Test #5 (____/30) **Name (print):** ___________________ **Student Number:** ___________________

Part A: Fill In The Blank/Multiple Choice (1 Mark Each)

Question 1: Consider a function f with a critical point at x= 2 with the following: f(2) = -1, f'(2) = 0, f''(2) = -3. Then this point is a:

a) Local maximum b) Local Minimum c) inflection point d) cannot be determined

Question 2: How can you recognize when a function's equation produces a hole?

When the function has a limit that is $\frac{0}{0}$ (thus not in the domain of the function), but when you solve the limit, it works out to a number (not infinity, or undefined).

Part B: Long Answer (Show all work)

Question 1: Solve the following limits:

a) $\lim_{x\to 0^+} x^2 lnx$ [3 Marks]

$$= \lim_{x\to 0^+} \frac{lnx}{x^{-2}} \quad (\infty)/(\infty) \quad \therefore l'hops$$

$$= \lim_{x\to 0^+} \frac{x^{-1}}{-2x^{-3}}$$

$$= \lim_{x\to 0^+} \frac{x^2}{-2}$$

$$= 0$$

b) $\lim_{x\to 0} \frac{x-\sin x}{3x^3}$ [3 Marks]

$$= \lim_{x\to 0} \frac{x-\sin x}{3x^3} \quad 0/0 \quad \therefore l'hops$$

$$= \lim_{x\to 0} \frac{1-\cos x}{9x^2} \quad 0/0 \quad \therefore l'hops$$

$$= \lim_{x\to 0} \frac{\sin x}{18x} \quad 0/0 \quad \therefore l'hops$$

$$= \lim_{x\to 0} \frac{\cos x}{18} = \frac{1}{18}$$

Question 2: Consider the function $(x) = \frac{lnx}{x-2}$:

a) Find all asymptotes that would appear on this graph: [5 Marks]

When considering vertical asymptotes, we spot that there is a division of zero at x = 2 [0.5 Marks]. We notice that this does not make a limit of 0/0 or ∞/∞ when we sub in x = 2, thus it is a vertical asymptote. [0.5 Marks]

We also spot that there is a ln function, which has a vertical asymptote at x = 0^+ (right hand side of zero) [0.5 Marks] Again, since this does not make 0/0 or ∞/∞, it remains a vertical asymptote at x=0. [0.5 Marks]

To find the horizontal asymptote, we check limits going to infinity, since the domain only includes positive numbers due to the ln function, we only need to consider the + infinity side: [1 Mark]

$$\lim_{x\to\infty} \frac{lnx}{x-2} = \infty/\infty \quad \therefore l'hops$$

$$= \lim_{x\to\infty} \frac{x^{-1}}{1} \quad \text{[1 Mark]}$$

$$= 0$$

Thus the horizontal asymptote is at $y = 0$. [1 Mark]

b) Find all intercepts (and if they exist) for this function: [2 Marks]

We notice that there is no y-intercept as we cannot allow x = 0 (it is not in the domain) [1 Mark]

To find the x-intercept, we require the numerator to be zero in this case, so the x-intercept is (1,0) [1 Mark]

Question 3: Consider a function $g(x) = \frac{(x^2-1)}{2x^2-8}$ that has the following properties.

$D = (-\infty, -2) \cup (-2,2) \cup (2, \infty)$

Intercepts: $(0,0.125), (-1, 0)$, and $(1, 0)$

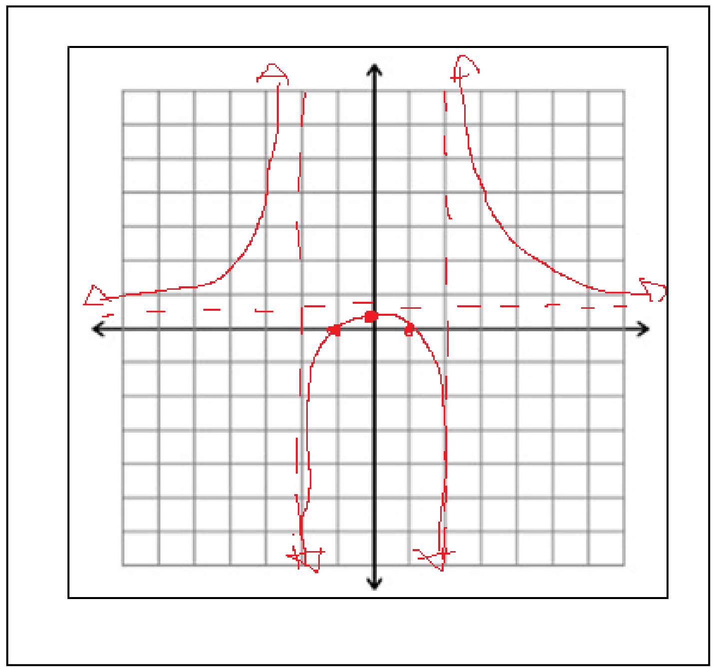
Critical point at $x = 0$

Increasing: $(-\infty, -2) \cup (-2,0)$; decreasing: $(0,2) \cup (2, \infty)$

concave up $(-\infty, -2) \cup (2, \infty)$; concave down $(-2,2)$

Vertical asymptotes: x=2 and x=-2; horizontal asymptotes: y = 0.5

Provide a sketch of the function. [6 Marks]



[1 Mark for each correct property]

Question 4: Given that $F(x)$ is the antiderivative of $f(x) = x + 4e^{2x} - \sin(2x)$ and $F(0) = 3$, find the antiderivative function $F(x)$: [4 Marks]

$F(x) = \int f(x)\,dx$ [1 Mark]

$= \int x + 4e^{2x} - \sin(2x)$

$= \frac{x^2}{2} + 2e^{2x} + \frac{\cos(2x)}{2} + c$ [1 Mark]

We now substitute in (0,3) to find our unknown constant:

$3 = \frac{0^2}{2} + 2e^{2(0)} + \frac{\cos(2(0))}{2} + c$

$3 = 2 + 0.5 + c$

$0.5 = c$ [1 Mark]

Thus $F(x) = \frac{x^2}{2} + 2e^{2x} - \frac{\cos(2x)}{2} + 0.5$ [1 Mark]

Question 5: Given the functions $f(x) = 3x^2 - x + 2$ and $g(x) = -x + 14$. Then

a) What are x-values of the points of intersection for the two functions? [2 Marks]

$f(x) = g(x)$

$3x^2 - x + 2 = -x + 14$ [1 Mark]

$3x^2 = 12$

$x^2 = 4$

$x = \pm 2$ [1 Mark]

b) Determine the area contained between the two functions. [3 Marks]

The intersection between two points means we must evaluate

$| \int_a^b f(x) - g(x) dx |$ [1 Mark]

$= | \int_{-2}^{2} 3x^2 - x + 2 - (-x + 14) dx |$

$= | \int_{-2}^{2} 3x^2 - 14 dx |$

$= |x^3 - 14x|_{-2}^{2}$ [1 Mark]

$= |8 - 24 - (-8 + 24)|$

$= 32$ [1 Mark]

Therefore the area in between the two curves is 32 $units^2$.