Faculty of Engineering and Computer Science
Concordia University
COEN231: Introduction to Discrete Mathematics

Fall 2015, Midterm 1

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1. (a) (8 points) Consider the two decimal numbers: (-74) and (+56). In binary 8-bit registers, using the 2's complement system, carry out the following operations (-74 + 56) and (-74 - 56). Explain your findings?

   (b) (4 points) Express -87.5×10^{-2} in IEEE 754 single precision format.

   (c) (4 points) Express 1/3 in binary using 10 bits to the right of the binary point.

2. (a) (5 points) Prove (without using truth table) that the following statement is a tautology: $(p \rightarrow q) \wedge (q \rightarrow p) \longleftrightarrow [(p \wedge q) \vee (\neg p \wedge \neg q)]$

   (b) (10 points) Let $R(x)$ be the statement "x can speak Russian" and $C(x)$ the statement"x knows C++". Consider the universe all students at your school. Express each of the following sentences in terms of $R(x)$, $C(x)$, quantifiers and logical connectives.

   i. There is a student at your school who can speak Russian and who knows C++
   ii. There is a student at your school who can speak Russian, but who does not know C++
   iii. Every student at your school either can speak Russian or know C++
   iv. No student at your school can speak Russian, or know C++
   v. Every student at your school who speaks Russian knows C++

   (c) (10 points) Consider the following argument in the context of logic without quantifiers:
   "If it does not rain or it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on." "If the sailing race is held, then the trophy will be awarded." "The trophy was not awarded. We conclude that it rained."
   Identify the primitive statements, and represent the argument in inference format. Then prove it or disprove it, showing clearly the steps of your reasoning.

   (d) (3 points) Let $P(n)$ be: "if $n > 1$, then $n^2 < n$". Determine the truth value of the proposition $P(0)$.

3. (a) (6 points) Consider the following sets: $A = \{a, b, \{a, b\}, \{\}\}$, $B = \{b, c, d\}$. What are the cardinalities of: A, B, $A \cap B$ and $A \cup B$?

   (b) (4 points) Find the cardinality of the following set: $\{x \in \mathbb{N} \mid 2 \leq x \leq 22 \wedge x$ is divisible by 3 $\}$

   (c) (4 points) Show that if A and B are sets , then $A - (A - B) = A \cap B$

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(1)

a) $-74 + 56$

$56 = 32 + 16 + 8 = 2^5 + 2^4 + 2^3$

$= \%\ 00111000$ in 8 bits

$74 = 64 + 8 + 2 = 2^6 + 2^3 + 2^1$

$= \%\ 01001010$

1's complement: 10110101

2's complement: $\underline{\quad\quad\quad 1}$ +

10110110

Now

  00111000
+ 10110110
____
  11101110

⇒ the answer is ⊖ :

2's complement: 00010001

$\underline{\quad\quad 1}$ +

$18 = 2^4 + 2^1 \Leftarrow \%\ 00010010$

⇒ the answer is **−18**

-74 -56 : we cannot do it using 8 bits since with 8 bits:

[-128 — +127]

and -74-56 is outside the range

b) $-87.5 * 10^{-2}$

$= -0.875 * 10^{0} = -0.875$

$0.875 = 0.5 + 0.25 + 0.125$

$= 2^{-1} + 2^{-2} + 2^{-3}$

$=_{2} 0.111$

$=_{2} 1.11\ 2^{-1}$

sign = 1 (negative number)

exponent = $-1 + 127 = 126 = 64 + 32 + 16 + 8 + 4 + 2$

$= 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1$

$=_{2} 01111110$ (8 bits)

Mantisa = 11000---00

23 bits

c) $\frac{1}{3}$

$\frac{1}{3} \times 2 = \frac{2}{3} = 0 + \frac{2}{3}$

$\frac{2}{3} \times 2 = \frac{4}{3} = 1 + \frac{1}{3}$

$\frac{1}{3} \times 2 = \frac{2}{3} = 0 + \frac{2}{3}$

$\frac{2}{3} \times 2 = \frac{4}{3} = 1 + \frac{1}{3}$

$\frac{1}{3} =_{2} 0.0101010101$

using 10 bits

2) a) $(p \to q) \wedge (q \to p) \leftrightarrow [(p \wedge q) \vee (\neg p \wedge \neg q)]$

$(p \to q) \wedge (q \to p) \equiv$

$(\neg p \vee q) \wedge (\neg q \vee p) \equiv$

$\neg p \wedge (\neg q \vee p) \vee q \wedge (\neg q \vee p) \equiv$

$(\neg p \wedge \neg q) \vee (\neg p \wedge p) \vee (q \wedge \neg q) \vee (q \wedge p) \equiv$

$\equiv F \qquad \equiv F$

$(\neg p \wedge \neg q) \vee F \vee F \vee (q \wedge p) \equiv$

$(\neg p \wedge \neg q) \vee (q \wedge p) \equiv$

$(q \wedge p) \vee (\neg p \wedge \neg q) \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

b) $R(x)$: x can speak Russian
$C(x)$: x knows C++
Domain: all students.

i). $\exists x [R(x) \wedge C(x)]$

ii). $\exists x [R(x) \wedge \neg C(x)]$

iii) - $\forall x [R(x) \vee C(x)]$

iv) - $\forall x \neg [R(x) \vee C(x)]$

v) - $\forall x [R(x) \to C(x)]$

c) "If it does not rain or it is not foggy, then the sailing race will be held and the life saving demonstration will go on".

"If the sailing race is held, then the trophy will be awarded".

"The trophy was not awarded"

Conclusion: "It rained".

r: it rains
f: foggy
s: sailing race will be held
l: life saving demonstration will go on
t: trophy awarded

1. $(\neg r \vee \neg f) \rightarrow (s \wedge l)$
2. $s \rightarrow t$
3. $\neg t$

$\therefore r$

$\frac{s \wedge l}{\therefore s}$

$\frac{(\neg r \vee \neg f) \rightarrow s \qquad s \rightarrow t}{\therefore (\neg r \vee \neg f) \rightarrow t}$

$\frac{(\neg r \vee \neg f) \rightarrow t \qquad \neg t}{\Rightarrow \therefore \neg(\neg r \vee \neg f) \equiv r \wedge f \equiv r}$

d)

P(n): " if $n>1$, then $n^2<n$ "

P(0): " if $0>1$, then $0^2<0$ " ?

p: $0>1$

q: $0^2<0$

$p \equiv F \Rightarrow p \to q \equiv T$

$\Rightarrow$ **P(0) is T**

3) a) $A = \{a, b, \{a,b\}, \{\}\}$

$B = \{b, c, d\}$

$|A| = 4$

$|B| = 3$

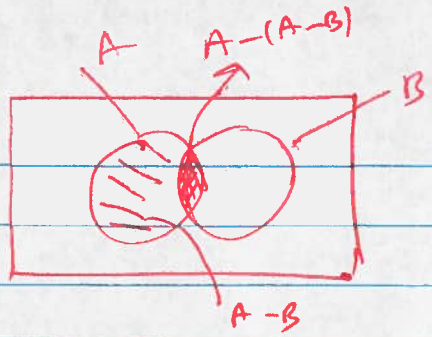
$A \cap B = \{b\} \Rightarrow |A \cap B| = 1$

$A \cup B = \{a, b, c, d, \{a,b\}, \{\}\}$

$\Rightarrow |A \cup B| = 6$

b) $\{x \in \mathbb{N} \mid 2 \leq x \leq 22 \wedge x \text{ divisible by } 3\}$

$\{3, 6, 9, 12, 15, 18, 21\}$ (7)



c) $A - (A - B) \stackrel{?}{=} A \cap B$

$$A - (A-B) = \{ x \mid x \in A \wedge x \notin (A-B) \}$$

$$= \{ x \mid (x \in A) \wedge \neg x \in (A-B) \}$$

$$= \{ x \mid (x \in A) \wedge \neg (x \in A \wedge x \notin B) \}$$

$$= \{ x \mid (x \in A) \wedge (x \notin A \vee x \in B) \}$$

$$= \{ x \mid (x \in A \wedge x \notin A) \vee (x \in A \wedge x \in B) \}$$

$$= \{ x \mid F \vee (x \in A \wedge x \in B) \}$$

$$= \{ x \mid x \in A \wedge x \in B \}$$

~~$x \in A \wedge$~~

$$= \{ x \mid x \in (A \cap B) \}$$

$$= A \cap B$$