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Econ 301- Intermediate Microeconomic Theory
Assignment #2
Due date: Nov 26, 2015 in class

NAME: KEY

Please show all the necessary calculations and give all necessary explanations that lead to your answer.

1. Ray Starr has the utility function $U(x, y) = y/(100 - x)$

5p

   a. Does Ray prefer more to less of both goods?
   b. Draw a diagram showing Ray's indifference curves corresponding to the utility levels $U = 1/2$, $U = 1$, and $U = 2$.
   c. How can you describe the set of indifference curves for Ray? Is the utility homothetic?
   d. If the price of x is \$1 and the price of y is \$1, find Ray's demand for x as a function of his income and draw a diagram showing his Engel curve for x.

a.) $u(1,1) = \frac{1}{100-1} = \frac{1}{99}$; $u(2,2) = \frac{2}{100-2} = \frac{2}{98} = \frac{1}{49} > \frac{1}{99}$

therefore more is preferred to less, $(2,2) \succ (1,1)$

The general approach would be to determine the sign of the total derivative of the U function, $\frac{dU(x,y)}{dxdy}$ when dx and $dy > 0$

$$\frac{dU(x,y)}{dxdy} = \frac{\partial U(x,y)}{\partial x}dx + \frac{\partial U(x,y)}{\partial y}dy \text{ where}$$

$$\underbrace{\frac{\partial U(x,y)}{\partial x} = y(100-x)^{-2}}_{>0} \text{ and } \underbrace{\frac{\partial U(x,y)}{\partial y} = (100-x)^{-1}}_{>0 \text{ for } x<100}$$

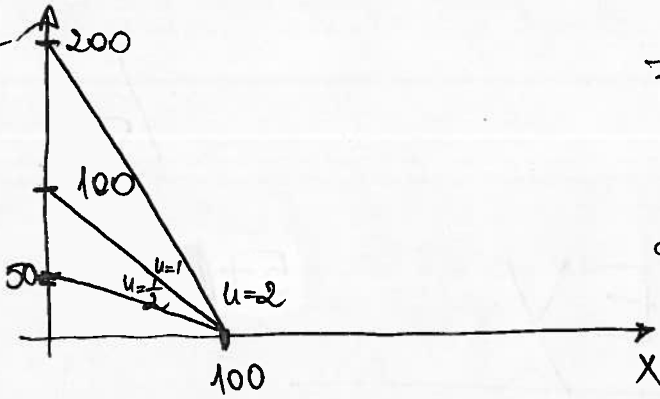
b.) $U = \frac{1}{2} \Rightarrow y = 50 - \frac{1}{2}x$

$U = 1 \Rightarrow y = 100 - x$

$U = 2 \Rightarrow y = 200 - 2x$

all linear functions (constant slope)

c.)



See p. 102-103 textbook.

For homothetic preferences,

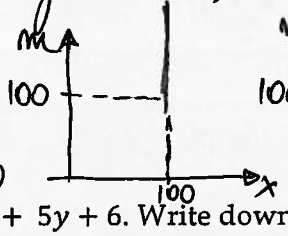
$(tx_1, ty_1) \succ (tx_2, ty_2)$, $t>0$

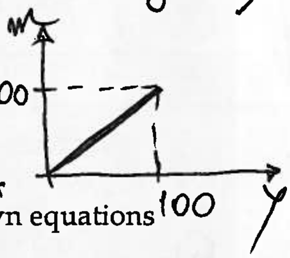
In other words, when income is scaled up/down, the demanded bundle scales up/down. (examples of homothetic preferences: perfect substitutes, perfect complements, Cobb Douglas)

d.) Given the 3 indifference curves, and the budget constraint $m = x+y$,

example, when $m=50$ and $y = 50 - \frac{1}{2}x$, $x=0$, $y=m$

$m=100$ and $y = 50 - \frac{1}{2}x$, $x=100$ (max), $y=0$





2. A competitive firm has the short-run cost function $c(y) = y^3 - 2y^2 + 5y + 6$. Write down equations for:

4p

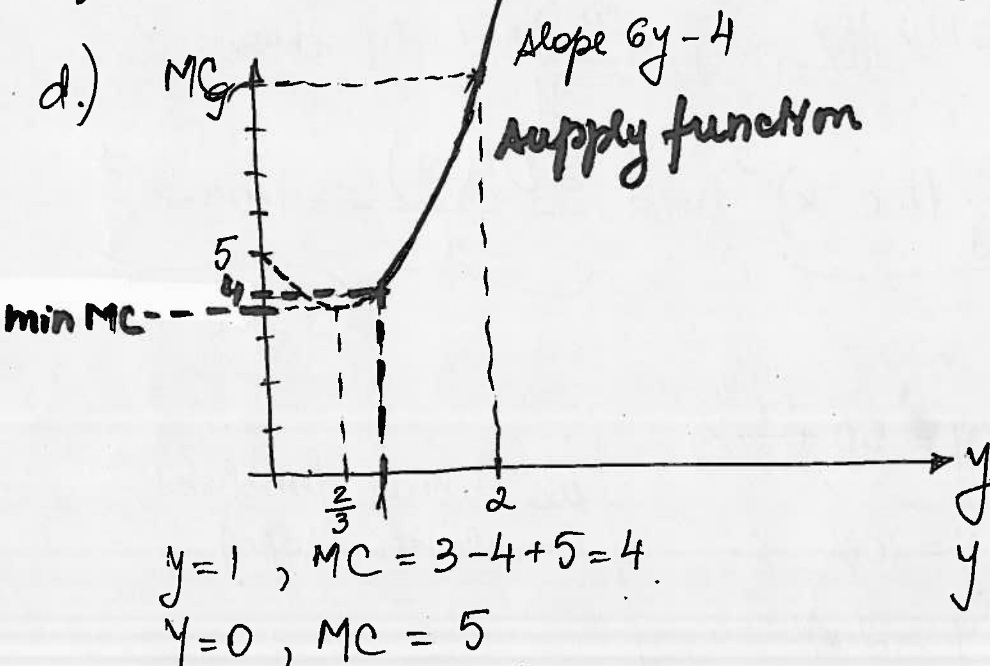
a. The firm's average variable cost function
b. The firm's marginal cost function
c. At what level of output is average variable cost minimized?
d. Graph the short-run supply function for this firm, being careful to label the key points on the graph with the numbers specifying the exact prices and quantities at these points. MC

a.) $AVC = (y^3 - 2y^2 + 5y)/y = y^2 - 2y + 5$

b.) $MC = \frac{dc(y)}{dy} = 3y^2 - 4y + 5$

c.) min AVC when $(AVC)' = 0 \Rightarrow 2y - 2 = 0$, $y^* = 1$

d.)



$y=1$, $MC = 3-4+5=4$

$y=0$, $MC = 5$

$y=2$, $MC = 3\times4 - 4\times2 + 5 = 9$

$y=5 \Rightarrow MC = 3\times25 - 4\times5 + 5$
$= 60$

Identify the choice that best completes the statement or answers the question. 5p

____ 1. In Ozone, California, people all have the same tastes and they all like hot tubs. Nobody wants more than one hot tub but a person with wealth $\$W$ will be willing to pay up to $\$.01W$ for a hot tub. The number of people with a wealth greater than $\$W$ for any given W in Ozone is approximately $1{,}000{,}000/W$. The price elasticity of demand for hot tubs in Ozone, California, is

a. 0.1.
b. 0.01.
c. 1. Actually −1; when the price of hot tubs doubles, the number of buyers decreases from $\frac{1{,}000{,}000}{W}$ to $\frac{1{,}000{,}000}{2W}$
d. 0.4.
e. none of the above.

(p increases from $0.01W$ to $2\times 0.01W$ or $0.01(2W)$)

____ 2. Dr. Social Science has recently figured out how to clone consumers. His first effort was done on the population of Walla, Washington. Each original citizen got a clone who had exactly the same income and preferences. Which of the following statements describes what happened to the demand function for tuna-fish casseroles in Walla?

a. The elasticity doubled and the slope remained constant.
b. The elasticity did not change at any price.
c. The elasticity of demand doubled and the slope doubled.
d. The elasticity halved and the slope remained constant.
e. none of the above.

____ 3. At the price of \$100, tourists demand 267 airplane tickets. At the same price, business travelers demand 237. At the price \$110, tourists demand 127 tickets and business travelers demand 127. Assuming that the demand curves of business travelers and tourists are both linear over this price range, what is the price elasticity of demand at the price \$100?

a. 4.96 if we take $|\varepsilon_p|$
b. 25
c. 5.46
d. 0.05
e. none of the above.

if we consider that $\varepsilon_p < 0$

$$|\varepsilon_p| = \frac{\Delta Q}{\Delta p} \cdot \frac{p}{Q} = \frac{254-504}{110-100} \cdot \frac{100}{504} = \frac{250}{10} \cdot \frac{100}{504} \approx 5$$

____ 4. A competitive firm produces a single output using several inputs. The price of output rises by \$4 per unit. The price of one of the inputs increases by \$4 and the quantity of this input that the firm uses increases by 16 units. The prices of all other inputs stay unchanged. From the weak axiom of profit maximization we can tell that

a. the inputs of the other factors must have stayed constant.
b. the inputs of at least one of the other factors must have decreased by at least 16 units.
c. the output of the good must have increased by at least 16 units.
d. the output of the good must have decreased by at least 8 units.
e. the inputs of at least one of the other factors must have increased by at least 16 units.

According to the law of supply $\Delta p \cdot \Delta y \geq 0$, an increase in price will justify the increase in output y. The increase in the quantity of one input (and the increase in its price) requires (conditions) that the increase in output be at least by 16 units to at least maintain the initial level of profits.

5. A competitive firm's production function is $f(x_1, x_2) = 8x_1^{1/2} + 8x_2^{1/2}$. The price of factor 1 is $1 and the price of factor 2 is $3. The price of output is $6. What is the profit-maximizing quantity of output?

a. 256
b. 512
c. 252
d. 516
e. 244

$\max \pi = \max\ pf(x_1,x_2) - w_1x_1 - w_2x_2$

$\max\ 6\left(8x_1^{1/2} + 8x_2^{1/2}\right) - x_1 - 3x_2$

FOC: $pMP_{x_1} = w_1 \Leftrightarrow MP_{x_1} = \frac{1}{6} \Leftrightarrow 8\frac{1}{2}x_1^{-\frac{1}{2}} = \frac{1}{6} \Rightarrow x_1^* = 24^2$

$pMP_{x_2} = w_2 \Leftrightarrow MP_{x_2} = \frac{1}{2} \Leftrightarrow 8\frac{1}{2}x_2^{-\frac{1}{2}} = \frac{1}{2} \Rightarrow x_2^* = 8^2$

from there, $y = 8(24) + 8(8) = 256$

True/False

Indicate whether the statement is true or false and explain.

1. A firm produces one output with one input and has decreasing returns to scale. The price that it pays per unit of input and the price it gets per unit of output are independent of the amount that this firm buys or sells. If the government taxes its net profits at some percentage rate and subsidizes its inputs at the same percentage rate, the firm's profit-maximizing output will not change.

3p

False The profit function for the initial levels of prices is

$\pi_{initial} = p \cdot y - wx$ (as there is only one input used)

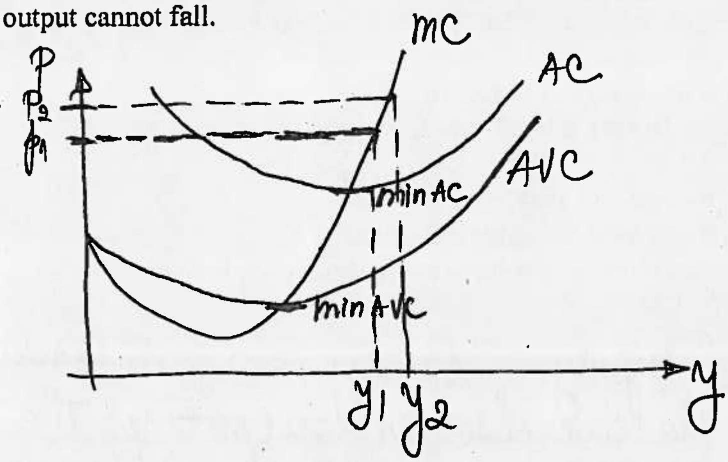
After the tax ~~on~~ net profits is applied and the subsidy is granted, the new π function will be

$\pi_{new} = [py - (1-t)wx](1-t) \neq \pi_{initial}$ (unless $py = 2wx - twx$)

2. If the price of the output of a profit-maximizing, competitive firm rises and all other prices stay constant, then the firm's output cannot fall.

True

3p.



An increase in the market price from p_1 to p_2 gives the incentive to the competitive firm to increase production and increase profits (as ~~the~~ input prices stay the same). That's consistent with the law of supply.