CONCORDIA UNIVERSITY
DEPARTMENT OF COMPUTER SCIENCE AND SOFTWARE ENGINEERING

COMP 232: MATHEMATICS FOR COMPUTER SCIENCE
FALL 2015

ASSIGNMENT 4

PROBLEM 1.

Use mathematical induction to show that

$$2^n \leq 2^{n+1} - 2^{n-1} - 1,$$

when n is a positive integer.

PROBLEM 2.

The sequence of Fibonacci numbers is defined by

$$f_0 = 0, f_1 = 1, \text{ and } f_n = f_{n-1} + f_{n-2}, \text{ for } n > 1.$$

The sequence of Lucas numbers is defined by

$$l_0 = 2, l_1 = 1, \text{ and } l_n = l_{n-1} + l_{n-2}, \text{ for } n > 1.$$

Prove that

$$f_n + f_{n+2} = l_{n+1},$$

whenever n is a positive integer, where f_i and l_i are the ith Fibonacci number and ith Lucas number, respectively.

PROBLEM 3.

For each of the following relations on the set **Z** of integers, determine if it is reflexive, symmetric, anti-symmetric, or transitive. On the basis of these properties, state whether or not it is an equivalence relation or a partial order.

(a) $R = \{(a, b) \mid a^2 = b^2\}$.
(b) $S = \{(a, b) \mid |a - b| \leq 1\}$.

PROBLEM 4.

(a) Prove that $\{(x, y) \mid x - y \in \mathbf{Q}\}$ is an equivalence relation on the set of real numbers, where **Q** denotes the set of rational numbers.
(b) Give [1], [1/2], and [π].

PROBLEM 5.

Prove or disprove the following statements:

(a) Let R be a relation on the set **Z** of integers such that xRy if and only if $xy \geq 1$. Then, R is irreflexive.
(b) Let R be a relation on the set **Z** of integers such that xRy if and only if $x = y + 1$ or $x = y - 1$. Then, R is irreflexive.
(c) Let R and S be reflexive relations on a set A. Then, $R - S$ is irreflexive.

PROBLEM 6.

Let R be the relation on $\mathbf{Z}^+$ defined by xRy if and only if $x < y$. Then, in the Set Builder Notation, $R = \{(x, y) \mid y - x > 0\}$.

(a) Use the Set Builder Notation to express the transitive closure of R.
(b) Use the Set Builder Notation to express the composite relation R^n, where n is a positive integer.

PROBLEM 7.

(a) Give the transitive closure of the relation $R = \{(a, c), (b, d), (c, a), (d, b), (e, d)\}$ on $\{a, b, c, d, e\}$.
(b) Give an example to show that when the symmetric closure of the reflexive closure of the transitive closure of a relation is formed, the result is not necessarily an equivalence relation.