CARLETON UNIVERSITY

A.M.

FINAL EXAMINATION APRIL 1999

457

No. of Students:

Department Name & Course Number: Mathematics & Statistics 69.204*C

Course Instructor(s) Dr. A. Monadi

AUTHORIZED MEMORANDA ANY NON-PROGRAMMABLE CALCULATOR IS PERMITTED.

Students **MUST** count the number of pages in this examination question paper before beginning to write, and report any discrepancy immediately to a proctor. This question paper has 12 pages.

This examination question paper MAY NOT be taken from the examination room. 10

This examination may be released to the Library.

Instructions: All answers must be written on this examination paper.

Part A: Twelve multiple choice questions: 2 marks each. Please circle only one answer to each question.

Part B: Do all six questions: 6 marks each. Part marks may be awarded.

The total of this examination is 60 marks.

Use the backs of the pages for rough work. If you are unable to complete a question in part B on the same page, continue on the last two blank pages.

Last Name (print) ____________ Given Names ____________ **Student Number** ____________

Question	Marks
Part A / 24	
Part B / 36	
B1	
B2	
B3	
B4	
B5	
B6	
Exam / 60	
Term / 40	
Final Marks / 100	

A. M

Part A: Multiple Choice Questions (2 marks each)

In each of the following questions, please circle only one answer. If you do not believe any of the answers to a given quesion, please provide your reason in the space given on this page.

A1. The equation $\rho = 4\cos\phi$ in spherical coordinate represents a sphere with center

(a) $(2,1,1)$, (b) $(-2,1,1)$, (c) $(2,0,1)$, (d) $(0,0,2)$.

A2. The area enclosed by the polar curve $r = 1 + \cos\theta$ is

(a) $\frac{3\pi}{4}$, (b) $\frac{5\pi}{2}$, (c) $\frac{3\pi}{2}$, (d) $\frac{\pi}{2}$.

A3. Let $w = f(u, v)$, $u = x + y$, $v = \frac{x^2+y^2}{2}$ then

(a) $w_x = f_u + xf_v$ and $w_y = f_u + yf_v$, (b) $w_x = f_u - xf_v$ and $w_y = yf_u + f_v$,
(c) $w_x = xf_u + f_v$ and $w_y = f_u + yf_v$, (d) $w_x = xf_u - f_v$ and $w_y = f_u - yf_v$.

A4. The rectangular coordinate of a point with cylindrical coordinate $(1, \frac{\pi}{2}, 5)$ is

(a) (0,1,5), (b) (1,0,5), (c) (5,0,1), (d) (5,1,1).

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A5. The speed $|r'(t)|$ of $r(t) = \mathbf{i} + (\tan t)\mathbf{j} + 12\mathbf{k}$ at $t = \frac{\pi}{4}$ is

(a) 2, (b) 3, (c) $\sqrt{2}$, (d) $\sqrt{3}$.

A6. The tangent plane to the surface $x^3 + y^3 + z^3 = 3$ at the point $(1,1,1)$ can be written as

(a) $x + y + z = 1$, (b) $x + y + z = 3$.

(c) $x - y + z = 1$, (d) $x + y - z = 3$.

A7. The Jacobian $J(u, v)$ of the transformation $u = -\frac{y}{x}$ and $v = x^2 + y^2$ is

(a) $\frac{1}{u^2+1}$, (b) $\frac{1}{2(u^2+1)}$, (c) $\frac{1}{v^2+1}$, (d) $\frac{1}{2(v^2+1)}$.

A8. Let C be the curve with parametric equations $x(t) = 1 + t^2, y(t) = 2t + 1$ with $0 \le t \le 1$. Then the line integral $\int_C x dy + y dx$ is equal to

(a) 2, (b) 3, (c) 4, (d) 5.

A9. The maximum directional derivative of the function $f(x,y) = x^3 + 4y + 15$ at the point $P(1,11)$ is

(a) 4, (b) 7, (c) 11, (d) 5.

A10. If the vectors $v = < 1,2,3 >$ and $w = < 1, -\frac{a}{2}, 0 >$ are perpendicular, then $a =$

(a) 1, (b) −1, (c) 2, (d) −2.

A11. The Fourier sine series of the function $f(t) = t$, $0 \le t < 3$ is

(a) $\frac{6}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\sin\frac{n\pi t}{3}}{n}$, (b) $\frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\sin\frac{n\pi t}{2}}{n}$,

(c) $\frac{3}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\sin\frac{n\pi t}{3}}{n}$, (d) $\frac{6}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}\sin\frac{n\pi t}{6}}{n}$.

A12. If $xz - xy - yz = 15$, then $\frac{\partial z}{\partial y}$ equal to

(a) $\frac{x+z}{x-y}$, (b) $\frac{x+z}{x+y}$, (c) $\frac{x-z}{x-y}$, (d) $\frac{x+y}{x-z}$.

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Part B: Do All Questions (6 Marks Each)

B1. By using Lagrange multipliers, find the minimum value of the function $f(x,y,z) = 2x + 4y + z$ subject to $z = x^2 + y^2$.

Solution.

A.M

B2. Find and classify all critical points of the function $f(x,y) = 2x^3 - 3x^2 + y^2 - 12x + 10$.

Solution.

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B3. Evaluate the double integral $\int_0^1 \int_{\frac{y}{2}}^{\frac{1}{2}} e^{-x^2}\,dx\,dy$.

Solution.

B4. By using polar coordinates, compute the polar moment of inertia $I_0 = \int\int_R (x^2+y^2)dxdy$, where R is the disk $x^2+y^2 \leq 9$.

Solution.

B5. Show that the vector field $F(x,y) = (x^3 + \frac{y}{x})\mathbf{i} + (y^2 + \ln x)\mathbf{j}$ is conservative. Find a potential function for this vector field.

Solution.

B6. Let $r(t) = (5\cos\frac{\pi t}{2})\mathbf{i} + (5\sin\frac{\pi t}{2})\mathbf{j}$ be the equation of a circular motion. Find the velocity and scalar acceleration of this motion at time $t = 1$.

Solution.