Solution

For any algorithm, "R" stands for "rejection".

Solution to Question 1

(a) To show that the matching $m = \langle Ae, Bd, Cb, Dc, Ea\rangle$ is unstable, we need to find a pair of boy and girl who are not matched in m but who like each other more than their respective matches in m. Consider boy e and girl C. Note that (1) boy e is matched with girl A in m, but e likes C more than A and (2) girl C is matched with boy b in m, but C likes e more than b.

(b) Boy proposing algorithm (bpa)

Girls	Day 1	R	Day 2
A			d
B	c, d	d	c
C			b
D	b, e	b	e
E	a		a

Bpa terminates in 2 days and results in stable matching $m_b = \langle Ad, Bc, Cb, De, Ea\rangle$.

(c) Girl proposing algorithm (gpa)

Boys	Day 1	R	Day 2	R	Day 3
a					E
b	A		A		A
c	B, E	E	B		B
d	C, D	C	D, E	E	D
e			C		C

Gpa terminates in 3 days and results in stable matching $m_g = \langle Ab, Bc, Ce, Dd, Ea\rangle$.

Note that bpa and gpa result in different matchings, but both of these matchings are stable.

Solution to Question 2

(a) College proposing algorithm (cpa):

Students	Day 1	R	Day 2	R	Day 3	R	Day 4
a	Y		Y		Y		Y
b	Y, Z	Z	Y		Y		Y
c	X, Z	Z	X, Y	X	Y		Y
d			Z, W	Z	W		W
e	W, X, Y	W, Y	X		X		X
f			Z		Z		Z
g					X, Z	Z	X
h							Z

Cpa terminates in 4 days, resulting in stable matching
$m_c = \langle W : (d), X : (e, g), Y : (a, b, c), Z : (f, h)\rangle$.

(b)Student proposing algorithm (spa):

Colleges	Day 1	R	Day 2	R	Day 3
(1) W	a, f, h	f, h	a		a
(2) X	b, d, e	b	d, e, h	h	d, e
(3) Y	c, g		c, g, b		c, g, b
(2) Z			f		f, h

Spa terminates in 3 days and results in stable matching $m_s = \langle W : (a), X : (d, e), Y : (c, g, b), Z : (f, h)\rangle$. For this example, cpa and spa result in different matchings. Both of these matchings are stable.

Solution to Question 3

(a) College proposing algorithm (cpa)

Students	Day 1	R	Day 2	R	Day 3	R	Day 4
a							Y
b					W		W
c	X		X		X		X
d	X		X, Y	Y	X		X
e	W, X	X	W		W		W
f			X		X, Y	Y	X
g	X, Y	X	Y		Y		Y
h	W, Y	Y	W, X	W	X		X

Cpa terminates in 4 days, resulting in stable matching $m_c = \langle W : (b, e), X : (c, d, f, h), Y : (a, g)\rangle$.

(b) Student proposing algorithm (spa):

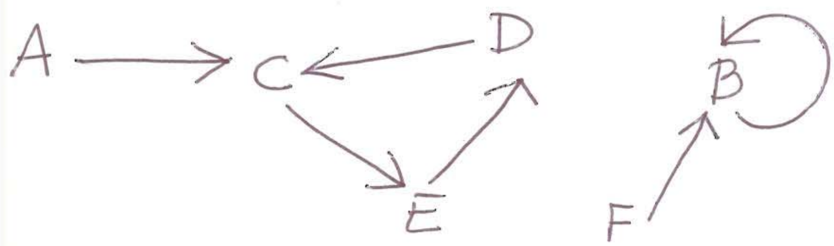
Colleges	Day 1	R	Day 2	R	Day 3	R	Day 4
(2) W	b, d, e, g	d, g	b, e		b, e, a	a	b, e
(4) X	a, c, f, h		a, c, f, h, d	a	c, f, h, d		c, f, h, d
(2) Y			g		g		g, a

Spa terminates in 4 days, resulting in stable matching $m_s = \langle W : (b, e), X : (c, f, h, d), Y : (g, a)\rangle$. For this example, cpa and spa result in the same stable matching.

Solution to Question 4

TTC algorithm

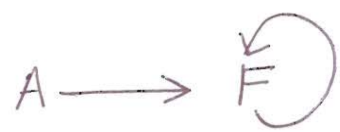
Day 1:



top trading cycles in day 1: [B], [CED]

Day 2: Traders B, C, E, D and their houses are out of the market:

A	f a
F	f a



top trading cycles in day 2: [F]

Day 3: Trader F & house f are out:

A	a

top trading cycle in day 3: [A]

Stable allocation from TTC algorithm:

Day 1: (Bb), (Ce, Ed, Dc)

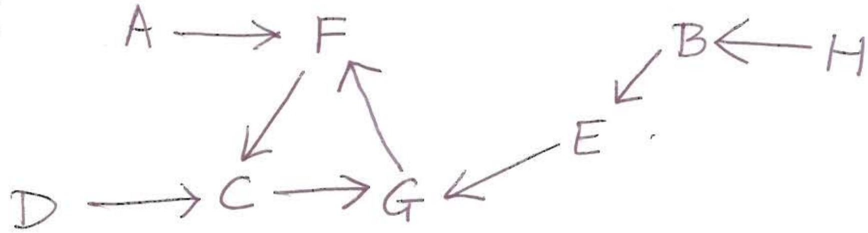
Day 2: (Ff)

Day 3: (Aa)

Stable allocation: ⟨Aa, Bb, Ce, Dc, Ed, Ff⟩

Solution to Question 5 TTC algorithm

Day 1:

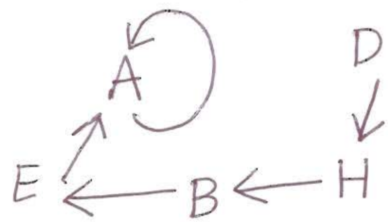


top trading cycles (TTC) in day 1: [CGF]

Day 2: Traders C, G, E and their houses are out:

A	a b d h e
B	e a b h d
D	h e d a b
E	a b h d e
H	b d e h a

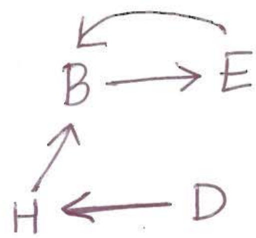
TTC in day 2: [A]



Day 3: Trader A, house are out:

TTC in day 3: [BE]

B	e b h d
D	h e d b
E	b h d e
H	b d h e



Day 4: Traders B, E and their houses are out:

TTC in day 4: [DH]

D	h d
H	d h

Stable allocation from TTC algorithm

Day 1: (Cg, Gf, Fc)
Day 2: (Aa)
Day 3: (Be, Eb)
Day 4: (Dh, Hd)
Stable allocation: <Aa, Be, Cg, Dh, Eb, Fc, Gf, Hd>