MAT2377C - Assignment 2

Solutions

Q1. (5 points)

Suppose that the random variable X has the following cumulative distribution function CDF:

$$F_X(x) = \begin{cases} 0, & x \leq 0 \\ x^3, & 0 \leq x \leq 1 \\ 1, & x \geq 1. \end{cases}$$

(a) Compute $P(X > 0.5)$ and $P(0.2 < X < 0.8)$.

(b) Find the probability density function of X.

(c) Find $\mathrm{E}[X]$ and $\mathrm{Var}[X]$.

Solution to Q1:

(a) $P(X > 0.5) = 1 - F_X(0.5) = 1 - 0.5^3 = 0.875$,
$P(0.2 < X < 0.8) = F_X(0.8) - F_X(0.2) = (0.8)^3 - (0.2)^3 = 0.504$.

(b)

$$f_X(x) = F_X'(x) = 3\,x^2,\ 0 < x < 1.$$

(c)

$$\mathrm{E}[X] = \int_{-\infty}^{\infty} x\, f_X(x)\, dx = \int_0^1 3\,x^3\, dx = \frac{3}{4}x^4\Big|_0^1 = \frac{3}{4}$$

and

$$\mathrm{E}[X^2] = \int_{-\infty}^{\infty} x^2\, f_X(x)\, dx = \int_0^1 3\,x^4\, dx = \frac{3}{5}x^5\Big|_0^1 = \frac{3}{5}.$$

Thus, $\mathrm{Var}[X] = \mathrm{E}[X^2] - \mu_X^2 = (3/5) - (3/4)^2 = 3/80 = 0.0375$.

Marking scheme for Q1:

1 point for each correct answer in part (a), 1 point for part (b), 1 point for each correct answer in part (c). Total - 5 points.

Q2. Assume that Z is a standard normal random variable. Calculate:

(a) $P(Z > -2)$;

(b) $P(1.7 < Z < 2.8)$;

(c) $P(-1 < Z < 1.5)$;

Solution to Q2:

(a) $P(Z > -2) = 1 - P(Z < -2) = 1 - 0.0228 = 0.9772$;

(b) $P(1.7 < Z < 2.8) = P(Z < 2.8) - P(Z < 1.7) = 0.9980 - 0.9554 = 0.0426$;

(c) $P(-1 < Z < 1.5) = P(Z < 1.5) - P(Z < -1) = 0.9332 - 0.1587 = 0.7745$;

Marking scheme for Q2:

This question will not be marked