Solutions

Student # ____________________ MAT1320 C Test II

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Total

Question 1. [2 point] Find the derivative of $f(x) = x^{\sin x}$.

$f(x) = e^{\sin x \ln x}$

$f'(x) = e^{\sin x \ln x}\left(\cos x \ln x + \frac{\sin x}{x}\right)$

$= x^{\sin x}\left(\cos x \ln x + \frac{\sin x}{x}\right)$

OR

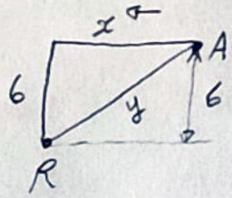
$\ln f(x) = \sin x \ln x$

$\frac{f'(x)}{f(x)} = \cos x \ln x + \frac{\sin x}{x}$

$f'(x) = x^{\sin x}\left(\cos x \ln x + \frac{\sin x}{x}\right)$

Answer: ____________

Question 2. [2 point] An airplane is flying on a horizontal flight path at a constant height of 6 km that will take it directly over a radar tracking station. If the distance of the airplane from the radar station is decreasing at a rate of 400 km per hour when the distance is 10 km, what is the speed of the plane?



$x^2 + 6^2 = y^2 \Rightarrow x = \sqrt{y^2 - 36}\Big|_{y=10} = \sqrt{100-36} = 8$

$\frac{d}{dt}(x^2 + 6^2) = \frac{d}{dt}(y^2) \Rightarrow 2xx' = 2yy'$

$x' = \frac{yy'}{x} = \frac{400(-10)}{8} = -500 \ km/h$

Answer: ____________

Question 3. [1 point] What is $\frac{d}{dx}\int_{\pi}^{x^2} e^{\tan t}\, dt$?

$e^{\tan x^2} \cdot 2x$

Answer: ____________

Question 4. [3 points] Evaluate the following integrals:

a) $\int_1^e \frac{\pi \sin(\ln x)}{x}\,dx = \int_{v=0}^{v=1} \pi \sin v \, dv = -\pi \cos v \Big|_0^1$

$v = \ln x$

$dv = \frac{1}{x}\,dx$

$= -\pi(\cos 1 - \cos 0)$

$= -\pi(\cos 1 - 1) = \pi(1 - \cos 1)$

b) $\int e^x \sin x \, dx = \int w\,dv = wv - \int v\,dw = e^x \sin x - \int e^x \cos x \, dx =$

By parts $\begin{cases} dv = e^x dx \\ w = \sin x \end{cases} \Rightarrow \begin{cases} v = e^x \\ dw = \cos x\, dx \end{cases}$ $\quad e^x \sin x - e^x \cos x - \int e^x \sin x \, dx$

2nd by parts $\begin{cases} dA = e^x dx \\ B = \cos x \end{cases} \Rightarrow \begin{cases} A = e^x \\ dB = -\sin x\, dx \end{cases}$ $\quad \therefore \int e^x \sin x \, dx = \frac{1}{2} e^x (\sin x - \cos x)$

c) $\int_0^{\pi/2} \tan^3 x \sec^3 x \, dx = \int_0^{\frac{\pi}{2}} \tan^2 x \sec^2 x (\sec x \tan x)\,dx =$

$\int_0^{\pi/2} (\sec^2 x - 1) \sec^2 x (\sec x \tan x)\,dx = \int_0^{\pi/2} \sec^4 x - \sec^2 x \;\; \sec x \tan x \, dx$

Subst. $\begin{cases} v = \sec x \\ dv = \sec x \tan x \, dx \end{cases}$ $\quad = \int_{v=1}^{v=\infty} v^4 - v^2 \, dv$ This is an improper integral that has not been discussed yet.

Question 5. [2 point] Find $\frac{dy}{dx}$ given that $y^3 + y^2 - 5y - x^2 = -4$.

$$\frac{d}{dx}\left(y^3+y^2-5y-x^2\right)=\frac{d}{dx}(-4)$$

$$3y^2y'+2yy'-5y'-2x=0$$

$$y'(3y^2+2y-5)=2x$$

$$y'=\frac{2x}{3y^2+2y-5}$$

Question 6. [2 points] Find the linear approximation of $f(x) = \frac{1}{\sqrt{1+x}}$ centered at $x = 0$ and use it to approximate $\frac{1}{\sqrt{2}}$.

$$f'(x)=\frac{d}{dx}(1+x)^{-1/2}=\frac{-1}{2}(1+x)^{-3/2} \qquad f'(0)=\frac{-1}{2} \qquad f(0)=1$$

Linear approximation: $L(x)=f(0)+f'(0)(x-0)$

$$L(x)=1-\frac{1}{2}x$$

$$\frac{1}{\sqrt{2}}=\frac{1}{\sqrt{1+1}}=f(1)\approx L(1)=1-\frac{1}{2}=\frac{1}{2}$$

Question 7. [2 points] Use L_4 (left-hand sum) to estimate the area under the curve $f(x) = \dfrac{x}{1+x}$ from $x = 0$ to $x = 1$.

$0 \quad \frac{1}{4} \quad \frac{2}{4} \quad \frac{3}{4} \quad 1$

$$L_4 = f(0)\frac{1}{4} + f(\tfrac{1}{4})\frac{1}{4} + f(\tfrac{2}{4})\frac{1}{4} + f(\tfrac{3}{4})\frac{1}{4}$$

$$= \frac{1}{4}\left(0 + \frac{1/4}{1+1/4} + \frac{2/4}{1+2/4} + \frac{3/4}{1+3/4}\right)$$

$$= \frac{1}{4}\left(\frac{1/4}{5/4} + \frac{2/4}{6/4} + \frac{3/4}{7/4}\right) = \frac{1}{4}\left(\frac{1}{5} + \frac{1}{3} + \frac{3}{7}\right) \approx 0.24$$

Question 8. [1 points] What is the area under the curve $f(x) = \dfrac{x}{1+x}$ from $x = 0$ to $x = 1$.

$$\int_0^1 \frac{x}{1+x}\,dx = \int_1^2 \frac{v-1}{v}\,dv = \int_1^2 1 - \frac{1}{v}\,dv = v - \ln v \Big|_1^2$$

$$\begin{cases} v = 1+x \\ dv = dx \end{cases}$$

$$= (2 - \ln 2) - (1 - \overset{0}{\cancel{\ln 1}})$$

$$= 1 - \ln 2$$