Université d'Ottawa | University of Ottawa

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Soil Mechanics II

Chapter 5
Earth Pressure

Course Instructor

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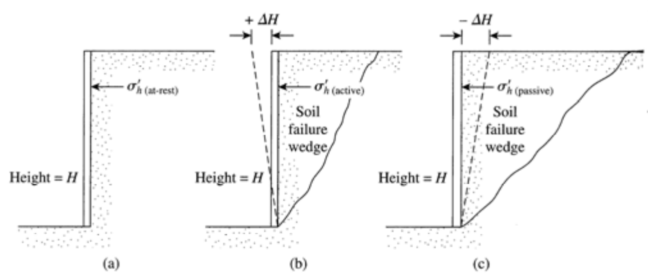
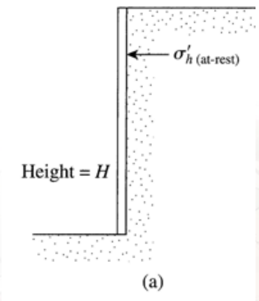
Introduction

- **Major design function of retaining walls (and foundation walls) is to *maintain the ground surfaces at different elevations* on either side of it and offer resistance to earth/water pressures (i.e., to provide lateral support).**
- **Lateral earth pressures are a function of *type* and *amount of wall movement*, *shear strength* properties, *weight of soil* and *drainage*.**



Introduction

- The earth pressure coefficient, ***K***, is defined as $K = \frac{\sigma'_h}{\sigma'_v}$



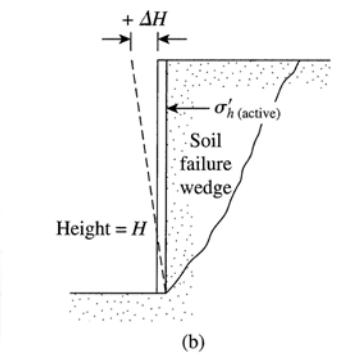
- ✓ **At rest: K_0**

$$K_0 = 1 - \sin\phi'$$

$$K_0 = (1 - \sin\phi')OCR^{\sin\phi'}$$

Introduction

- The earth pressure coefficient, ***K***, is defined as $K = \frac{\sigma'_h}{\sigma'_v}$



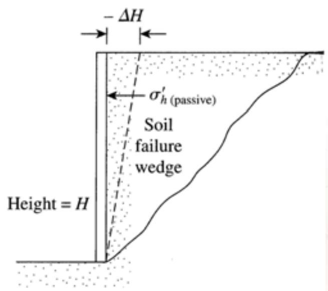
- ✓ **Active: K_a**

$$K_a = \frac{1 - \sin\phi'}{1 + \sin\phi'} = \tan^2\left(45 - \frac{\phi'}{2}\right)$$

- ✓ **vertical stress, σ_v is the maximum principal stress (i.e. σ_1) and horizontal stress, σ_h will be equal to σ_3**

Introduction

❑ **The earth pressure coefficient, *K*, is defined as** $K = \dfrac{\sigma'_h}{\sigma'_v}$



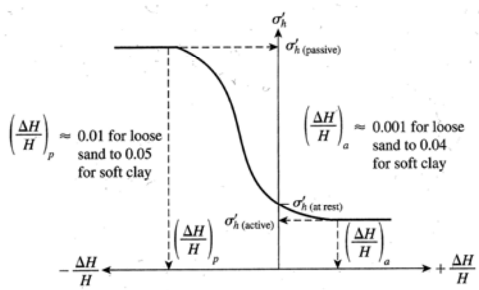
(c)

✓ **Passive: K_p**

$$K_a = \frac{1+\sin\phi'}{1-\sin\phi'} = \tan^2\left(45+\frac{\phi'}{2}\right)$$

✓ **vertical stress, σ_v will be minor principal minor stress (i.e. σ_3) and horizontal stress, σ_h will major principal stress, σ_1**

Introduction



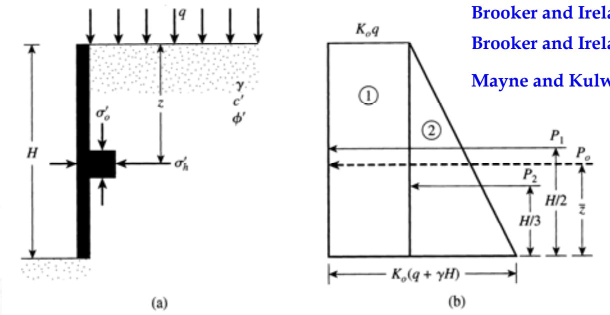
Active earth pressure coefficient, $K_a = \dfrac{1-\sin\phi'}{1+\sin\phi'} = \tan^2\left(45-\dfrac{\phi'}{2}\right)$

Passive earth pressure coefficient, $K_p = \dfrac{1+\sin\phi'}{1-\sin\phi'} = \tan^2\left(45+\dfrac{\phi'}{2}\right)$

$K_a < K_o < K_p$

What is the relationship between K_a and K_p?

Lateral earth pressure at rest



Jaky, 1944 $K_o = 1 - \sin\phi$

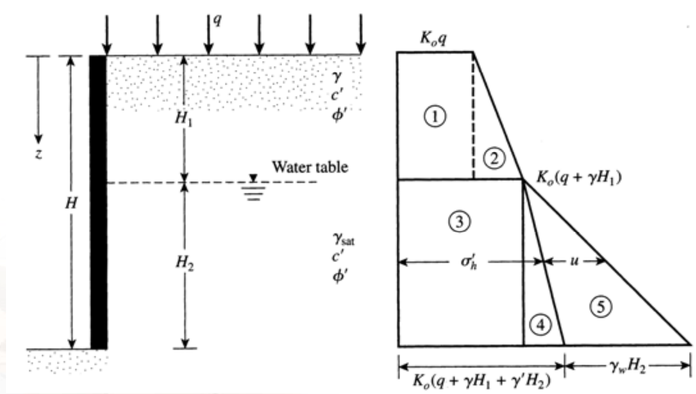
Brooker and Ireland, 1965 $K_o = 0.95 - \sin\phi$

Brooker and Ireland, 1965 $K_{o(OC)} = K_{o(NC)}\sqrt{OCR}$

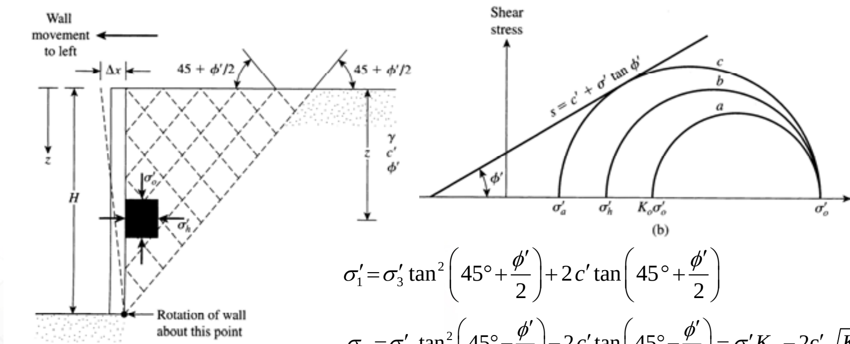
Mayne and Kulway, 1982 $K_0 = (1 - \sin\varphi')OCR^{\sin\varphi'}$

$$\bar{z} = \frac{P_1\left(\frac{H}{2}\right) + P_2\left(\frac{H}{3}\right)}{P_o}$$

Lateral earth pressure at rest



Rankine active earth pressure

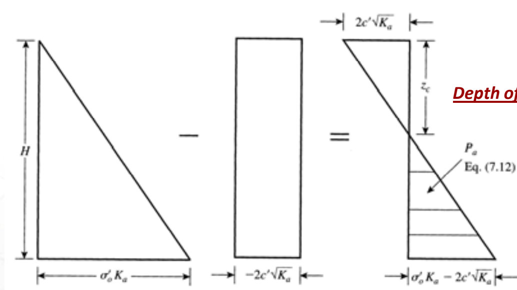


$$\sigma_1' = \sigma_3' \tan^2\left(45° + \frac{\phi'}{2}\right) + 2c' \tan\left(45° + \frac{\phi'}{2}\right)$$

$$\sigma_a = \sigma_0' \tan^2\left(45° - \frac{\phi'}{2}\right) - 2c' \tan\left(45° - \frac{\phi'}{2}\right) = \sigma_0' K_a - 2c'\sqrt{K_a}$$

Rankine active earth pressure

$$\sigma_a = \sigma_0' \tan^2\left(45° - \frac{\phi'}{2}\right) - 2c' \tan\left(45° - \frac{\phi'}{2}\right) = \sigma_0' K_a - 2c'\sqrt{K_a}$$

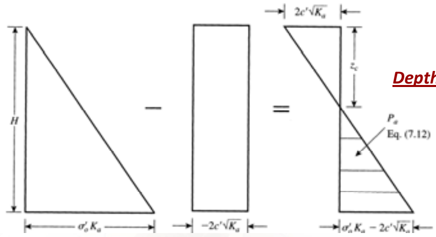


Depth of tension crack

$$z_c = \frac{2c'}{\gamma\sqrt{K_a}}$$

$$\sigma_a = \sigma_0 \tan^2\left(45° - \frac{\phi}{2}\right) - 2c \tan\left(45° - \frac{\phi}{2}\right)$$

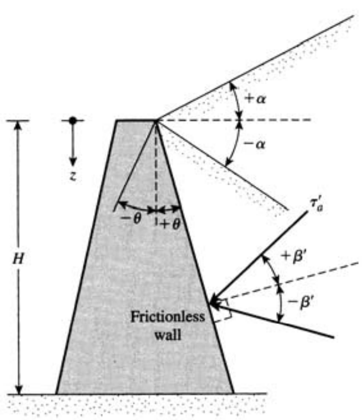
Rankine active earth pressure



Depth of tension crack $z_c = \dfrac{2c'}{\gamma \sqrt{K_a}}$

- **Before tension crack** $P_a = \int_0^H \sigma_a' \, dz = \int_0^H \gamma z \sqrt{K_a} \, dz - \int_0^H 2c' \sqrt{K_a} \, dz = \frac{1}{2} \gamma H^2 K_a - 2c' H \sqrt{K_a}$
- **After tension crack** $P_a = \frac{1}{2}(H - z_c)\left(\gamma H K_a - 2c' \sqrt{K_a}\right) = \frac{1}{2}\left(H - \frac{2c'}{\gamma \sqrt{K_a}}\right)\left(\gamma H K_a - 2c' \sqrt{K_a}\right)$

Generalize case for Rankine active pressure



- **Granular backfill**

$$K_a = \frac{\cos(\alpha - \theta)\sqrt{1 + \sin^2 \phi' - 2\sin\phi' \cos\psi_a}}{\cos^2\theta\left(\cos\alpha + \sqrt{\sin^2\phi' - \sin^2\alpha}\right)}$$

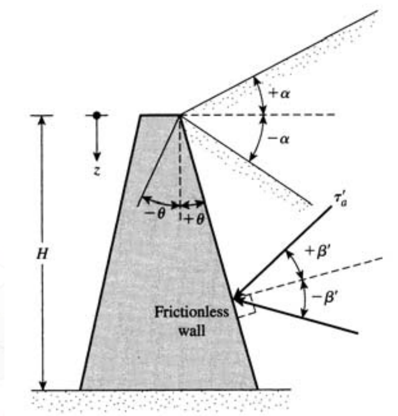
$$\psi_a = \sin^{-1}\left(\frac{\sin\alpha}{\sin\phi'}\right) - \alpha + 2\theta$$

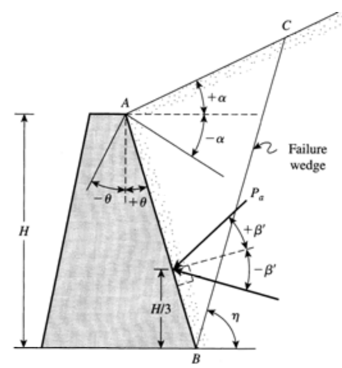
$$\beta' = \tan^{-1}\left(\frac{\sin\phi' \sin\psi_a}{1 - \sin\phi' \cos\psi_a}\right)$$

$$P_a = \tfrac{1}{2}\gamma H^2 K_a$$

Generalize case for Rankine active pressure

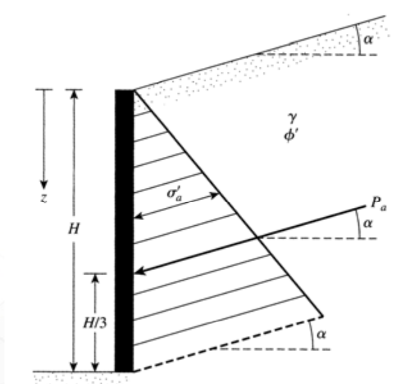
- Granular backfill





$$\eta = \frac{\pi}{4} + \frac{\phi'}{4} + \frac{\alpha}{2} - \frac{1}{2}\sin^{-1}\left(\frac{\sin\alpha}{\sin\phi'}\right)$$

Generalize case for Rankine active pressure

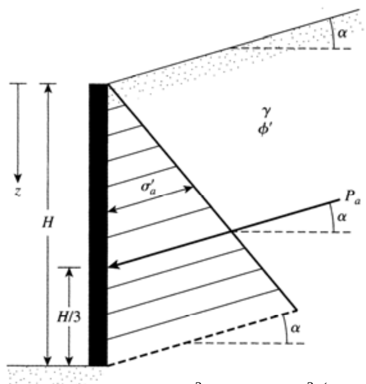


- Granular backfill with vertical back face (c' = 0)

$$K_a = \cos\alpha \frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi'}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi'}}$$

$$P_a = \tfrac{1}{2}\gamma H^2 K_a$$

Generalize case for Rankine active pressure



$$P_a = \tfrac{1}{2}\gamma H^2 K_a = \tfrac{1}{2}\gamma H^2 \left(K_a' \cos\alpha\right)$$

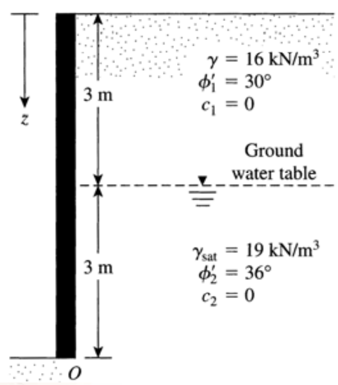
- Granular backfill with vertical back face (c', ϕ')

$$K_a' = \frac{1}{\cos^2\phi'}\left\{\begin{array}{l} 2\cos^2\alpha + 2\left(\dfrac{c'}{\gamma z}\right)\cos\phi'\sin\phi' \\ -\sqrt{4\left(\cos^2\alpha - \cos^2\phi'\right) + 4\left(\dfrac{c'}{\gamma z}\right)\cos^2\phi' + 8\left(\dfrac{c'}{\gamma z}\right)\cos^2\alpha\sin\phi'\cos\phi'} \end{array}\right\} - 1$$

		$\frac{c'}{\gamma z}$			
ϕ' (deg)	α (deg)	0.025	0.05	0.1	0.5
15	0	0.550	0.512	0.435	−0.179
	5	0.566	0.525	0.445	−0.184
	10	0.621	0.571	0.477	−0.186
	15	0.776	0.683	0.546	−0.196
20	0	0.455	0.420	0.350	−0.210
	5	0.465	0.429	0.357	−0.212
	10	0.497	0.456	0.377	−0.218
	15	0.567	0.514	0.417	−0.229
25	0	0.374	0.342	0.278	−0.231
	5	0.381	0.348	0.283	−0.233
	10	0.402	0.366	0.296	−0.239
	15	0.443	0.401	0.321	−0.250
30	0	0.305	0.276	0.218	−0.244
	5	0.309	0.280	0.221	−0.246
	10	0.323	0.292	0.230	−0.252
	15	0.350	0.315	0.246	−0.263

- **Example**

Refer to the figure shown below. Assume that the wall can yield sufficiently and determine the Rankine active force per unit length of the wall. Also determine the location of the resultant line of action

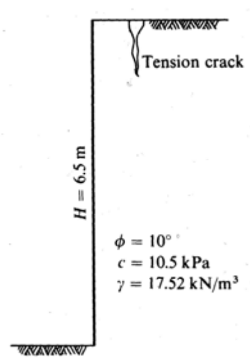


❑ Example

A 6-m-high retaining wall is to support a soil with unit weight γ = 17.4 kN/m^3. Determine the Rankine active force per unit length of the wall both before and after the tensile crack occurs, and determine the line of action of the resultant in both cases. (c' = 14.36 kPa, ϕ' = 26°)

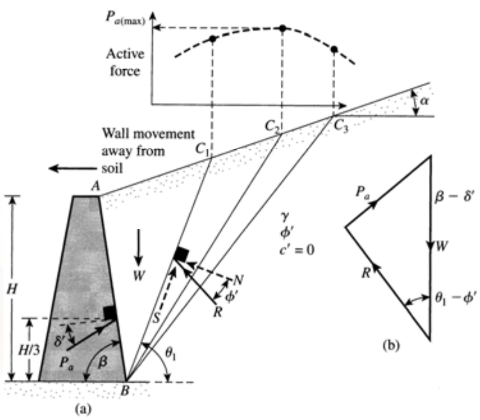
❑ Example

Draw the pressure diagram for a unit width of wall for the conditions shown below. Compare the several possible alternatives that are produced this problem (tension crack, how the diagram might me modified, and water in tension crack).



Coulomb's active earth pressure

- Coulomb's theory allows to determine active earth pressure using a ***graphical procedure*** for ***an inclined backfill***, ***inclined retaining structure*** and ***wall friction*** as shown below:



Coulomb's active earth pressure

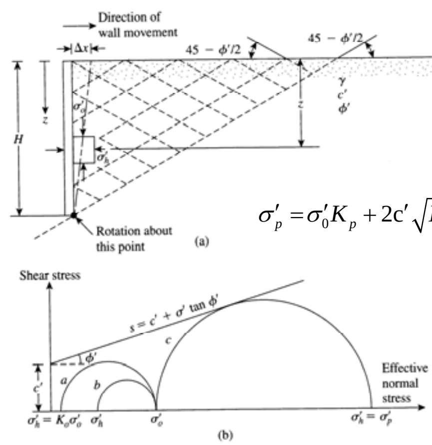
$$K_a = \frac{\sin^2(\beta+\phi')}{\sin^2\beta\sin(\beta-\delta')\left[1+\sqrt{\dfrac{\sin(\phi'+\delta')+\sin(\phi'-\alpha)}{\sin(\beta-\delta')+\sin(\alpha+\beta)}}\right]^2}$$

- The friction between the soil and the retaining structure is usually set to: $1/2\phi' < \delta' < 2/3\phi'$

- Coulomb's theory is more ***general than Rankine's*** as it can be applied to different shapes of retaining walls with no generalized assumptions; however in some circumstances it ***underestimates active earth pressure*** and ***overestimates passive resistance***.

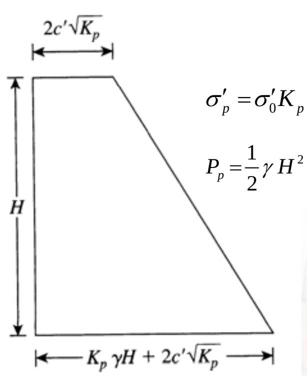
Rankine passive earth pressure

- ***frictionless vertical wall*** **of height, H, that exhibits movement into the soil.**
- **$\sigma_h = \sigma_1$ at failure**
- **Failure planes will develop in the soil wedge as shown in the figure.**



$$\sigma'_p = \sigma'_0 K_p + 2c'\sqrt{K_p}$$

Rankine passive earth pressure



$$\sigma'_p = \sigma'_0 K_p + 2c'\sqrt{K_p}$$

$$P_p = \frac{1}{2}\gamma H^2 K_p + 2c' H\sqrt{K_p}$$

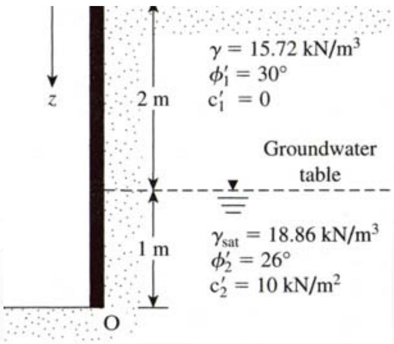
- **For an inclined backfill at an angle of α, the Rankine passive pressure is inclined at an angel of α, and the Rankine passive coefficient**

$$K_p = \cos\alpha \frac{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi'}}{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi'}}$$

- **Note that a sufficient amount of lateral displacement must occur for passive conditions to develop, the amount is more than that for active conditions and may not occur in field applications.**

Example

A 3-m high wall is shown below. Determine the Rankine passive force per unit length of the wall

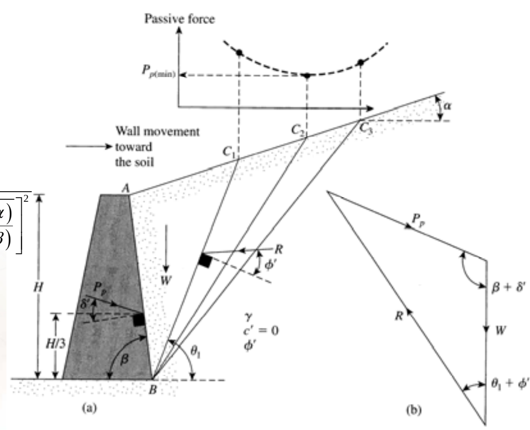


Coulomb's passive earth pressure

- passive earth pressure of an ***inclined backfill***, ***inclined retaining structure*** and ***wall friction***

$$K_a = \frac{\sin^2(\beta - \phi')}{\sin^2\beta \sin(\beta + \delta')\left[1 - \sqrt{\frac{\sin(\phi' + \delta') + \sin(\phi' + \alpha)}{\sin(\beta + \delta') + \sin(\alpha + \beta)}}\right]^2}$$

$$\tfrac{1}{2}\phi' < \delta' < \tfrac{2}{3}\phi'$$




Thank You
for
Your Attention