CONCORDIA UNIVERSITY
DEPARTMENT OF COMPUTER SCIENCE AND SOFTWARE ENGINEERING

COMP 232: MATHEMATICS FOR COMPUTER SCIENCE
FALL 2015

ASSIGNMENT 3

PROBLEM 1.

Let A and B be sets. Prove that $A \subseteq B$ if and only if $P(A) \subseteq P(B)$.

PROBLEM 2.

Let A, B, C, and D be sets. Prove or disprove the following:

$$(A \cap B) \cup (C \cap D) = (A \cap D) \cup (C \cap B).$$

PROBLEM 3.

Give an example of two uncountable sets A and B such that $A - B$ is

(a) Countably Infinite.
(b) Uncountable.

PROBLEM 4.

Prove that $\lfloor 3x \rfloor = \lfloor x \rfloor + \lfloor x + 1/3 \rfloor + \lfloor x + 2/3 \rfloor$.

PROBLEM 5.

(a) Give an example of a function from $\mathbf{Z}^+$ to $\mathbf{Z}^+$ that is neither one-to-one nor onto.
(b) Let $g : A \to B$ and $f : B \to C$ be functions. Let $f \circ g$ be onto. Are both f and g necessarily onto?
(c) Let f be a function from $\mathbf{R}$ to $\mathbf{R}$ defined by $f(x) = x^2$. Find $f^{-1}(\{x \mid 0 < x < 1\})$.

PROBLEM 6.

Draw the graph of $\lceil x/2 \rceil \cdot \lfloor x/2 \rfloor$.

PROBLEM 7.

Let a and b be integers, and m be a positive integer. Prove that

$$ab \equiv [\ (a \text{ (mod m)}) \cdot (b \text{ (mod m)})\ ] \text{ (mod m)}.$$

PROBLEM 8.

Prove that $a^3 \equiv a \pmod 3$ for every positive integer a.

PROBLEM 9.

Prove that if p is a prime number greater than 3, then $p^2 = 6k + 1$, for some integer k.

PROBLEM 10.

Let a, b, and d be integers such that $d \geq 2$ and $a \equiv b \pmod d$. Prove that $\gcd(a, d) = \gcd(b, d)$.