 & -24 & 17 \\ 0 & 0 & & \end{array}\right]$$

$43 - \frac{24 \cdot 34}{26}$ $\qquad$ $1 + \frac{17 \cdot 34}{26}$

$\frac{302}{26} i_3 = \frac{604}{26}$

$\boxed{i_3 = 2\text{ A}}$

$26i_2 - 24(2) = 17$

$\boxed{i_2 = 5/2\text{ A}}$

$2i_1 + 6(5/2) - 9(2) = 0$

$\boxed{i_1 = 3/2\text{ A}}$

Which direction does current flow across

R1 (4Ω) $i_1 > 0$ L to R →

R2 (4Ω) $i_1 - i_2 = -1$ L to R →

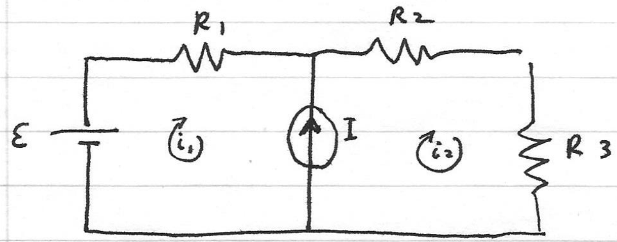
R3 (4Ω) $i_2 > 0$ R to L ←

R4 (2Ω) $i_3 - i_1 > 0$ down to up ↗

R5 (6Ω) $i_3 - i_2 < 0$ up to down ↘

R6 (1Ω) $i_3 > 0$ up to down ↓

HW 5.7



$R_1 = 2\Omega$ $\mathcal{E} = 52V$

$R_2 = 5\Omega$ $I = 1A$

$R_3 = 2\Omega$

outer loop: contains R_1, R_2, R_3, $\mathcal{E}$

$$\mathcal{E} = i_1 R_1 + i_2(R_2 + R_3)$$

$$52 = 2i_1 + 7i_2$$

relationship b/w the current src and the loop currents.

$$I = i_2 - i_1$$

$$1 = i_2 - i_1$$

Find i_1, i_2 (+ve for CW direction)

$$\left[\begin{array}{cc|c} 2 & 7 & 52 \\ -1 & 1 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} -1 & 1 & 1 \\ 0 & 9 & 54 \end{array}\right]$$

$(6) - (i_1) = 1 \Rightarrow i_1 = 5A$

$9i_2 = 54 \Rightarrow i_2 = 6A$

Use KVL to find voltage gain across the current src I.

loop 1: $\mathcal{E} = \underset{A}{(5)}\underset{\Omega}{(2)} + V$, $V = 42V$

Suppose I is adjustable so it's no longer 1A. What should I be changed to so there is no current through the cell ε?

$I = i_2 - \cancel{i_1}$

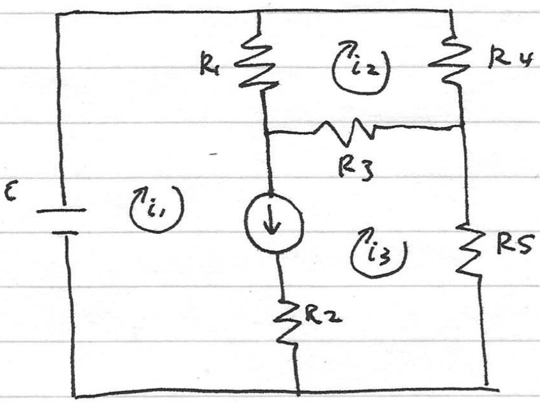
outer loop: $52 = 0R_1 + (R_2 + R_3)i_2$

$52 = 7i_2 \Rightarrow i_2 = 52/7$

$I = 52/7\ A$

-or-

ww 5.8



$R_1 = 4\Omega$ $\quad \varepsilon = 16\ V$

$R_2 = 6\Omega$ $\quad I = 7A$

$R_3 = 2\Omega$

$R_4 = 2\Omega$

$R_5 = 3\Omega$

a. Find 3 independent linear equations that can be used to solve for the loop currents.

$\varepsilon + V = (4\Omega)(i_1 - i_2) + (6\Omega)(i_1 - i_3)$ ~~loop 1~~

$(4)(i_2 - i_1) + (2)i_2 + (2)(i_2 - i_3) = 0$ loop 2

$-4i_1 + 8i_2 - 2i_3 = 0$

$-2i_1 + 4i_2 - i_3 = 0$

contains V ~~loop 3~~

$16 = (2)i_2 + (3)i_3$ outer loop

$7 = i_1 - i_3$ current src

b. Find the loop currents i_1, i_2, i_3 (+ve for cw direction)

$$\left[\begin{array}{ccc|c} -2 & 4 & -1 & 0 \\ 0 & 2 & 3 & 16 \\ 1 & 0 & -1 & 7 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} -2 & 4 & -1 & 0 \\ 0 & 2 & 3 & 16 \\ 0 & 2 & -1.5 & 7 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} -2 & 4 & -1 & 0 \\ 0 & 2 & 3 & 16 \\ 0 & 0 & -4.5 & -9 \end{array}\right]$$

$\boxed{i_3 = 2}$

$2i_2 + 3(2) = 16$

$\boxed{i_2 = 5}$

$-2i_1 + 4(5) - (2) = 0$

$\boxed{i_1 = 9}$

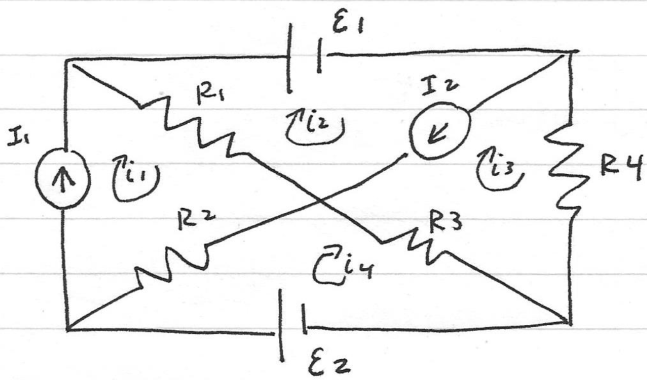
c. Use KVL to find the voltage gain across the src I.

$16 - (4)(i_1 - i_2) + V - (6)(i_1 - i_3) = 0$

$16 - (4)(4) + V - (6)(7) = 0$

$V = 42\ V$

ww 5.9



$R_1 = 6\Omega$ $\quad \mathcal{E}_1 = 25V$

$R_2 = 5\Omega$ $\quad \mathcal{E}_2 = 10V$

$R_3 = 2\Omega$ $\quad I_1 = 2A$

$R_4 = 3\Omega$ $\quad I_2 = 4A$

Find i_1, i_2, i_3, i_4.

current srces: $4 = i_2 - i_3$ *

$\boxed{i_1 = 2}$

loop 4: $10 - (5)(i_4 - \cancel{i_1}^{2}) - (2)(i_4 - i_3) = 0$

$20 = 7i_4 - 2i_3$ *

$25 - (6)(\cancel{i_1}^{2} - i_2) - (2)(i_4 - i_3) + (3)i_3 = 0$

$13 + 6i_2 - 2i_4 + 5i_3 = 0$

$13 = -6i_2 - 5i_3 + 2i_4$ *

$$\begin{array}{ccc} i_2 & i_3 & i_4 \end{array}$$

$$\left[\begin{array}{ccc|c} -6 & -5 & 2 & 13 \\ 1 & -1 & 0 & 4 \\ 0 & -2 & 7 & 20 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 0 & 4 \\ 0 & -11 & 2 & 37 \\ 0 & -2 & 7 & 20 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 0 & 4 \\ 0 & 11 & -2 & -37 \\ 0 & 0 & 7-\frac{4}{11} & 20-\frac{74}{11} \end{array}\right]$$

$\frac{73}{11} i_4 = \frac{146}{11}$

$\boxed{i_4 = 2}$

$11 i_3 - 2(2) = -37$

$\boxed{i_3 = -3}$

$i_2 - (-3) = 4$

$\boxed{i_2 = 1}$

current flow through

R_1 $i_1 - i_2 > 0$ $\searrow$

R_2 $i_4 - i_1 = 0$ no current

R_3 $i_4 - i_3 > 0$ $\searrow$

R_4 $i_3 < 0$ $\uparrow$