WW 4.11 The reduced row echelon forms of the augmented matrices of some systems are given below. How many sol'ns does each system have?

a) $\left[\begin{array}{ccc|c} 1 & 0 & -16 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$ None, ← contradiction

b) $\left[\begin{array}{ccc|c} 0 & 1 & 0 & -17 \\ 0 & 0 & 1 & 9 \end{array}\right]$ 3 vars, 2 basic, 1 free. free ∴ infinite sol'ns.

c) $\left[\begin{array}{ccc|c} 1 & 0 & 11 & 0 \\ 0 & 1 & 14 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$ None, ← contradiction

d) $\left[\begin{array}{cc|c} 1 & 0 & 6 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right]$ 1; No contradictions, no free vars. 2 variables, both basic

e) $\left[\begin{array}{cc|c} 1 & 0 & 12 \\ 0 & 1 & -16 \end{array}\right]$ Unique sol'n No contradiction No free

f) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 18 \\ 0 & 0 & 1 & -19 \end{array}\right]$ 3 vars: 2 basic, 1 free free ∴ infinite sol'ns.

WW. 4.12 How many free variables does each augmented matrix have?

a) $\left[\begin{array}{cc|c} 1 & 0 & 9 \\ 0 & 1 & -9 \\ 0 & 0 & 0 \end{array}\right]$ 0

b) $\left[\begin{array}{cccc|c} 1 & 0 & 0 & -9 & 8 \\ 0 & 1 & 5 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$ 2

c) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 9 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 10 \end{array}\right]$ 0

d) $\left[\begin{array}{ccc|c} 1 & -5 & 10 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$ 2

e) $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 4 & -8 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 9 \end{array}\right]$ 1

e) $\left[\begin{array}{cc|c} 1 & 5 & 10 \\ 0 & 0 & -9 \\ 0 & 0 & 0 \end{array}\right]$ 0

$x+5y=10$

$0=-9$??

ww 4 3 If the following system has infinitely many sol'ns

$$-6x + 5y + 3z = 6$$
$$5x - 5y - 9z = -9$$
$$-8x + 5y + hz = k$$

then $h =$, $k =$

Need free variables

$$\left[\begin{array}{ccc|c} -6 & 5 & 3 & 6 \\ 5 & -5 & -9 & -9 \\ -8 & 5 & h & k \end{array}\right] \quad R_2 \to \tfrac{5}{6}R_1 + R_2$$

$$\begin{array}{cccc|c} & -5 & 25/6 & 15/6 & 5 \\ + & 5 & -5 & -9 & -9 \\ \hline & 0 & -5/6 & -39/6 & -4 \end{array} \quad \times 6 \Rightarrow$$

$$\left[\begin{array}{ccc|c} -6 & 5 & 3 & 6 \\ 0 & -5 & -39 & -24 \\ -8 & 5 & h & k \end{array}\right] \quad R_3 \to \tfrac{-4}{3}R_1 + R_3$$

$$\begin{array}{cccc|c} & 8 & \frac{-20}{3} & -4 & -8 \\ + & -8 & 5 & h & k \\ \hline & 0 & -5/3 & h-4 & k-8 \end{array} \quad \times 3$$

$$\left[\begin{array}{ccc|c} -6 & 5 & 3 & 6 \\ 0 & -5 & -39 & -24 \\ 0 & -5 & 3(h-4) & 3(k-8) \end{array}\right] \quad R_3 \to R_3 - R_2$$

$$\begin{array}{cccc|c} & 0 & -5 & 3(h-4) & 3(k-8) \\ -(& 0 & -5 & -39\ ) & -24 \\ \hline & 0 & 0 & 3(h-4)+39 & 3(k-8)+24 \end{array}$$

$$\left[\begin{array}{ccc|c} -6 & 5 & 3 & 6 \\ 0 & -5 & -39 & -24 \\ 0 & 0 & 3(h-4)+39 & 3(k-8)+24 \end{array}\right]$$

$3h - 12 + 39$ $\quad$ $3k - 24 + 24$

$3h + 27$ $\quad$ $3k$

Make z a free variable:

$3h + 27 = 0$ $\quad$ $3k = 0$

$h = -27/3$ $\quad$ $k = 0$

$h = -9$

WW 4.10 The system $\begin{aligned} -8x + 8y &= 10 \\ -20x + ky &= 27 \end{aligned}$

has a unique solution if k $\underline{\neq 20}$

The solution is $x =$ and $y =$

Your answers should depend on the value of k.

$$\left[\begin{array}{cc|c} -8 & 8 & 10 \\ -20 & k & 27 \end{array}\right] \xrightarrow{R_2 \to R_2 - \frac{20}{8}R_1} \left[\begin{array}{cc|c} -8 & 8 & 10 \\ 0 & k-20 & 2 \end{array}\right]$$

No free variables.

No contradiction unless $k-20=0$

Unique sol'n unless $k=20$

$$\begin{aligned} -8x + 8y &= 10 \\ y(k-20) &= 2 \end{aligned}$$

$$y = \frac{2}{k-20} \qquad -8x + 8\left(\frac{2}{k-20}\right) = 10$$

$$8x = 8\left(\frac{2}{k-20}\right) - 10$$

$$x = \left(\frac{2}{k-20}\right) - \frac{5}{4}$$

ww 4 (1) Determine whether the following systems have no solution

a unique solution -or-

an infinite amount of solutions.

a) $\begin{cases} -16x - 16y = 16 \\ 8x + 8y = -8 \\ -28x - 28y = 28 \end{cases}$ can all be modified into $-x - y = 1$ or $x + y = -1$: same line

$\left[\begin{array}{cc|c} -1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{array}\right] \begin{matrix} R_2 \to R_2 + R_1 \\ R_3 \to R_3 + R_1 \end{matrix} \left[\begin{array}{cc|c} -1 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$, free var : infinite sol'ns.

b) $\begin{cases} 7x - 3y = 7 & (A) \\ -2x - 7y = 6 & (B) \\ -29x - 19y = 3 & (C) \end{cases}$

(B) $2x = -6 - 7y$

$x = -3 - \frac{7}{2}y$

(A) $7(-3 - \frac{7}{2}y) = 7$

$-21 - \frac{49}{2}y = 7$

$-\frac{49}{2}y = 28$

$y = \frac{56}{49}$

(B II) $x = -3 - \frac{7}{2}\left(\frac{56}{49}\right)$

$x = -7$

$\Rightarrow$ (C) $-29(-7) - 19\left(\frac{56}{49}\right) \neq 3$ contradiction

$\Rightarrow$ (B) $7y = -2x - 6$

$y = -\frac{2}{7}x - \frac{6}{7}$

(A) $7x - 3(-\frac{2}{7}x - \frac{6}{7})$

$7x + \frac{6}{7}x + \frac{18}{7} = 7$

$\frac{55}{7}x = \frac{31}{7}$

$x = \frac{31}{55}$

$y = -\frac{2}{7}\left(\frac{31}{55}\right) - \frac{6}{7}$

$= \frac{-62}{55 \times 7} - \frac{330}{55 \times 7}$

$= -\frac{392}{385}$

$\Rightarrow$ (C) $-29\left(\frac{31}{55}\right) - 19\left(\frac{-392}{385}\right) = 3$ ✓

One unique sol'n. $(x, y) = \left(\frac{31}{55}, \frac{-392}{385}\right)$

c) $\begin{cases} 7x - 3y = 7 \\ -2x - 7y = 6 \\ -29x - 19y = 0 \end{cases}$ see back: (II) $\left[\begin{array}{cc|c} 7 & -3 & 7 \\ 0 & -55 & 56 \\ 0 & -220 & 203 \end{array}\right]$

$y = \frac{-56}{55}$ contradiction

$y = \frac{203}{220}$

No solution

MV4. 1 Continue from front

b) $\begin{cases} 7x - 3y = 7 \\ -2x - 7y = 6 \\ -29x - 19y = 3 \end{cases}$

$\left[\begin{array}{cc|c} 7 & -3 & 7 \\ -2 & -7 & 6 \\ -29 & -19 & 3 \end{array}\right]$ $R_2 \to R_2 + \frac{2}{7}R_1$

$\begin{array}{cc|c} -2 & -7 & 6 \\ 2 & -6/7 & 2 \\ \hline 0 & -55/7 & 8 \end{array}$ $\times 7$

II $\left[\begin{array}{cc|c} 7 & -3 & 7 \\ 0 & -55 & 56 \\ -29 & -19 & 3 \end{array}\right]$ $R_3 \to R_3 + R_1 \frac{29}{7}$

$\begin{array}{cc|c} -29 & -19 & 3 \\ 29 & -87/7 & 29 \\ \hline 0 & -220/7 & 32 \end{array}$ $\times 7$

$\left[\begin{array}{cc|c} 7 & -3 & 7 \\ 0 & -55 & 56 \\ 0 & -220 & 224 \end{array}\right]$ $R_3 \to R_3 - 4R_2$

$\begin{array}{cc|c} 0 & -220 & 224 \\ 0 & 220 & -224 \\ \hline 0 & 0 & 0 \end{array}$

$\left[\begin{array}{cc|c} 7 & -3 & 7 \\ 0 & -55 & 56 \\ 0 & 0 & 0 \end{array}\right]$ No contradictions

One unique sol'n

$7x - 3y = 7$

$-55y = 56$

$y = -56/55$

$7x - 3(-56/55) = 7$

$7x + 168/55 = 7$

$7x = 7 - 168/55$

$x = 1 - \frac{168}{55 \times 7}$

WW3 (9) Write $\vec{v}$ as a linear combination of $\vec{u}_1 = \begin{bmatrix} -3 \\ 1 \\ 3 \end{bmatrix}$, $\vec{u}_2 = \begin{bmatrix} -2 \\ 0 \\ -4 \end{bmatrix}$, $\vec{u}_3 = \begin{bmatrix} 1 \\ -2 \\ -4 \end{bmatrix}$

$\vec{v} = [7, -9, -43]$

$$\begin{bmatrix} 7 \\ -9 \\ -43 \end{bmatrix} = \begin{bmatrix} -3s - 2t + u \\ s - 2u \\ 3s - 4t - 4u \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix}$$

$$\left[\begin{array}{ccc|c} -3 & -2 & 1 & 7 \\ 1 & 0 & -2 & -9 \\ 3 & -4 & -4 & -43 \end{array}\right] \begin{matrix} R_2 \to R_2 + \frac{1}{3}R_1 \\ R_2 \to 3R_2 \\ R_3 \to R_3 + R_1 \end{matrix} \left[\begin{array}{ccc|c} -3 & -2 & 1 & 7 \\ 0 & -2 & -5 & -20 \\ 0 & -6 & -3 & -36 \end{array}\right] R_3 \to R_3 - 3R_2 \left[\begin{array}{ccc|c} -3 & -2 & 1 & 7 \\ 0 & -2 & -5 & -20 \\ 0 & 0 & 12 & 24 \end{array}\right]$$

$-3s - 2t + u = 7$ $\quad -3s - 2(5) + (2) = 7, \quad s = -5$

$-2t - 5u = -20$ $\quad -2t - 5(2) = -20, \quad t = 5$

$12u = 24$ $\quad u = 2$

$(s, t, u) = (-5, 5, 2)$

$[7, -9, -43] = -5[-3, 1, 3] + 5[-2, 0, 4] + 2[1, -2, -4]$

WW3 (10) Solve the following system:

$$\begin{aligned} 6x + 5y + 5z &= -24 \\ 3x - 2y - 6z &= 0 \\ -4x + 3y - 4z &= 37 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 6 & 5 & 5 & -24 \\ 3 & -2 & -6 & 0 \\ -4 & 3 & -4 & 37 \end{array}\right] \begin{matrix} R_2 - \frac{1}{2}R_1 \\ R_2 \times 2 \end{matrix} \left[\begin{array}{ccc|c} 6 & 5 & 5 & -24 \\ 0 & -9 & -17 & 24 \\ -4 & 3 & -4 & 37 \end{array}\right] \begin{matrix} R_3 + \frac{2}{3}R_1 \\ R_3 \times 3 \end{matrix} \left[\begin{array}{ccc|c} 6 & 5 & 5 & -24 \\ 0 & -9 & -17 & 24 \\ 0 & 19 & -2 & 63 \end{array}\right]$$

$$\begin{matrix} R_3 + \frac{19}{9}R_2 \\ R_3 \times 9 \end{matrix} \left[\begin{array}{ccc|c} 6 & 5 & 5 & -24 \\ 0 & -9 & -17 & 24 \\ 0 & 0 & -341 & 1023 \end{array}\right] \quad \left[\begin{array}{ccc|c} 6 & 5 & 5 & -24 \\ 0 & -9 & -17 & 24 \\ 0 & 0 & -1 & 3 \end{array}\right]$$

$6x + 5y + 5z = -24$ $\quad 6x + 5(3) + 5(-3) = -24, \quad x = -4$

$-9y - 17z = 24$ $\quad -9y - 17(-3) = 24, \quad y = 3$

$-z = 3$ $\quad z = -3$

$(x, y, z) = (-4, 3, -3)$

ww 2 (14) Find the value for a for which $\vec{v} = \begin{bmatrix} -10 \\ 12 \\ 21 \\ a \end{bmatrix}$

is a linear combination of $\begin{bmatrix} -5 \\ 4 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 5 \\ -5 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 5 \\ 2 \end{bmatrix}$

$$\begin{bmatrix} -10 \\ 12 \\ 21 \\ a \end{bmatrix} = s\begin{bmatrix} 0 \\ 0 \\ 5 \\ 2 \end{bmatrix} + t\begin{bmatrix} 0 \\ 2 \\ 5 \\ -5 \end{bmatrix} + u\begin{bmatrix} -5 \\ 4 \\ 3 \\ 2 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 0 & 0 & -5 & -10 \\ 0 & 2 & 4 & 12 \\ 5 & 5 & 3 & 21 \\ 2 & -5 & 2 & a \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 5 & 5 & 3 & 21 \\ 0 & 2 & 4 & 12 \\ 0 & 0 & -5 & -10 \\ 2 & -5 & 2 & a \end{array}\right]$$

$R4 \to R4 - \frac{2}{5}R_1$ $\left[\begin{array}{ccc|c} 5 & 5 & 3 & 21 \\ 0 & 2 & 4 & 12 \\ 0 & 0 & -5 & -10 \\ 0 & -7 & 4/5 & a - 42/5 \end{array}\right]$

$R_4 \to R_4 + \frac{7}{2}R_2$ $\left[\begin{array}{ccc|c} 5 & 5 & 3 & 21 \\ 0 & 2 & 4 & 12 \\ 0 & 0 & -5 & -10 \\ 0 & 0 & 74/5 & a - 42/5 + 42 \end{array}\right]$

$R_4 \to R_4 + \left(\frac{74}{25}\right)R_3$ $\left[\begin{array}{ccc|c} 5 & 5 & 3 & 21 \\ 0 & 2 & 4 & 12 \\ 0 & 0 & -5 & -10 \\ 0 & 0 & 0 & a - 42/5 + 42 - \frac{740}{25} \end{array}\right]$

$a = 42/5 - 42 + 740/25$

$a = -4$

WW 4.4 Solve the system

WW Sec 2.12

$$\begin{cases} x_1 + x_2 - 2x_3 = 7 \\ 4x_1 + 5x_2 - 5x_3 = -1 \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 36 \\ -29 \\ 0 \end{bmatrix} + t \begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & 7 \\ 4 & 5 & -5 & -1 \end{array}\right] \quad R_2 \to R_2 - 4R_1 \qquad \begin{array}{cccc} 4 & 5 & -5 & -1 \\ -4 & -4 & 8 & -28 \\ 0 & 1 & 3 & -29 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -2 & 7 \\ 0 & 1 & 3 & -29 \end{array}\right] \qquad \begin{array}{ll} x + y + -2z = 7 & (A) \\ y + 3z = -29 & (B) \end{array}$$

z is a free variable. Let s be a parameter, $z = s$

$y + 3s = -29$; $y = -3s - 29$

$\Rightarrow$ (A) $x + (-3s - 29) - 2s = 7$; $x = 5s + 36$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 36 \\ -29 \\ 0 \end{bmatrix} + s \begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix}$$

WW 4.5 Solve the system

$$\begin{aligned} x_1 + x_2 \qquad\qquad\qquad &= -4 \\ x_2 + x_3 \qquad\quad &= -1 \\ x_3 + x_4 &= 2 \\ x_1 \qquad\qquad + x_4 &= -1 \end{aligned}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix} + s \begin{bmatrix} \\ \\ \\ \\ \end{bmatrix}$$, where s is a free variable.

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & -4 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 & 2 \\ 1 & 0 & 0 & 1 & -1 \end{array}\right] \xrightarrow{R_4 \to R_4 - R_1} \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & -4 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & -1 & 0 & 1 & 3 \end{array}\right] \xrightarrow{R_4 \to R_4 + R_2} \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & -4 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 & 2 \end{array}\right]$$

$$\xrightarrow{R_4 \to R_4 - R_3} \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & -4 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

x_4 a free variable.

Let s be a parameter. $s = x_4$

(A) $x_1 + x_2 = -4$

(B) $x_2 + x_3 = -1$

(C) $x_3 + x_4 = 2$

(C) $x_3 + s = 2$; $x_3 = 2 - s$

(B) $x_2 + (2 - s) = -1$; $x_2 = s - 3$

(A) $x_1 + (s - 3) = -4$; $x_1 = -1 - s$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \\ 2 \\ 0 \end{bmatrix} + s \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

WW 4.6 Find the general sol'n of the following system

$$x - y + 2z - w = 0$$
$$2x + 2y + z - 3w = 0$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = s\begin{bmatrix} -5/4 \\ 3/4 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 5/4 \\ 1/4 \\ 0 \\ 1 \end{bmatrix}$$, where s & t are free variables

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & 0 \\ 2 & 2 & 1 & -3 & 0 \end{array}\right] \xrightarrow{R_2 \to R_2 - 2R_1} \left[\begin{array}{cccc|c} 1 & -1 & 2 & -1 & 0 \\ 0 & 4 & -3 & -1 & 0 \end{array}\right]$$

z, w free

(A) $x - y + 2z - w = 0$

(B) $4y - 3z - w = 0$

Let s and t be parameters:

$s = z, \quad t = w$

(B) $4y - 3s - t = 0$, $\quad 4y = 3s + t$

$y = \frac{3}{4}s + \frac{1}{4}t$

(A) $x - (\frac{3}{4}s + \frac{1}{4}t) + 2s - t = 0$

$x = (\frac{3}{4})s - 2s + \frac{1}{4}t + t$

$x = \frac{-5}{4}s + \frac{5}{4}t$

ww 4 7 Find the general sol'n of the system

$$A = \left[\begin{array}{cccccc|c} 1 & 2 & -3 & -4 & -4 & -4 & 0 \\ 0 & 0 & 1 & 5 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \end{array}\right] \begin{array}{c} A \\ B \\ C \\ \\ \end{array}$$

$s_1 \quad s_2 \quad s_3 \quad s_4 \quad s_5 \quad s_6$

free vars: s_2, s_4, s_6

C $\quad s_5 - 3s_6 = 0 \; ; \quad s_5 = 3s_6$

B $\quad s_3 + 5s_4 - 3s_6 = 0$

$s_3 = 3s_6 - 5s_4$

A $\quad s_1 + \overset{F}{2s_2} - 3s_3 \overset{F}{-4s_4} - 4s_5 \overset{F}{-4s_6} = 0$

$s_1 - 3(3s_6 - 5s_4) - 4(3s_6) = 4s_6 + 4s_4 - 2s_2$

$s_1 - 9s_6 + 15s_4 - 12s_6 = 4s_6 + 4s_4 - 2s_2$

$s_1 = -2s_2 - 11s_4 + 25s_6$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = s_2 \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s_4 \begin{bmatrix} -11 \\ 0 \\ -5 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s_6 \begin{bmatrix} 25 \\ 0 \\ 3 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

WW4 (8) Solve the system $\begin{cases} x_1 - x_2 + 5x_3 = -4 \\ 5x_1 - 6x_2 + 5x_3 = -4 \\ -3x_1 \quad\quad -75x_3 = 60 \end{cases}$

$$\left[\begin{array}{ccc|c} 1 & -1 & 5 & -4 \\ 5 & -6 & 5 & -4 \\ -3 & 0 & -75 & 60 \end{array}\right] \xrightarrow{R_2 \to R_2 - 5R_1} \left[\begin{array}{ccc|c} 1 & -1 & 5 & -4 \\ 0 & -1 & -20 & 16 \\ -3 & 0 & -75 & 60 \end{array}\right] \xrightarrow{R_3 \to R_3 + 3R_1} \left[\begin{array}{ccc|c} 1 & -1 & 5 & -4 \\ 0 & -1 & -20 & 16 \\ 0 & -3 & -60 & 48 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 3R_2} \left[\begin{array}{ccc|c} 1 & -1 & 5 & -4 \\ 0 & -1 & -20 & 16 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

x_3 is free

(A) $x_1 - x_2 + 5x_3 = -4$

(B) $-x_2 - 20x_3 = 16$

(B) $x_2 = -16 - 20x_3$

(A) $x_1 - (-16 - 20x_3) + 5x_3 = -4$

$x_1 + 16 + 20x_3 + 5x_3 = -4$

$x_1 = -20 - 25x_3$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -20 \\ -16 \\ 0 \end{bmatrix} + s\begin{bmatrix} -25 \\ -20 \\ 1 \end{bmatrix}$$

WW4 (9) Solve the system $\begin{cases} x_1 + 5x_2 - 2x_3 \quad\quad -3x_5 - 3x_6 = 0 \\ \quad\quad -x_4 + 3x_5 - 5x_6 = -3 \\ x_1 + 5x_2 \quad\quad -7x_5 - 5x_6 = -2 \end{cases}$

$$\left[\begin{array}{cccccc|c} 1 & 5 & -2 & 0 & -3 & -3 & 0 \\ 0 & 0 & 0 & -1 & 3 & -5 & -3 \\ 1 & 5 & 0 & 0 & -7 & -5 & -2 \end{array}\right] \xrightarrow{R_3 \to R_3 - R_1} \left[\begin{array}{cccccc|c} 1 & 5 & -2 & 0 & -3 & -3 & 0 \\ 0 & 0 & 0 & -1 & 3 & -5 & -3 \\ 0 & 0 & 2 & 0 & -4 & -2 & -2 \end{array}\right]$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{cccccc|c} 1 & 5 & -2 & 0 & -3 & -3 & 0 \\ 0 & 0 & 2 & 0 & -4 & -2 & -2 \\ 0 & 0 & 0 & -1 & 3 & -5 & -3 \end{array}\right]$$

(A) $x_1 + 5x_2 - 2x_3 - 3x_5 + 3x_6 = 0$

(B) $2x_3 - 4x_5 - 2x_6 = -2$

(C) $-x_4 + 3x_5 - 5x_6 = -3$

free: x_2, x_5, x_6

(C) $x_4 = 3x_5 - 5x_6 + 3$

$x_4 = 3 + 3x_5 - 5x_6$

(B) $2x_3 = -2 + 4x_5 + 2x_6$

$x_3 = -1 + 2x_5 + x_6$

(A) $x_1 + 5x_2 - 2(-1 + 2x_5 + x_6) - 3x_5 - 3x_6 = 0$

$x_1 + 5x_2 + 2 - 4x_5 - 2x_6 - 3x_5 - 3x_6 = 0$

$x_1 = -2 - 5x_2 + 7x_5 + 5x_6$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ -1 \\ 3 \\ 0 \\ 0 \end{bmatrix} + x_2\begin{bmatrix} -5 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5\begin{bmatrix} 7 \\ 0 \\ 2 \\ 3 \\ 1 \\ 0 \end{bmatrix} + x_6\begin{bmatrix} 5 \\ 0 \\ 1 \\ -5 \\ 0 \\ 1 \end{bmatrix}$$

95 e.g. Do the lines L_1, L_2, in $\mathbb{R}^3$ intersect?

$$L_1 \begin{cases} x+y+z=0 \\ 2x+2y-z=1 \end{cases} \qquad L_2 \begin{cases} -x+2y-3z=4 \\ 2y-z=2 \end{cases}$$

intersect if they have a solution; point is on both lines; satisfy all eq'ns

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 2 & -1 & 2 \\ 2 & 2 & -1 & 1 \\ -1 & 2 & -3 & 4 \end{array}\right] \xrightarrow{R_3 \to R_3 + 2R_4} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 2 & -1 & 2 \\ 0 & 6 & -7 & 9 \\ -1 & 2 & -3 & 4 \end{array}\right] \xrightarrow{R_3 \to R_3 - 3R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 2 & -1 & 2 \\ 0 & 0 & -4 & 3 \\ -1 & 2 & -3 & 4 \end{array}\right]$$

$$\xrightarrow{R_4 \to R_4 + R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 2 & -1 & 2 \\ 0 & 0 & -4 & 3 \\ 0 & 3 & -2 & 4 \end{array}\right] \xrightarrow[R_4 \to 2R_4]{R_4 \to R_4 - \frac{3}{2}R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 2 & -1 & 2 \\ 0 & 0 & -4 & 3 \\ 0 & 0 & \cancel{-1/2}\ -1 & \cancel{1}\ 2 \end{array}\right] \xrightarrow{R_4 \to R_4 - \frac{1}{4}R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 2 & -1 & 2 \\ 0 & 0 & -4 & 3 \\ 0 & 0 & 0 & 5/4 \end{array}\right]$$

Contradiction. No intersection

e.g. Find parametric form of L and P below

$$L \begin{cases} x+y+z=1 \\ x \qquad -z=3 \end{cases} \qquad P: x+2y+3z=6$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 0 & -1 & 3 \end{array}\right]$$

$R_3 \to R_3 - R_1$

$$\left[\begin{array}{ccc|c} \boxed{1} & 1 & 1 & 1 \\ 0 & \boxed{-1} & -2 & 2 \end{array}\right]$$

free: z

$x+y+z=1$ (A)

$-y-2z=2$ (B)

(B) $y=-2-2z$

(A) $x-2-2z+z=1$

$x=z+3$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3+s \\ -2-2s \\ 0+s \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix} + s\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$x+2y+3z=6$. free: y, z (Let them be parameters)

$x=6-2y-3z$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6-2y-3z \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix} + y\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + z\begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$$

WW5.1 Find the cubic polynomial such that $f(2)=-23$, $f'(2)=-29$, $f''(2)=-28$ & $f'''(2)=-12$

$f(x) = Ax^3 + Bx^2 + Cx + D$ $\quad$ $f(2) = A(2)^3 + B(2)^2 + C(2) + D = -23$

$f'(x) = 3Ax^2 + 2Bx + C$ $\quad$ $f'(2) = 3A(2)^2 + 2B(2) + C = -29$

$f''(x) = 6Ax + 2B$ $\quad$ $f''(2) = 6A(2) + 2B = -28$

$f'''(x) = 6A$ $\quad$ $f^{(3)}(2) = 6A = -12$

$$\begin{array}{cccccc} D & C & B & A & & \\ \end{array}$$
$$\left[\begin{array}{cccc|c} 1 & 2 & 4 & 8 & -23 \\ 0 & 1 & 4 & 12 & -29 \\ 0 & 0 & 2 & 12 & -28 \\ 0 & 0 & 0 & 6 & -12 \end{array}\right]$$

$8(-2)+4(-2)+2(3)+D=-23$ $\quad$ $D=-5$

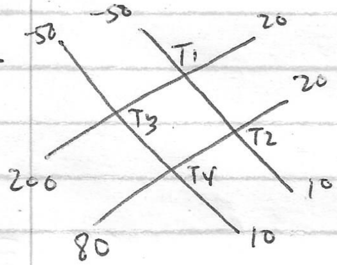
$12(-2)+4(-2)+C=-29$ $\quad$ $C=3$

$12(-2)+2B=-28$ $\quad$ $B=-2$

$A=-2$

$f(x) = (-2)x^3 + (-2)x^2 + (3)x + (-5)$

WW 5.2



$4T_1 = T_2 + T_3 - 30$ $\quad$ $4a - b - c = -30$

$4T_2 = T_1 + T_4 + 30$ $\quad$ $-a + 4b - d = 30$

$4T_3 = T_1 + T_4 + 150$ $\quad$ $-a + 4c - d = 150$

$4T_4 = T_2 + T_3 + 90$ $\quad$ $-b - c + 4d = 90$

$$\left[\begin{array}{cccc|c} 4 & -1 & -1 & 0 & -30 \\ 1 & -4 & 0 & 1 & -30 \\ 1 & 0 & -4 & 1 & -150 \\ 0 & 1 & 1 & -4 & -90 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 4 & -1 & -1 & 0 & -30 \\ 0 & -4 & 4 & 0 & 120 \\ 0 & 1 & -15 & 4 & -570 \\ 0 & 1 & 1 & -4 & -90 \end{array}\right]$$

$T_4 = 40$

$T_3 = 50$

$T_2 = 20$

$T_1 = 10$

$$\Rightarrow \left[\begin{array}{cccc|c} 4 & -1 & -1 & 0 & -30 \\ 0 & -4 & 4 & 0 & 120 \\ 0 & 0 & -16 & 8 & -480 \\ 0 & 0 & 8 & -16 & -240 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 4 & -1 & -1 & 0 & -30 \\ 0 & -4 & 4 & 0 & 120 \\ 0 & 0 & 8 & -16 & -240 \\ 0 & 0 & 0 & -24 & -960 \end{array}\right]$$