

Econ 301: Assignment 1

Fall 2015, Concordia University

Due: October 20, 2015, by 4:00 p.m. in my mailbox on the 11th floor of the Hall building, ECON Department

Please show all the necessary calculations and explanations that lead to your answer. Provide your answers in the order the problems are given.

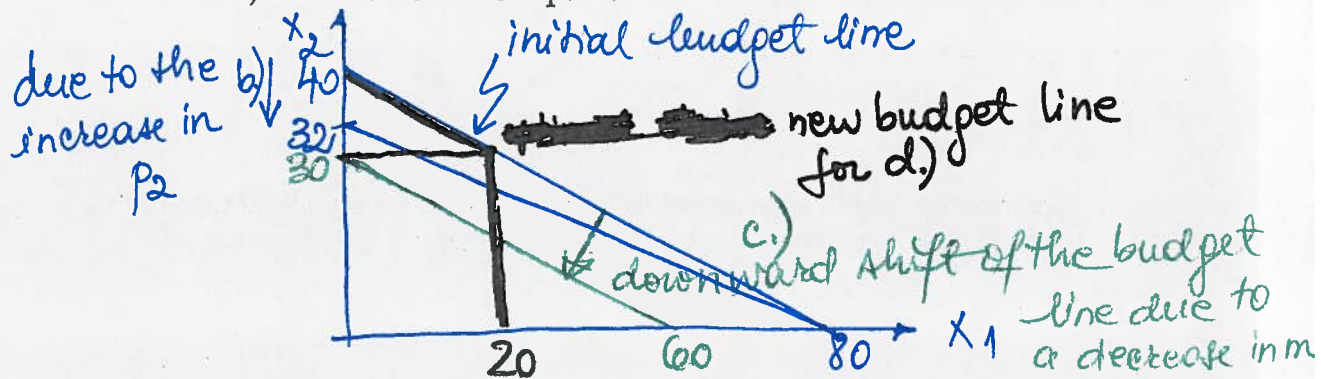
1. Suppose the consumer consumes two goods: pizzas and CDs. Her monthly income is $m = \$1600$. The price of a pizza is $p_1 = \$20$ and the price of a CD is $p_2 = \$40$.

- a) Write down her monthly budget constraint. Illustrate graphically the budget set. Calculate the slope of the budget line.

The budget constraint is $20x_1 + 40x_2 = 1600$ with vertical intercept $\frac{1600}{40} = 40$ and horizontal intercept $\frac{1600}{20} = 80$ and slope $-\frac{p_1}{p_2} = -\frac{20}{40} = -0.5$.

- b) Suppose the government levies a 25% tax on each CD the consumer purchases. What is the new monthly budget constraint? Draw the new budget line.

The new price of CD's is $p'_2 = 40(1 + 0.25) = 50$ for a CD. The budget constraint is $20x_1 + 50x_2 = 1600$, with the same horizontal intercept as in a) and a vertical intercept 32.



- c) Now suppose that instead of the 25% tax on CDs each consumer pays a fixed sum of \$400 in taxes per month. What is the monthly budget constraint in this case? Draw the new budget line.

The consumer has a smaller income, \$1,200. The budget constraint is $20x_1 + 40x_2 = 1200$.

- d) Now suppose that instead of imposing any tax, the government introduces a new policy that each consumer is allowed to eat at most 20 pizzas per month. What is the monthly budget set in this case? Illustrate the new budget set in a graph.

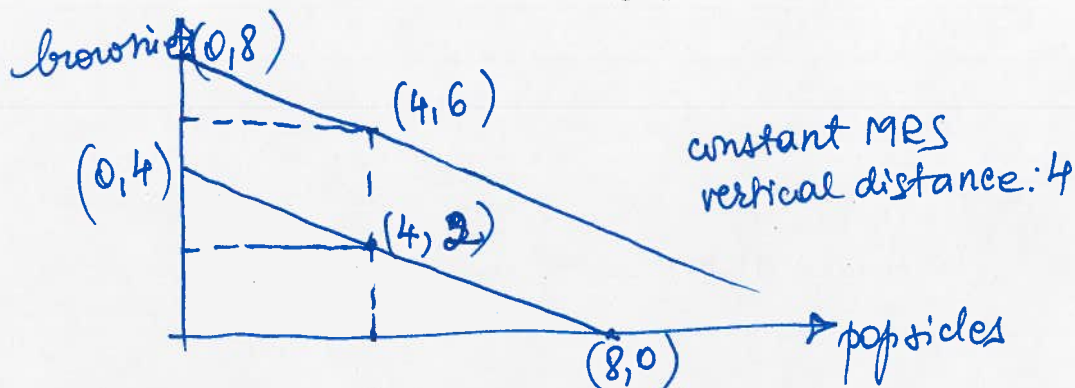
Rationing pizzas to 20 a month means that the budget line drops at $(20, 30)$.

2. Azalia likes both popsicles and brownies. She is willing to substitute exactly two popsicles for each brownie. For example, Azalia is indifferent among the bundles $(8, 0)$, $(6, 1)$, $(4, 2)$, $(2, 3)$ and $(0, 4)$, where each bundle is given as (x_p, x_b) , and x_p denotes the number of popsicles, while x_b denotes the number of brownies.

- a) Which bundles make Azalia indifferent between them and the bundle $(0, 8)$? Identify a few such bundles.

The consumer is indifferent between $(0, 8)$ and $(2, 7)$, $(4, 6)$, for example. The vertical distance between the indifference curve and the new one is 4.

- b) Plot some of Azalia's indifference curves in a graph.



- c) What is the MRS for these indifference curves at different points? Do these indifference curves exhibit a diminishing marginal rate of substitution?

The MRS is constant at $-1/2$, not diminishing. An easy way to figure it out is to compare max x_1 and max x_2 , or the intercepts.

- d) Are Azalia's preferences monotonic?

Monotonic preferences: more is better, negatively sloped indifference curve.

3. Art is a history major. He has to take two courses, one in philosophy and the other in mathematics. He is relatively good at philosophy, but not so much at math. He needs to spend 1 hour studying for philosophy in order to improve his grade by one point, while he needs to spend 3 hours studying for math in order to do the same. Art has a total of 60 hours to study for the two subjects. Suppose that he would get 50 points in each course if he didn't study at all.

- a) We want to analyze Art's problem with consumption theory. What are the goods, prices, and income? Write down the budget constraint. Illustrate it with a graph. (Hint: consider the improvements on Art's grades. That is, the goods are improvements in philosophy and math, and the prices are the cost of improvement measured in hours.)

The prices are measured in terms of the time spent studying in order to

improve the grades for each subject, by 1 point. Therefore, $p_1 = 1$ and $p_2 = 3$. The income is measured by the total time available to study, 60 hours. The budget constraint is: $x_1 + 3x_2 = 60$.

- b) Art's satisfaction from his academic record is solely determined by the lower of his grades in philosophy and mathematics. For example, he is indifferent between 60 in philosophy and 70 in math, and 60 in philosophy and 60 in math. What kind of preferences does Art have over improvements in his philosophy and mathematics grades? Find a utility representation for these preferences. Derive his optimal choice of grades. Illustrate it in a graph.

Art needs to improve the marks for both subjects, not only for one, therefore x_1 and x_2 are complements. The utility function form is

$$u = \min \{x_1, x_2\}$$

The optimal choice of grades is (65,65), with 15 additional points obtained through studying, and 50 points obtained without studying.

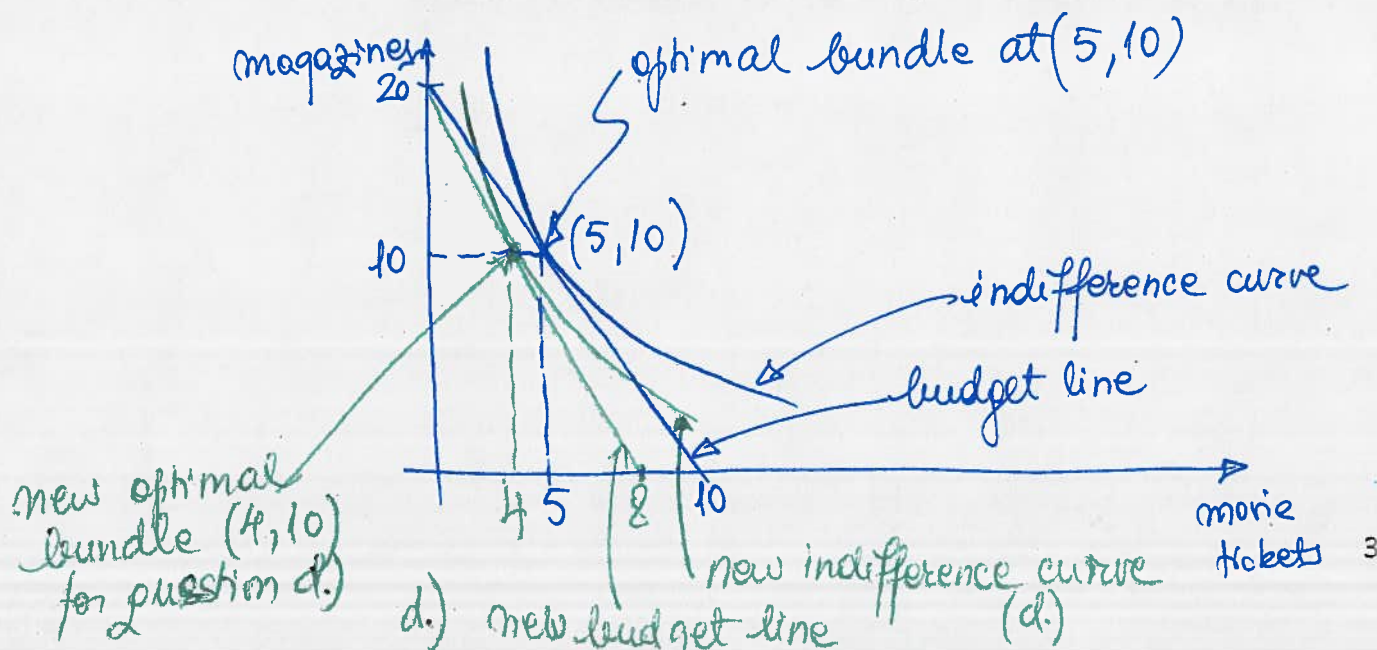
- c) Now suppose that Art has to pass both subjects by obtaining at least 60 points in both courses. Assuming that this is a "budget restriction," illustrate Art's new budget set in a graph. What is Art's optimal choice in this case?

The original optimal combination is still feasible, at (65,65).

4. Jacob spends all his income on movie tickets (good 1) and magazines (good 2). His utility function can be represented by $u(x_1, x_2) = x_1x_2$. His income is $m = \$100$. The prices are $p_1 = \$10$ and $p_2 = \$5$.

- a) Derive Jacob's optimal consumption bundle. Show the complete calculation, and illustrate your answer graphically (draw the indifference curves and the budget constraint).

$MRS = -\frac{MU_{x1}}{MU_{x2}} = -\frac{x_2}{x_1} = -\frac{p_1}{p_2}$ at optimum point. Using this result and the budget constraint equation, we get $x_1 = 5$ and $x_2 = 10$.



- b) How would your answer change to part a. if Jacob's utility function were given by $u(x_1, x_2) = 5x_1^2x_2^2$?

The new function is a monotonic transformation of the equation above, the result in terms of MRS does not change:

$$MRS = -\frac{MU_{x_1}}{MU_{x_2}} = -\frac{10x_1x_2^2}{10x_2x_1^2} = -\frac{x_2}{x_1}$$

- c) What is Jacob's marginal utility of movie tickets when he buys 5 movie tickets and 8 magazines? Compute this marginal utility for both utility functions given above.

At (5,8), the marginal utilities given the function form from a) is: $MU_{x_2}=8$ and from b) is: $MU_{x_2}=10x_1x_2^2=10*5*64=3200$.

- d) Now suppose the government levies a 25% tax on each movie ticket purchased. Derive Jacob's consumption bundle and illustrate it in the same graph that you drew in part a.

$p'_1 = 10(1 + 0.25) = 12.5$ per ticket. The new budget line is steeper with the slope $-p'_1/p_2 = -2.5$. Given the new price of movie tickets, the two equations for finding the solution are:

$$-\frac{x'_1}{x'_2} = -\frac{p'_1}{p_2} = -2.5 \text{ and } 12.5x'_1 + 5x'_2 = 100. \text{ From here, } x'_1 = 4 \text{ and } x'_2 = 10.$$

- e) Would Jacob prefer a \$10 income tax (i.e., lump-sum tax) to the 25% value tax on movie tickets? Which scenario would the government prefer? Explain your answers.

According to the result from d), Jacob buys 4 movie tickets, therefore he pays \$10 in taxes ($4*\2.5). The government collects the same amount in either case.