MATH 3355A Assignment 1 September 30, 2015

Due on Wednesday October 14 at 11:30am in class.
Each question is 5 marks, and a total of 30 marks.

1. Let $a = 5472$ and $b = 2760$. Let $d = (a, b)$ be the greatest common divisor of a and b.

(a) Find d by using the Euclidean Algorithm.

(b) Find integers m and n such that $d = am + bn$.

2. Solve the system of congruences $\begin{cases} x \equiv 3 \pmod{5} \\ x \equiv 4 \pmod{11} \\ x \equiv 42 \pmod{61} \end{cases}$

3. Let m and n be two positive integers such that $(m, n) = 1$. Show that

$$m^{\phi(n)} + n^{\phi(m)} \equiv 1 \pmod{mn}.$$

4. Determine all the primitive roots modulo 31.

5. You are given that 7, 14 and 19 are primitive roots modulo 23.
Find $\text{ind}_7(3)$, $\text{ind}_{14}(3)$ and $\text{ind}_{19}(3)$.

6. **(a)** For each positive integer n, show that

$$\mu(n)\mu(n+1)\mu(n+2)\mu(n+3) = 0.$$

(b) For any integer $n \geq 3$, show that $\displaystyle\sum_{k=1}^{n} \mu(k!) = 1$.