

Midterm 2 Review Answers

1. Assuming independence we have: $P(A)P(B) = 0.5(0.2) = 0.1$ and
 $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = 0.5 + 0.2 - 0.1 = 0.6$
But this calculation contradicts: $P(A \text{ or } B) = 0.68$. So, A and B are **not independent**.

2. No, disjoint events are never independent. They are always dependent.

3. First, let's define events and write the information provided in terms of our events.

Let $A = \{\text{first alarm goes off}\}$ with $P(A) = 0.99$, and

$B = \{\text{second alarm goes off}\}$ with $P(B) = 0.98$. A and B are independent.

$$P(A^c \text{ and } B^c) = P(A^c)P(B^c) = (1 - 0.99)(1 - 0.98) = 0.0002. \quad (\text{used multiplication rule})$$

4. (a) X can take on values $\{2, 3, 4, \dots, 12\}$, i.e. any integer between 2 and 12.

(b) Define $A = \{\text{sum is odd}\} = \{X \text{ is } 3, 5, 7, 9, \text{ or } 11\}$. Any given outcome occurs with probability $1/36$, and half the outcomes give rise to an odd number. So $P(A) = 0.5$.

(c) $A = \{X \text{ is odd}\}$ and $B = \{X \geq 9\}$. The checkerboard below highlights these events, where rows correspond to value on die 1, columns correspond to values on die 2, and cells correspond to the sums of the top faces that are possible. Dark grey corresponds to A only, light grey corresponds to B only, black corresponds to $(A \text{ and } B)$, and white corresponds to neither A nor B .

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

From the checkerboard it is evident that $P(A \text{ or } B) = 22/36$, and $P(A \text{ and } B) = 6/36$.

(d) **No.** $P(A)P(B) = (18/36)(10/36) = 180/36^2 = 5/36 \neq P(A \text{ and } B)$.

5. Let $X = \text{number of heads in 5 tosses}$. On a single toss, $P(H) = 0.3446$.

$$A = \{4 \text{ or more heads in 5 tosses}\} = \{X \geq 4\}$$

Out of the 5 tosses, there are 5 possible ways of observing 4 heads:

$$\{HHHHT\}, \{HHHTH\}, \{HHTHH\}, \{HTHHH\}, \{THHHH\}$$

Since tosses are independent, the probabilities of observing each of these outcomes are equal, and given by, $P(HHHHT) = (0.3446)^4(0.6554) = 0.009242$.

There is only one way of observing 5 heads, and this happens with probability

$$P(HHHHH) = (0.3446)^5 = 0.0049.$$

ALTERNATIVELY: recognize that $X \sim \text{Bin}(n = 5, p = 0.3446)$

So,

$$P(A) = P(4 \text{ heads}) + P(5 \text{ heads})$$

$$= 5 \times 0.009242 + (0.3446)^5$$

$$= 0.0462 + 0.0049 = 0.0511$$

6. Let $X =$ student's mark on test 1. $X \sim N(71, 10)$. $n = 5$.
 $A = \{5 \text{ scored higher than } 75\}$

For a single, randomly selected student:

$$P(X > 75) = P(Z > (75 - 71)/10) = P(Z > 0.4) = 1 - 0.6554 = 0.3446$$

Since students' scores are independent, we have:

$$P(A) = P(\text{all 5 students scored higher than } 75) = 0.3446^5 = 0.0049.$$

7. Let $X =$ student's mark on test 1. $X \sim N(71, 10)$. $n = 5$.
 $A = \{\text{at least 4 scored higher than } 75\}$, From Q2, $P(X > 75) = 0.3446$.

From here on in, this question is analogous to independently tossing a biased coin 5 times, in which the probability of getting a head (H) is 0.3446, and we can use the results from Q1.

i.e.,

$$P(A) = P(\text{at least 4 score higher than } 75) \quad \text{where } P(X > 75) = 0.3446$$

$$= P(\text{at least 4 heads in biased coin}) \quad \text{where } P(H) = 0.3446$$

$$= 0.0511 \quad \text{from Q1.}$$

8. (A) Since the sum of the probabilities equals 1.

9. (B) Discrete

$$10. E[X] = 1(.3) + 2(.1) + 3(.3) + 4(.3) = 2.6$$

$$11. \sigma_X^2 = (1 - 2.6)^2(.3) + \dots + (4 - 2.6)^2(.3) = 1.44 \quad \text{so } \sigma_X = \sqrt{1.44} = 1.2$$

$$12. P(X < 4) = P(1) + P(2) + P(3) = 0.3 + 0.1 + 0.3 = 0.7$$

13. (B).

14. (C). i. is false because $\rho = 0$ for independent random variables, and iv. is false because the standard deviation is scaled by a factor of b . It would be b^2 for the variance.

$$15. (A) \text{ since } 1.0 - 0.3 - 0.12 - 0.18 - 0.2 = 0.2$$

16. (B)

$$17. (B). \text{ Since } X \sim N(110, 5), \text{ we have } P(X < 100) = P(Z < -2) = 0.0228.$$

18. (B). $Y \sim N(170, 10)$. From the Normal table, we see that $P(Z < -1.28) = 0.1003$ and $P(Z < -1.29) = 0.0985$. Since we want the z -score associated with $(1 - 0.9) = 0.1$, we will use the closest score, which is $Z = -1.28$. Unstandardizing this value, we get $X = 157.2$ lbs.

$$19. (D) \mu_W = 110 + 170 = 280 \text{ lbs}$$

$$20. (B) \sigma_W = \sqrt{25 + 100} = 11.18 \text{ lbs}$$

21. (C) $\sigma_W = \sqrt{25 + 100 + 2(.77)(5)(10)} = 14.21 \text{ lbs}$

22. (C)

23. No more than 3 means that 0,1,2, or 3 cars fail the emissions test. Using the binomial distribution equation, with a probability of success of 0.05, and $n=25$ trials we can find $P(X=0)$, $P(X=1)$, $P(X=2)$, $P(X=3)$ and add these together to find $P(X \leq 3)$.

$$P(X = 0) = \binom{25}{0} (0.05)^0 (1 - 0.05)^{25-0} = 0.2773896$$

$$P(X = 1) = \binom{25}{1} (0.05)^1 (1 - 0.05)^{25-1} = 0.3649863$$

$$P(X = 2) = \binom{25}{2} (0.05)^2 (1 - 0.05)^{25-2} = 0.2305177$$

$$P(X = 3) = \binom{25}{3} (0.05)^3 (1 - 0.05)^{25-3} = 0.09301589$$

So, we have $P(X \leq 3) = 0.9659094$

24. From the table, $P(Z > 2) = 0.0228$. If we want the probability of two independently sampled numbers being greater than two we can multiply the two probabilities together, i.e. $P(\text{both} > 2) = 0.0228^2 = 0.00051984$.

25. (B) only (i) is true

26. To get $P(X \geq 2)$ we can take $1 - [P(X = 0) + P(X = 1)]$.

$$P(X = 0) = \binom{5}{0} (0.6)^0 (1 - 0.6)^{5-0} = 0.01024$$

$$P(X = 1) = \binom{5}{1} (0.6)^1 (1 - 0.6)^{5-1} = 0.0768$$

$$P(X \geq 2) = 1 - (0.01024 + 0.0768) = 0.91296$$

27. Recall the mean of binomial distribution is $\mu = np$, solving for p we find that $p=0.24$.

$$P(X = 2) = \binom{10}{2} (0.24)^2 (1 - 0.24)^{10-2} = 0.2884986$$

28. (D) The mean of a sampling distribution is equal to the mean of the population.

29. (A) The standard deviation of the sampling distribution is $\frac{\sigma}{\sqrt{n}} = \frac{6}{\sqrt{36}} = 1.0$

30. (C)

31. (B)

32. (B) $0.01 = \frac{1}{\sqrt{n}}$ Solving for n we find $n = \left(\frac{1}{.01}\right)^2 = 100$

33. (B) $Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{8 - 8.1}{.1/\sqrt{10}} = -3.162278$. From table $P(Z \leq -3.16) = .0008$

34. mean: $\mu = np = 100 \left(\frac{1}{200}\right) = 0.5$

variance: $\sigma^2 = np(1 - p) = 100 \left(\frac{1}{200}\right) \left(\frac{199}{200}\right) = 0.4975$

So, the standard deviation is $\sigma = \sqrt{0.4975} = 0.7053368$

35. X follows a binomial distribution with $n = 100$ trials and probability of success $p = \frac{1}{200} = 0.005$.

36. No, $np = (1/200)(100) = 0.5$. The Normal approximation can be used when $np \geq 10$ and $n(1 - p) \geq 10$.
37. (D) $1/2$.
38. (C) The centre will remain the same, but the spread will decrease.
39. (C) The sample proportion from University of Guelph will have about the same sampling variability as that from University of Toronto. Although the population level proportions may differ between the two universities, since the sample sizes are the same, the
40. (B) The sample proportion from University of Guelph will have more sampling variability than that from University of Toronto.