Final Exam Review Solutions

1. B, C, and F are false.

2. n =100. $C = 0.95 \Rightarrow Z_{(1-C)/2} = 1.96$.
The margin of error is (61.83074 – 61.1) = 0.73074 = $1.96(\sqrt{13.9/n})$.

3. $C = 0.85 \Rightarrow Z_{(1-C)/2} = Z_{0.075} = 1.44$ for a 85% interval.
Interval is $61.1 \pm 1.44\sqrt{13.9/100}$ = (60.5631 , 61.6369)

4. Let X = monthly rent for a UoG student
$H_0 : \mu = 400$ vs. $H_a: \mu \neq 400$.
$t = (\bar{x} - 400)/(s/\sqrt{64}) \sim t_{63}$ and $t = (360 - 400)/(75/8) = -4.267$
p-value = $2P(T > |t|) < 2(.0005) < .001$ (need to look at t_{60} in Table D).

5. We can reject the null hypothesis at the 5% level of significance, and conclude that there is strong evidence that the mean rent for all off campus University of Guelph students is not 400. Actually, we can even say that there is evidence that the mean off-campus rent is *less* than 400 (since the test statistic is negative). For the exam I'll explicitly give you a significance level for the most part. Often a 5% level of significance is used, but keep in mind that there is nothing magical about the 5% number.

6. Data is the difference in reaction times:

Student	**Before**	**After**	**Diff**
1	.41	.78	-.37
2	.38	.63	-.25
3	.52	.94	-.42
4	.41	.47	-.06

Sample mean diff. is -0.275, std. deviation 0.1601.
$H_0 : \mu = 0$ vs. $H_a: \mu < 0$.
$t = \bar{x}/(s/\sqrt{4}) \sim t_3$ and $t = -.275/(.1601/2) = -3.435$
p-value = $P(T < t) = P(T > |t|)$ by symmetry

$0.02 < p\text{-value} < 0.025$ (need to look at t_3 in Table D).

7. $0.02 < p\text{-value} < 0.025$

8. Reject Ho at the 5% significance level. There is strong evidence that alcohol tends to slow reaction times.

9. Like always, we need a random sample. We also need the population of *differences* to be normally distributed.

10. $t = (2.1–1.0)/(1.1/\sqrt{10}) = 3.16$. df = 10–1 = 9. $.01 < p\text{-value} < .02$.

11. D.

12. C.

13. D.

14. E. None of the statements are true.

15. C. III is the only true statement.

16. B. This is a one-sided, lower tail test, with test statistic, $t_0 = \frac{4100-5000}{9000/\sqrt{200}} = -1.41$. The *p*-value is calculated assuming the null hypothesis and given by $P(T < –1.41) = P(T > 1.41)$. The test statistic has 199 degrees of freedom so we need to consult the df=100 row of the *t*-table, which tells us that $0.05 < p < 0.10$. So, 0.08 is the best approximation.

17. A. and D. are true.

18. D. Here, the researcher's hypothesis is a null hypothesis rather than an alternative hypothesis because it states that $\mu = 10$. Statistical evidence can't prove the null hypothesis is true, but rather we decide if there is sufficient evidence to falsify it. With 95% confidence we believe the true value of μ is somewhere in the confidence interval. Since the C.I. includes $\mu = 10$, the null hypothesis can't be rejected, so I is true. But insufficient evidence to reject the null hypothesis doesn't prove the null hypothesis is true, so II is false. Indeed, we can't reject a null hypothesis for any value of p which is included in the C.I. For example, values of $\mu = 12$ and $\mu = 13$ are also included in the C.I., so III is also true.

19. A. For those that know hypothesis testing basics this will be obvious and the rest of the alternatives are simply there to confuse the confused!

20. C. The universal decision rule with a p-value is to reject the null hypothesis if the p-value $\leq \alpha$.

21. C. For proportions, we use the Normal approximation, especially for large sample sizes. Take a look at the standard deviation for a binomial proportion. It depends on the sample size *n*, but the size of the population does not enter into the equation – the populations of both countries are so large that we can treat them as "infinite" populations.

22. D. The quick solution is to recognize that since the margin of error is proportional to $1/\sqrt{n}$, reducing the margin to ¼ of its previous size requires increasing the sample size by $4^2 = 16$ times.

More formally, we can set $m_2 = m_1/4$, where $m_1 = z_{\alpha/2}\, \sigma/\sqrt{n_1}$ is the original margin of error, and $m_2 = z_{\alpha/2}\, \sigma/\sqrt{n_2}$ is the smaller margin. Taking ratios and cancelling constant terms, we get

$$\frac{m_1}{m_2} = 4 = \frac{\frac{1}{\sqrt{n_1}}}{\frac{1}{\sqrt{n_2}}} = \sqrt{\frac{n_2}{n_1}}$$

which, by solving for n_2, tells us $n_2 = 16n_1$.

23. C. I is false; almost certainly less than 95% of possible sample means will fall in the interval. II is true, because the value 55 is not included in the C.I. III simply must be true – how could it ever be false?

24. C. The value of the test statistic is extreme enough to lead to *p*-values small enough to be significant at the 5% level of significance, but it may or may not be extreme enough to be significant at the 1% level of significance.

25. B. A binomial r. v. *X* can take on values x = 0, 1, ..., *n*, so it can't be negative. Normal can be defined with any mean, and *t* random variables can take on any value, more or less between –4 and +4, so these can be negative. A uniform random variable can be defined over any interval, so it could be negative too.

26. C. The lower tail area for $t_{16} = -2.183$ is between 0.025 and 0.01, since $t_{16} = 2.183$ falls between $t_{16;\,0.025} = 2.120$ and $t_{16;\,0.02} = 2.235$. Because this is a two-tailed test we double the lower tail area to get the p-value, so the p-value falls between 2(0.02) = 0.04 and 2(0.025) = 0.05. The only interval that covers these values is 0.02 < *p*-value < 0.05.

27. D. Remember that 95% confidence intervals are constructed in such a way that in repeated sampling, we expect 95% of the 95% confidence intervals constructed will capture the population parameter, and 5% will not. This holds even though we don't know the value of the population parameter.

28. B. Under the alternative hypothesis, the normal rats have a lower mean blood viscosity, which would lead to negative differences.

29. E. This is a paired data experiment, so we want $t_{n-1;\,\alpha/2} = t_{7;\,0.025} = 2.365$

30. B. If affected eyes have thicker corneas on average, and differences are taken as affected eye minus unaffected eye, the mean difference will be positive so the alternative hypothesis is Ha: $\mu_d > 0$. The null hypothesis would be H_0: $\mu_d = 0$.

31. D. This is a paired experiment, with predicted and actual grades being recorded on each student. The test is one-tailed, as the experimenter is predicting grades will, on average, be overestimated.

32**.** C. It doesn't really matter whether you call this a one-sample *t* confidence interval or a paired *t* confidence interval, since the paired *t* procedure is simply a one-sample *t* procedure after you take the differences. Here the differences are already calculated. Using the notation for the paired *t* procedure and using the statistics function of your calculator to get the sample mean and standard deviation efficiently, we get the summary statistics: $\mu_d = 16.5$, $s_d = 5.066$. So, $m = t_{3,.025}\left(\frac{s}{\sqrt{n}}\right)$.

33. D. The margin of error is $Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) = 0.25$. Substituting σ = 1.61 and *n* = 225, solve for $Z_{\alpha/2} = 2.33$. This is a *Z* value with α/2 = 0.01, thus α = 0.02 and the confidence coefficient is 98%.

34. B. $P(\bar{X} < 0) = P(Z < (0-\mu)/(\sigma/\sqrt{n})) = P(Z < (0-4)/(15/3)) = P(Z < -0.80) = 0.2119$.

35. B. The margin of error is $m = Z_{\alpha/2}(\sigma/\sqrt{n})$, so the width of the C.I. is simply 2m. For a 95% C.I. for μ, m = .042/2 = 0.021. Further, $Z_{\alpha/2}$ = 1.96 so the standard deviation $(\sigma/\sqrt{n})$ is simply $m/Z_{\alpha/2}$ = 0.021/1.96 = 0.0107. Finally, since $Z_{\alpha/2}$ = 1.645 for the 90% C.I., the width of the 90% C.I. for μ is: 2(1.645)(0.0107) = 0.035.

If you are comfortable with your math skills, then simply note the only difference between the two C.I. is the *Z*-value used in the margin of error. Then, set

$$\text{(Length of 90\% C.I)/(Length of 95\% C.I.)} = \frac{2(1.645)\sigma/\sqrt{n}}{2(1.96)\sigma/\sqrt{n}} = 1.645/1.96$$

Finally, solve for: (Length of 90% C.I.) = (1.645/1.96)(Length of 95% C.I.) = (1.645/1.96)(0.042) = 0.035.

36. A. A *t*-value rather than a *z*-value must be used here, because σ is unknown. First use the statistics function on your calculator to efficiently get the summary statistics: $\bar{x}$ = 33.167, s = 13.819. The 95% confidence interval for μ has the limits: $\bar{x} \pm t_{n-1,\,\alpha/2}\, s/\sqrt{n}$ where $t_{n-1,\,\alpha/2} = t_{5,\,0.025}$ = 2.571. Hence, we obtain 33.167 ± $2.571(13.819/\sqrt{6})$ ⟺ 33.167 ± 14.504 ⟺ (18.7, 47.7).

37. E. The 95% confidence interval for μ has the limits: $\bar{x} \pm t_{n-1,\,\alpha/2}\, s/\sqrt{n}$ where $t_{n-1,\,\alpha/2} = t_{7,\,0.025}$ = 2.365. Set: $2.365(s/\sqrt{8})$ = 1.66 and solve for $s = 1.66\sqrt{8}/2.365$ = 1.985.

38. By setting the margin of error to 2: $m = Z_{\frac{\alpha}{2}}\left(\frac{\sigma}{\sqrt{n}}\right) = Z_{.005}\left(\frac{\sigma}{\sqrt{64}}\right) = 2$, we can solve for σ. Since $Z_{0.005}$ = 2.576, solving for the standard deviation we find: $\sigma = 2(8)/2.576$ = 6.2.

39. A. Let σ^2 be the variance of the population. This doesn't change as the sample size n increases. The variance of the sample mean is σ^2/n, which becomes smaller as n increases. Similarly, its estimate, s^2/n, will also on average decrease as n increases. The value of $t_{n-1,\,0.025}$ decreases as n increases (converging to a value of 1.96 as n tends to positive infinity).