Midterm 2 Review Questions

These questions review material covered since Midterm 1 even though Midterm 2 is cumulative. However, it does not provide an exhaustive coverage of these topics. Not all questions given here are multiple choice.

1. If A and B are two events such that P(A) = 0.5, P(B) = 0.2 and P(A or B) = 0.68, are A and B independent?

2. Two events A and B are disjoint. If P(A) = 0.5 and P(B) = 0.3, are A and B independent?

3. A student is worried about sleeping in and missing her final exam, so she sets two battery powered alarm clocks. The probability the first alarm will go off is .99, and the probability the second alarm will go off is 0.98. If these clocks work independently, what is the probability that neither alarm will go off?

4. Two fair and independent dice are tossed. Let X be a random variable representing the sum of the numbers on the faces.
(a) What is the sample space of X?
(b) What is the probability that X is odd?
(c) Let A be the event that X is odd. Let B be the event that X is greater than or equal to 9. Find $P(A$ or $B)$. Find $P(A$ and $B)$.
(d) Are events A and B from part (c) independent events?

5. Suppose a biased coin, for which the probability of heads is 0.3446 and the probability of tails is 0.6554, is tossed independently 5 times. What is the probability of observing at least 4 heads out of the five tosses?

The next two questions refer to the following information. Suppose that marks on Midterm 1 were approximately normally distributed with a mean of 71, and variance 100. The scores of 5 students are randomly selected.

6. What is the probability that all 5 students scored higher than 75?
[**Hint:** first find the probability that a single student scores above 75, and then exploit that the 5 students selected are independent].

7. What is the probability that at least 4 students scored higher than 75?

The next 5 questions refer to the following information. The probability distribution of random variable, X, is defined as follows:

X	0	1	2	3	4
Probability	0	0.3	0.1	0.3	0.3

8. Is the above a valid probability model?

(A) Yes.
(B) No.
(C) Not information is provided to answer the question.

9. The random variable X is _____.

(A) Continuous (B) Discrete (C) Both discrete and continuous (D) None of the above

10. Find the expected value of X.
11. Find the standard deviation of X.
12. Find $P(X < 4)$.

13. Consider the following three scenarios and determine if the random variable described in each is either discrete or continuous.

I. The increase in length of life of a cancer patient following chemotherapy.
II. The volume of gasoline lost due to evaporation during the filling of a gas tank.
III. The number of cracks that exceed 1.5 centimeters in 10 kilometers of a major highway.

The random variables in scenarios I, II, and III, respectively, are:

(A) continuous, discrete, discrete.
(B) continuous, continuous, discrete.
(C) continuous, continuous, continuous.
(D) discrete, continuous, discrete.
(E) discrete, discrete, continuous.

14. How many of the following statements are TRUE?

i. The correlation between two independent random variables is 0.5.
ii. The variance of a constant number, a, is zero.
iii. Adding a constant a to a random variable will shift the mean of the random variable by a value of a.
iv. Multiplying a random variable by a factor b will scale the variance of the random variable by a factor of b.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

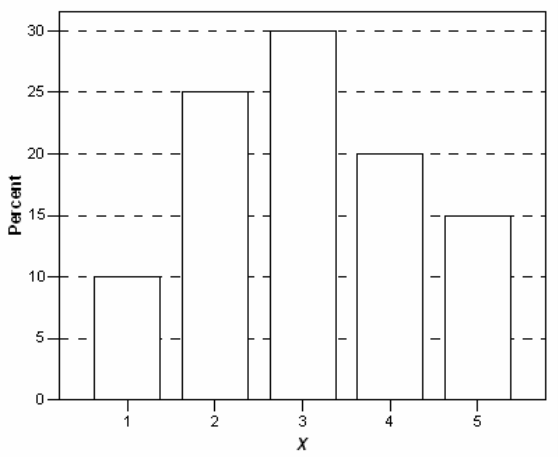
15. Let the random variable X be the number of repair calls that an appliance repair shop may receive during an hour. The distribution of X is given below:

Value of X	0	1	2	3	4
Probability	?	0.3	0.12	0.18	0.2

What is the value of the missing probability, associated with the question mark?

(A) 0.2
(B) 0.25
(C) 0.3
(D) 0.35
(E) None of the above.

16. Consider the following probability histogram for a discrete random variable X:



This probability histogram corresponds to which of the following distributions for X?

(A)

Value of X	1	2	3	4	5
Probability	0.06	0.25	0.38	0.25	0.06

(B)

Value of X	1	2	3	4	5
Probability	0.10	0.25	0.30	0.20	0.15

(C)

Value of X	1	2	3	4	5
Probability	0.10	0.25	0.30	0.25	0.10

(D) None of the above.

The next 5 questions refer to the following information. When figure skaters need to find a partner for "pair figure skating," it is important to find a partner who is compatible in weight. The weight of figure skaters can be modeled by a normal distribution. For male skaters, the mean is 170 lbs with a standard deviation of 10 lbs. For female skaters, the mean is 110 lbs with a standard deviation of 5 lbs. Let the random variable X = the weight of female skaters and the random variable Y = the weight of male skaters.

17. What is $P(X< 100)$?

(A) 0.0143
(B) 0.0228
(C) 0.0494
(D) 0.1587
(E) 0.9772

18. Approximately 90% of the male skaters weigh more than how many pounds?

(A) 142.1 lbs
(B) 157.2 lbs
(C) 163.3 lbs
(D) 176.4 lbs
(E) 183.5 lbs

19. The total weight of a pair of figure skaters (a male and a female) can be thought of as a new random variable. Let the random variable $W = X + Y$. What is the mean of this new random variable W?

(A) μ_W = 110 lbs
(B) μ_W = 140 lbs
(C) μ_W = 170 lbs
(D) μ_W = 280 lbs
(E) μ_W = 300 lbs

20. Suppose we consider the weights of the male partner and the female partner to be independent. What is the standard deviation of the random variable $W = X + Y$?

(A) σ_W = 3.87 lbs
(B) σ_W = 11.18 lbs
(C) σ_W = 14.21 lbs
(D) σ_W = 15.00 lbs
(E) σ_W = 16.73 lbs

21. Suppose the correlation between X and Y is $\rho = 0.77$. What is the standard deviation of the random variable $W = X + Y$?

(A) σ_W = 6.93 lbs
(B) σ_W = 11.18 lbs
(C) σ_W = 14.21 lbs
(D) σ_W = 15.77 lbs
(E) σ_W = 16.93 lbs

22. Which one of the following statements is TRUE?

(A) If X is a binomial random variable with $n = 40$ and $p = .10$, then $P(X = 3) > P(X = 4)$.
(B) The sample mean is a biased estimator of the population mean when n is small.
(C) The normal distribution can be strongly skewed, but only for small sample sizes.
(D) For skewed distributions, the mean is always smaller than the standard deviation.
(E) None of the above.

23. Suppose that 5% of cars that take the emissions test fail. In a sample of 25 of these cars, what is the probability that no more than 3 fail?

24. If two values are sampled independently from the standard normal distribution, what is the probability that both of the values are larger than 2?

25. Which, if any, of the following statements is/are TRUE?

i. The sampling distribution of the sample mean is exactly normally distributed if the population that is sampled from is exactly normally distributed.

ii. The standard deviation of the sampling distribution of the sample mean increases as the sample size increases.

iii. The Central Limit Theorem states that the sample mean is an unbiased estimator of the population mean.

(A) 0 (B) 1 (C) 2 (D) 3

26. If 60% of Canadian adults voted in the last federal election, what is the probability that in a random sample of 5 Canadian adults, two or more voted in the last federal election?

27. If a binomial random variable X has a mean of 2.4, and $n = 10$, what is the $P(X = 2)$?

The next three questions refer to the following information. The scores of individual students on the American College Testing (ACT) Program Composite College Entrance Examination follow a normal distribution with mean 18.6 and standard deviation 6.0. At Northside High, 36 seniors take the test. Assume the scores at this school have the same distribution as national scores.

28. What is the mean of the sampling distribution for the sample mean score based on a random sample of 36 students?

(A) 1.0 (B) 3.1 (C) 6.0 (D) 18.6

29. What is the standard deviation of the sampling distribution for the sample mean score based on a random sample of 36 students?

(A) 1.0 (B) 3.1 (C) 6.0 (D) 18.6

30. What is the sampling distribution of the sample mean score based on a random sample of 36 students?

(A) Approximately normal, but the approximation is poor.
(B) Approximately normal, and the approximation is good.
(C) Exactly normal.
(D) Neither normal nor non-normal. It depends on the particular 36 students selected.
(E) We cannot say anything about the sampling distribution based on the information provided.

The next four questions refer to the following information. Chocolate bars produced by a certain machine are labeled 8.0 oz. The distribution of the actual weights of these chocolate bars is claimed to be normal with a mean of 8.1 oz and a standard deviation of 0.1 oz. The quality control manager plans to take a simple random sample of *n* bars from the production line.

31. If he were to double his sample size (to 2*n*), by what factor would the standard deviation of the sampling distribution of $\bar{X}$ change?

(A) 1/2 (B) $1/\sqrt{2}$ (C) $\sqrt{2}$ (D) 2

32. How big should *n* be so that the sampling distribution of $\bar{X}$ has standard deviation 0.01 oz?

(A) 10 (B) 100 (C) 1000
(D) Cannot be determined unless we know the population follows a Normal distribution.

33. If *n* = 10, then what is the probability that the sample mean weight of the 10 sampled chocolate bars will be less than 8.0 oz?

(A) 0.0
(B) 0.00078
(C) 0.0316
(D) 0.1587

The next three questions refer to the following information. Chromosome defect A occurs in only one out of 200 adult males. A random sample of 100 adult males is selected. Let the random variable *X* represent the number of males in the sample who have this chromosome defect.

34. What are the mean and standard deviation of the random variable *X*?

35. What is the exact distribution of the random variable *X*?

36. Can we use the normal approximation to answer probability questions about the random variable *X*? Briefly explain why or why not.

The next two questions refer to the following information. Suppose you are going to roll a fair six-sided die 60 times and record $\hat{p}$, the proportion of times that an even number (2, 4, or 6) is showing on the top face.

37. On what value should the sampling distribution of $\hat{p}$ be centred?

(A) 1/6
(B) 1/4
(C) 1/3
(D) 1/2
(E) 30

38. Suppose you decide to roll the die 200 times instead of 60 times. How will this affect the centre and spread of the sampling distribution of $\hat{p}$?

(A) Both the centre and the spread will remain the same.
(B) The centre will remain the same, but the spread will increase.
(C) The centre will remain the same, but the spread will decrease.
(D) Both the centre and the spread will decrease.
(E) We cannot tell how the centre and spread of the sampling distribution will change.

The next two questions refer to the following information. The number of graduate students at the University of Guelph is approximately 2500, while the number at the University of Toronto is approximately 16,500. Researchers are interested in estimating the proportion of students who feel that drinking is a problem among university students. Suppose the true proportion at the University of Guelph is the same as that at the University of Toronto. A simple random sample of graduate students is drawn from each university.

39. Suppose that a simple random sample of size 50 will be drawn from each university. What can we conclude about the sampling variability in the sample proportion, $\hat{p}$, calculated from the sample at the University of Guelph compared to that calculated from the sample at the University of Toronto?

(A) The sample proportion from University of Guelph will have less sampling variability than that from University of Toronto.
(B) The sample proportion from University of Guelph will have more sampling variability than that from University of Toronto.
(C) The sample proportion from University of Guelph will have about the same sampling variability as that from University of Toronto.
(D) It is impossible to make any statements about the sampling variability of the two samples because the students surveyed were different.
(E) We cannot tell how the centre and spread of the sampling distribution will change.

40. Suppose, instead, that a simple random sample of about 3% of each institution's graduate student population will be drawn, respectively. What can we conclude about the sampling variability in the sample proportion, $\hat{p}$, calculated from the sample at the University of Guelph compared to that calculated from the sample at the University of Toronto?

(A) The sample proportion from University of Guelph will have less sampling variability than that from University of Toronto.
(B) The sample proportion from University of Guelph will have more sampling variability than that from University of Toronto.
(C) The sample proportion from University of Guelph will have about the same sampling variability as that from University of Toronto.
(D) It is impossible to make any statements about the sampling variability of the two samples because the students surveyed were different.
(E) We cannot tell how the centre and spread of the sampling distribution will change.