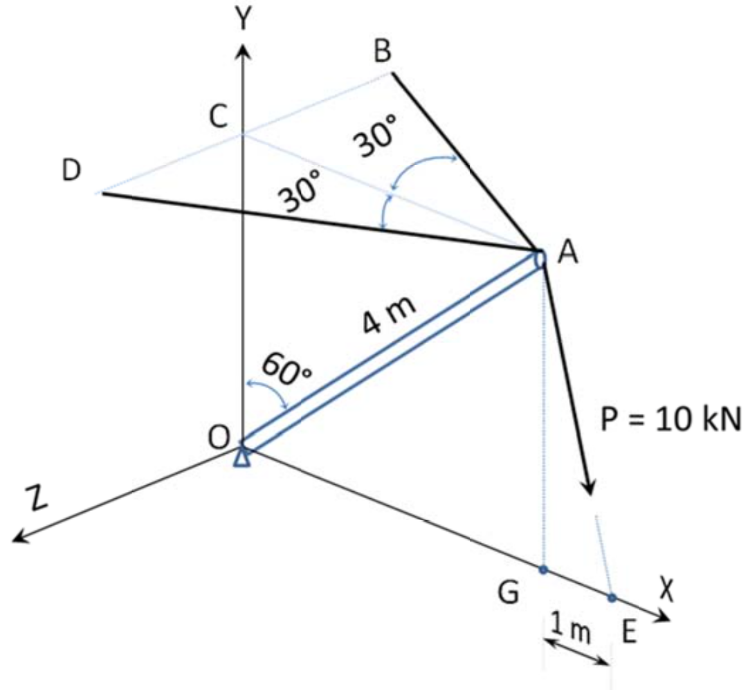
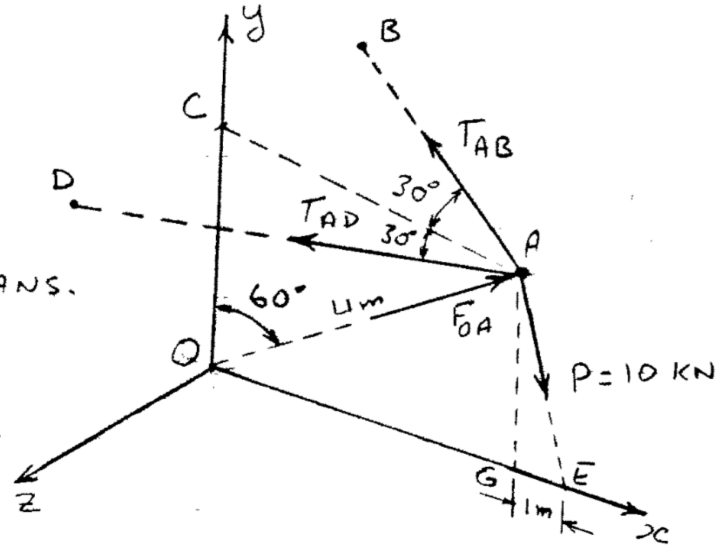
Problem 1. (14/60)

Load ***P*** = 10kN is supported by rod ***OA*** and cables ***AB*** and ***AD*** as shown in the diagram. Rod OA lies in the ***xy*** plane and the force in it is along its longitudinal axis. Cables ***AB*** and ***AD*** lie in a plane parallel to the ***xz*** plane. Rod ***OA*** is supported by a ball and socket joint at ***O***. Rod ***OA*** is of negligible weight.



a) Write the forces in rod ***OA***, cables ***AB*** and ***AD*** and the force ***P*** in **<u>vector form</u>**.
b) Determine the magnitudes of the force in rod ***OA*** and the tensions in cables ***AB*** and ***AD***.

1. a) FBD – point A



$\bar{F}_{OA} = F_{OA}\dfrac{\overline{OA}}{OA}$

$\overline{OA} = 4\sin 60°\,\bar{i} + 4\cos 60°\,\bar{j}$

$\therefore\ \bar{F}_{OA} = \dfrac{F_{OA}}{4}(4\sin 60°\,\bar{i} + 4\cos 60°\,\bar{j})$

i.e $\bar{F}_{OA} = F_{OA}(0.866\bar{i} + 0.5\bar{j})$ —— ANS.

$\bar{T}_{AB} = T_{AB}\dfrac{\overline{AB}}{AB}$

$AB = \dfrac{AC}{\cos 30°} = \dfrac{4\sin 60}{\cos 30°} = 4m.$

$\overline{AB} = -4\cos 30°\,\bar{i} - 4\sin 30°\,\bar{k}$

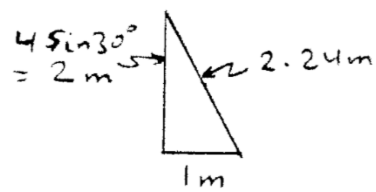
$\therefore\ \bar{T}_{AB} = \dfrac{T_{AB}}{4}(-4\cos 30°\,\bar{i} - 4\sin 30°\,\bar{k})$

i.e $\bar{T}_{AB} = T_{AB}(-0.866\bar{i} - 0.5\bar{k})$ —— ANS.

Similarly, $\bar{T}_{AD} = T_{AD}(-0.866\bar{i} + 0.5\bar{k})$ —— ANS.

$\bar{P} = \dfrac{P}{2.24}(+1\bar{i} - 2\bar{j}) = 10(0.45\bar{i} - 0.89\bar{j})$

$\therefore\ \bar{P} = 4.5\bar{i} - 8.9\bar{j}$ —————— ANS.



b) Add vectors $\bar{F}_{OA}, \bar{T}_{AB}, \bar{T}_{AD}$ & $\bar{P}$.

Then equate coefficients $\bar{i}, \bar{j}$ & $\bar{k}$ to zero.

$\Sigma F_x = 0;\ 0.866F_{OA} - 0.866T_{AB} - 0.866T_{AD} + 4.5 = 0$ —— (1)

$\Sigma F_y = 0;\ 0.5F_{OA} - 8.9 = 0;\ \therefore F_{OA} = \dfrac{8.9}{0.5} = \underline{\underline{17.8\text{ KN}}}$ —— ANS.

$\Sigma F_z = 0;\ -0.5T_{AB} + 0.5T_{AD} = 0;\ \therefore T_{AB} = T_{AD}$

Insert $F_{OA} = 17.8$ kN and $T_{AB} = T_{AD}$ in (1):

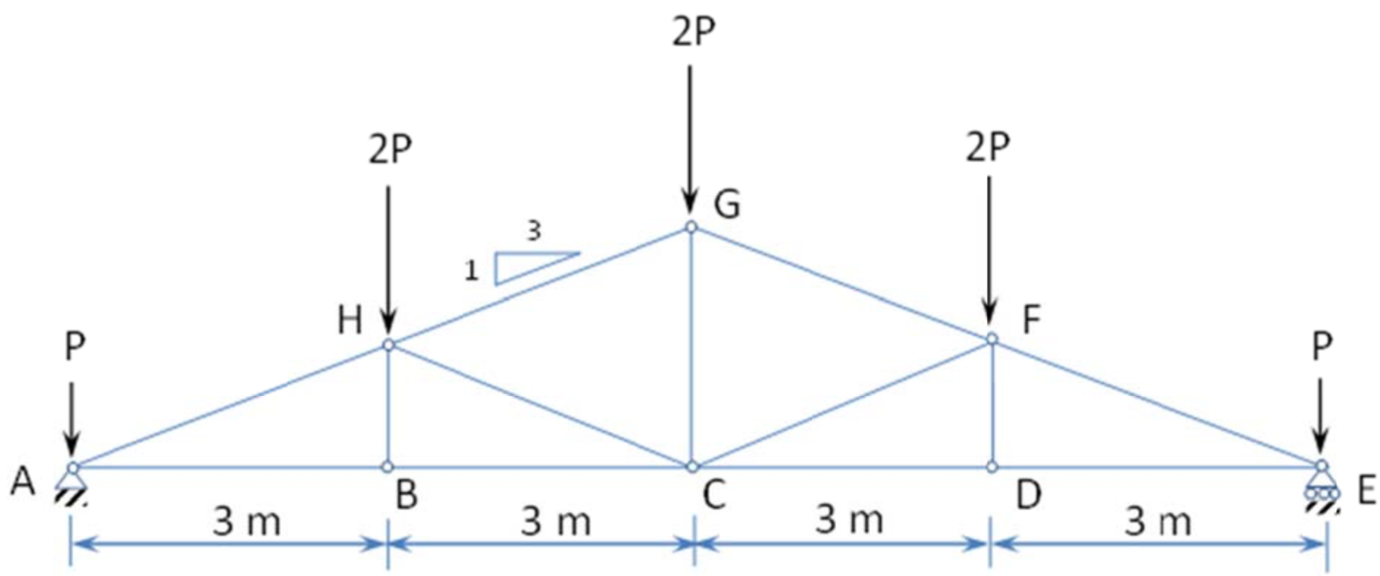
$0.866\times 17.8 - 0.866T_{AB} - 0.866T_{AB} + 4.5 = 0$

$\therefore\ 1.732T_{AB} = 19.92$. Hence, $T_{AB} = T_{AD} = \dfrac{19.92}{1.732} = \underline{\underline{11.5\text{ KN}}}$ ANS.

Problem 2. (12/60)

The sketch shows a standard Howe roof truss. It is subjected to a snow loading which results in ***P*** = 1.5 kN. The support at ***A*** is a pin joint and that at ***E*** is a roller. Determine:

a) The reactions at the supports.
b) The forces in members ***BC***, ***HC*** and ***HG***, by the **method of sections**, stating whether each is in tension or compression.



2. a)

FBD – Whole Truss

$\xrightarrow{+} \Sigma F_x = 0$

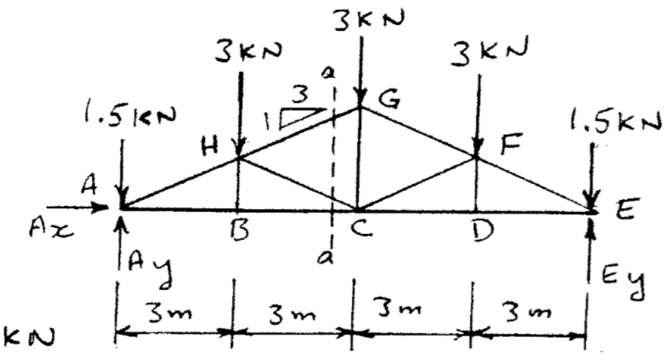
$\therefore A_x = 0$

Because of Symmetry, $A_y = E_y$

$+\uparrow \Sigma F_y = 0$

$A_y + E_y = 1.5 + 3 + 3 + 3 + 1.5 = 12\,kN$

$\therefore A_y = E_y = \frac{12}{2} = \underline{\underline{6\,kN\uparrow}}$



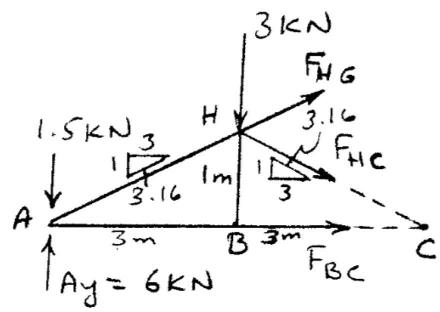
ANS.

b) FBD – Section left of aa

$+\circlearrowleft \Sigma M_H = 0$

$F_{BC} \times 1m + 1.5KN \times 3m - 6KN \times 3m = 0$

$F_{BC} = 18 - 4.5 = \underline{\underline{13.5\,KN\,(T)}}$



ANS.

$+\circlearrowleft \Sigma M_A = 0$

$-3KN \times 3m + F_{HC} \times \frac{1}{3.16} \times 3m + F_{HC} \times \frac{3}{3.16} \times 1m = 0$

$-9 + 0.95 F_{HC} + 0.95 F_{HC} = 0$

$\therefore F_{HC} = \frac{9}{1.9} = \underline{\underline{4.74\,KN\,(T)}}$ ANS.

$+\circlearrowleft \Sigma M_C = 0$

$1.5KN \times 6m - 6KN \times 6m + 3KN \times 3m - F_{HG} \times \frac{1}{3.16} \times 3m - F_{HG} \times \frac{3}{3.16} \times 1m = 0$

$9 - 36 + 9 - 0.95 F_{HG} - 0.95 F_{HG} = 0$

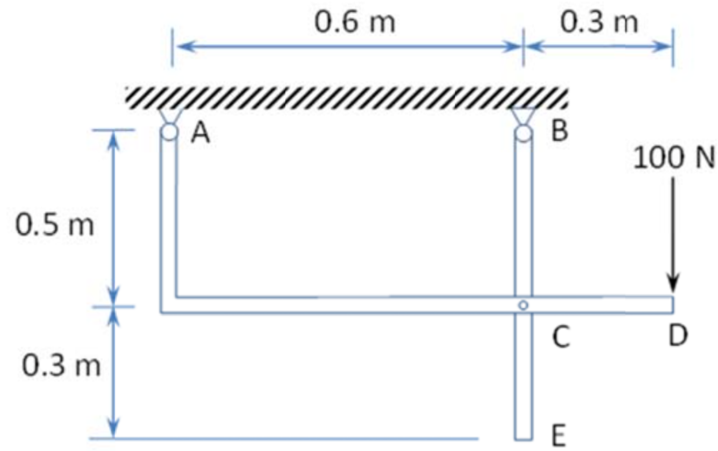
$-1.9 F_{HG} = 18$

$\therefore F_{HG} = -\frac{18}{1.9} = -9.47\,KN = \underline{\underline{9.47\,KN\,(C)}}$ ANS.

Problem 3. (12/60)

The sketch to the right shows a frame which is supported by pin joints at ***A*** and ***B***. The two members (***ACD*** & ***BCE***) are joined by a pin at ***C***.

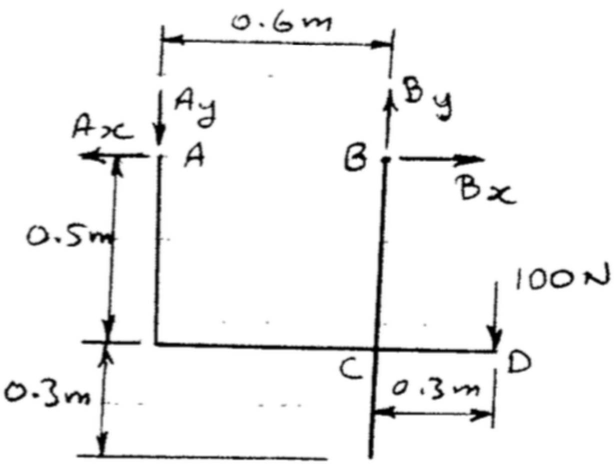
Note: Members ***ACD*** and **BCE** are continuous.



Determine the reactions at ***A*** and ***B***.

3.

FBD – Entire Frame



$\xrightarrow{+} \Sigma F_x = 0$

$B_x - A_x = 0$

$\therefore A_x = B_x$

$\circlearrowleft + \Sigma M_A = 0$

$B_y \times 0.6m - 100N \times 0.9m = 0$

$\therefore 0.6 B_y = 90$

$\therefore B_y = \frac{90}{0.6} = 150N \uparrow$

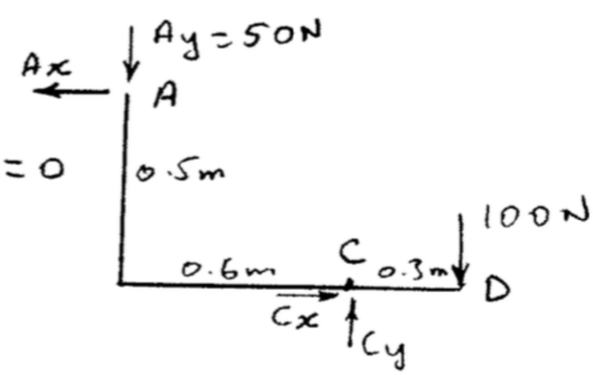
$\circlearrowleft + \Sigma M_B = 0$

$A_y \times 0.6m - 100N \times 0.3m = 0$

$0.6 A_y = 30$

$\therefore A_y = \frac{30}{0.6} = 50N \downarrow$

FBD – Member ACD



$\circlearrowleft + \Sigma M_C = 0$

$50N \times 0.6m + A_x \times 0.5m - 100N \times 0.3m = 0$

$30 + 0.5A_x - 30 = 0$

$0.5 A_x = 0;$

$\therefore A_x = 0$

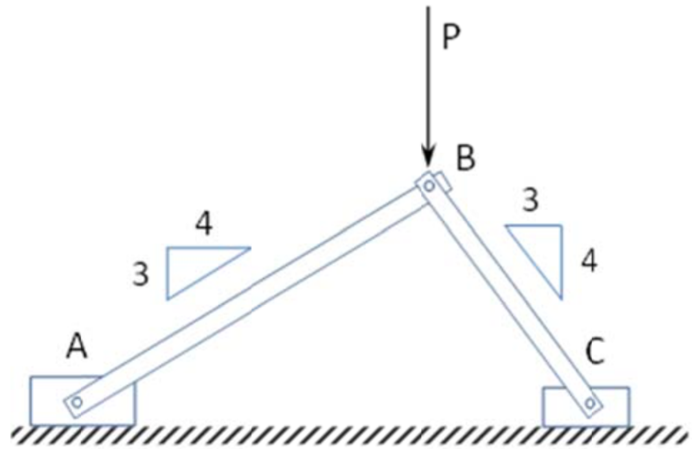
But $A_x = B_x$

$\therefore B_x = 0$

Problem 4. (12/60)

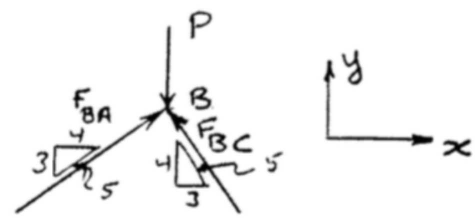
Block ***A*** weighs ***200N*** and block ***C*** weighs ***120N***. The coefficient of static friction between each block and the horizontal surface is $\boldsymbol{\mu_s}$ = 0.3.
Bars ***AB*** and ***BC*** are of negligible weight. The applied force ***P*** is vertical and all joints are pinned.

a) Write the force in ***BA*** and that in ***BC*** as functions of ***P***.
   Hint: you may take point ***B*** as a free-body diagram.
b) Find the magnitude of the applied load ***P*** for which motion will be impending.
   Hint: assume that slipping impends at ***C*** but not at ***A***.
c) Verify that slipping does not occur at ***A***.



4. a) AB and BC are 2-force members.

FBD - Joint B



$\xrightarrow{+} \Sigma F_x = 0$

$$F_{BA} \times \frac{4}{5} - F_{BC} \times \frac{3}{5} = 0$$

$$F_{BA} = F_{BC} \times \frac{3}{\cancel{5}} \times \frac{\cancel{5}}{4}$$

$$\therefore F_{BA} = \frac{3}{4} F_{BC} \quad \text{——— (1)}$$

$+\uparrow \Sigma F_y = 0$

$$F_{BA} \times \frac{3}{5} + F_{BC} \times \frac{4}{5} - P = 0$$

Insert $F_{BA} = \frac{3}{4} F_{BC}$, $\therefore \frac{3}{4} F_{BC} \times \frac{3}{5} + F_{BC} \times \frac{4}{5} = P$

$$\frac{9}{20} F_{BC} + \frac{4}{5} F_{BC} = P$$

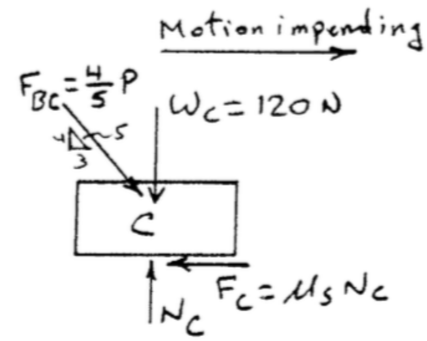
Mult. by 20; $9 F_{BC} + 16 F_{BC} = 20 P$

$$25 F_{BC} = 20 P. \quad \therefore F_{BC} = \frac{20P}{25} = \frac{4}{5} P$$

Insert in eq. (1): $F_{BA} = \frac{3}{4} \times \frac{4}{5} P = \frac{3}{5} P.$

Hence, $\underline{\underline{F_{BA} = \frac{3}{5} P}}$ and $\underline{\underline{F_{BC} = \frac{4}{5} P}}$ ANS.

b) FBD - Block C



$\xrightarrow{+} \Sigma F_x = 0$

$$F_{BC} \times \frac{3}{5} - F_C = 0 \; ; \quad F_C = \mu_s N_C$$

$$\therefore \frac{3}{5} F_{BC} - \mu_s N_C = 0$$

$$\therefore N_C = \frac{3}{5\mu_s} F_{BC} = \frac{3}{5 \times 0.3} F_{BC} = 2 F_{BC}$$

$+\uparrow \Sigma F_y = 0$

$$- F_{BC} \times \frac{4}{5} + N_C - W_C = 0$$

$$- \frac{4}{5} F_{BC} + 2 F_{BC} - 120 = 0$$

Mult. by 5:

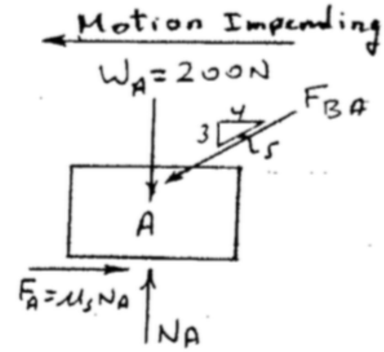
$$-4 F_{BC} + 10 F_{BC} = 600$$

$$6 F_{BC} = 600, \quad \therefore F_{BC} = \frac{600}{6} = 100$$

But, $F_{BC} = \frac{4}{5} P$, $\therefore \frac{4}{5} P = 100$; Hence, $P = \frac{500}{4} = \underline{\underline{125 \text{ N}}}$ ANS

4. (Cont'd)

c) FBD - Block A



$\xrightarrow{+} \Sigma F_x = 0$

$-F_{BA} \times \frac{4}{5} + F_A = 0$

But $F_{BA} = \frac{3}{5} P$

$\therefore -\frac{3}{5} P \times \frac{4}{5} + F_A = 0$

$-\frac{12}{25} P + F_A = 0$

Insert $P = 125N$

$\therefore -\frac{12}{25} \times 125 + F_A = 0$

$\therefore F_A = 60N$

$+\uparrow \Sigma F_y = 0$

$-\frac{3}{5} F_{BA} - W_A + N_A = 0$

$F_{BA} = \frac{3}{5} P$

$\therefore -\frac{3}{5} \times \frac{3}{5} P - 200N + N_A = 0$

$-\frac{9}{25} P - 200N + N_A = 0$

Insert $P = 125N$

$\therefore -\frac{9}{25} \times 125 - 200N + N_A = 0$

$-45 - 200 + N_A = 0$

$\therefore N_A = 245N$

$\mu_s = \frac{F_A}{N_A} = \frac{60}{245} = 0.245$

$\underline{\underline{0.245 < 0.3}}$, $\therefore$ $\underline{\underline{\text{A will not slip.}}}$ ANS.

4. (Cont'd)

c) Another method. If A slips, find the corresponding value of P.

FBD - Block A (See previous page)

$\xrightarrow{+} \Sigma F_x = 0$

$-F_{BA} \times \frac{4}{5} + F_A = 0$; $F_A = \mu_s N_A$

$\therefore -\frac{4}{5} F_{BA} + \mu_s N_A = 0$

$\therefore N_A = \frac{4}{5\mu_s} F_{BA} = \frac{4}{5 \times 0.3} F_{BA} = 2.67 F_{BA}$

$+\uparrow \Sigma F_y = 0$

$-F_{BA} \times \frac{3}{5} - W_A + N_A = 0$

$-\frac{3}{5} F_{BA} - 200N + 2.67 F_{BA} = 0$

Mult. by 5

$-3 F_{BA} - 1000 + 13.35 F_{BA} = 0$

$10.35 F_{BA} = 1000$

$\therefore F_{BA} = \frac{1000}{10.35} = 96.62 N$

But, $F_{BA} = \frac{3}{5} P$

i.e. $\frac{3}{5} P = 96.62$, $\therefore P = 161.03 N$

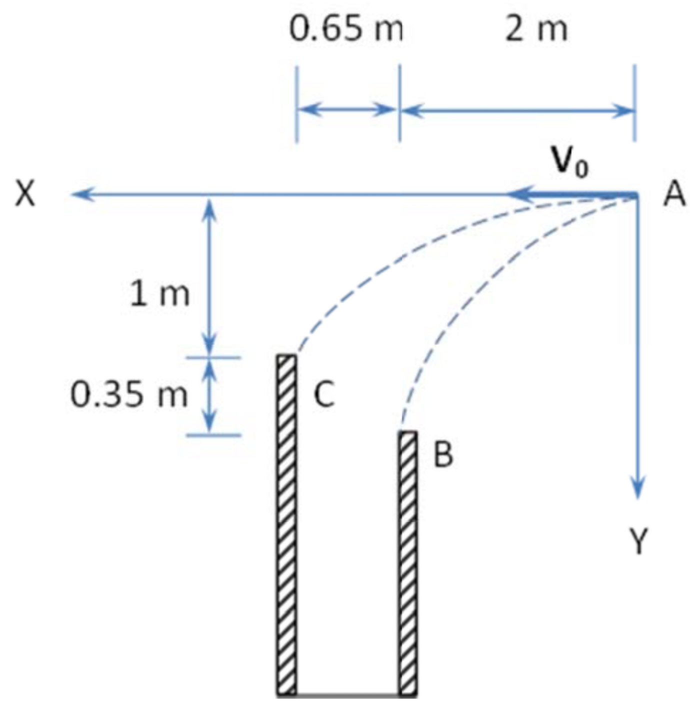
Since $P = 161.03 < P = 125 N$.

$\therefore$ Block C will slip first.

Hence, slipping does not occur at A.

Problem 5. (10/60)

Sand is discharged at ***A*** from a horizontal conveyor belt with an initial velocity $\boldsymbol{V_o}$. Determine the range of values of $\boldsymbol{V_0}$ for which the sand will enter the vertical chute down (*i.e.* between ***B*** and ***C***).



__Useful Equations:__

$$x = x_0 + vt$$
$$v = v_0 + at$$
$$x = x_0 + v_0 t + \frac{1}{2}at^2$$
$$v^2 = v_0^2 + 2a(x - x_0)$$

$$\bar{F} = m\bar{a}$$
$$\sum \bar{F} = m\bar{a}$$

5.

*- <u>Sand hits C</u> : <u>Largest value of v_o</u>

$x_c = 2.65m$, $y_c = 1m$.

$y_c = \cancel{(v_o)_y}^{\,0} t + \frac{1}{2} g t^2$

$y_c = \frac{1}{2} g t^2$

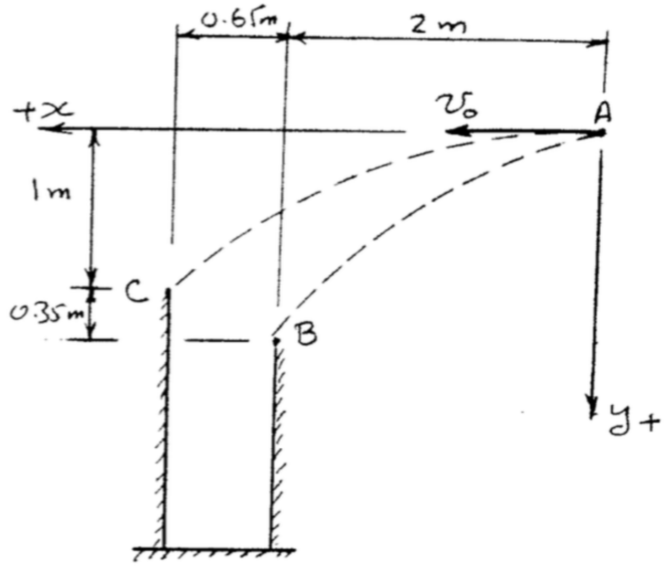
$1m = \frac{1}{2} \times 9.81 m/s^2 \, t^2$

$\therefore t = 0.4515\ s.$

$x_c = v_o t$

$2.65m = v_o \times 0.4515$

$\therefore v_o = 5.87 m/s$



*- <u>Sand hits B</u> : <u>Smallest value of v_o</u>

$x_B = 2m$, $y_B = 1.35m$

$y_B = \cancel{(v_o)_y}^{\,0} t + \frac{1}{2} g t^2.$

$y_B = \frac{1}{2} g t^2$

$1.35m = \frac{1}{2} \times 9.81 t^2$

$\therefore t = 0.5246\ s.$

$x_B = v_o t$

$2m = v_o \times 0.5246\ s$

$\therefore v_o = 3.81 m/s$

<u>For sand to enter the chute</u> : $\underline{\underline{3.81 m/s \leq v_o \leq 5.87 m/s}}$ ANS

END